University of Oum El Bouaghi
12/05/2026 $11:00 \Rightarrow 12:30$

Applied Sciences and Technolohgies Institute
Departement : Netwoks and Telecoms - Applied Industry
Exam : Algebra 2

Exercise 1

Check whether the set $V = \{(x;y) | x, y \in \mathbb{R}\}$ with the vector addition (internal law) defined by : $(x_1;y_1) + (x_2;y_2) = (x_1y_2; x_2y_1)$; and the scalar multiplication (external law) defined by : $k(x;y) = (kx;ky) \setminus (k \in \mathbb{R})$ is an $\mathbb{R}$ vector space.

Exercise 2

Let S be the following linear system :

$$(S)\begin{cases} -x+z=1 \\ y-2z=2 \\ x+z=1 \end{cases} ; x,y,z \in \mathbb{R}$$

1. Give the equivalent matrix system to the system S.
2. Resolve this system if possible by calculating A^{-1}.

Exercise 3

Let be the matix $A \in \mathbb{M}_3(\mathbb{R})$:

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 4 & 2 \\ -1 & 0 & 3 \end{pmatrix}$$

1. Determine $P_A(\lambda)$ the caracteristique polynom of the martix A.
2. Determine the eigen values and vectors of the matrix A.
3. Is the matrix A diagonalisable? if yes, determine the matrix P , D and P^{-1}.

Typical correction of Algebra 2 exam

1st Year RT+MIP

Exo 1:

$$\begin{cases} V = \{(x;y) \mid x;y \in \mathbb{R}\}. \\ +: (x_1;y_1)+(x_2;y_2) = (x_1y_2\,;\,x_2y_1) \\ \cdot: K(x;y) = (Kx;Ky). \end{cases}$$

This is not a vector space over $\mathbb{R}$. (1)

because:

The commutativity of the internal law is not verified.

$$v_2+v_1 = (x_2;y_2)+(x_1;y_1) = (x_2y_1\,;\,x_1y_2) \neq (x_1y_2\,;\,x_2y_1) \quad (2)$$

$$\Rightarrow v_2+v_1 \neq v_1+v_2.$$

Exo 2:

1) Matrix form of the linear system (S) (1,5)

$$AX = b \iff \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

2) Resolution of the system:

$$\det(A) = -1\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1-1 \quad (1,5)$$

$\det(A) = -2 \neq 0$; (S) is resolvable (admits one only solution)

$X = A^{-1}b$ (0,5)

$$A^{-1} = \frac{1}{\det(A)}\, {}^{t}\mathrm{com}(A) \quad (0,5)$$

$$\mathrm{com}(A) = \begin{pmatrix} 1 & -2 & -1 \\ 0 & -2 & 0 \\ -1 & -2 & -1 \end{pmatrix} \quad (1,5)$$

$$A^{-1} = -\frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ -2 & -2 & -2 \\ -1 & 0 & -1 \end{pmatrix} \quad (0,5)$$

$$A^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 1 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \quad (0,5)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ -2 & -2 & -2 \\ -1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} \quad (1,5)$$

Exo 3:

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 4 & 2 \\ -1 & 0 & 3 \end{pmatrix}$$

1) Caracteristic polynom of Matrix A:

$P_A(\lambda) = \det(A - \lambda I)$

$P_A(\lambda) = -\lambda^3 + 10\lambda^2 - 32\lambda + 32$ (2,5)

2) Eigen values: $P_A(\lambda) = 0$ (0,5)

$\lambda_1 = 2$ (0,25) ; $\lambda_2 = 4$ (0,25) (double value).

Eigen vectors.

$(A - \lambda_i I)v_i = 0$

* $\lambda_1 = 2$: $v_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

$$(A-2I)v_1 = 0 \iff \begin{cases} x - z = 0 \\ 2x + 2y + 2z = 0 \\ -x + z = 0. \end{cases} \quad (0,25)$$

$x = z \rightarrow 2x + 2y + 2x = 0$ (0,5)

$2x + y = 0 \Rightarrow y = -2x.$

$$v_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ -2x \\ x \end{pmatrix} = x\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \Rightarrow v_1 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad (0,25)$$

* $\lambda_2 = 4$: $v_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

$$(A - 4I)v_2 = 0 \iff \begin{cases} -x - z = 0 \\ 2x + 2z = 0 \\ -x - z = 0 \end{cases}$$

(0,25)

$\Rightarrow -x = +z$.

$$v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -z \\ y \\ +z \end{pmatrix} = \begin{pmatrix} -z \\ 0 \\ +z \end{pmatrix} + \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$$

(0,25)

$$= z\begin{pmatrix} -1 \\ 0 \\ +1 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(0,25)

$$\Rightarrow v_2 = \begin{pmatrix} -1 \\ 0 \\ +1 \end{pmatrix} ; v_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

1

Then A is diagonalisable.

3). $A = PDP^{-1}$

$$P = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & 1 \end{pmatrix} ; D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

(0,75) (0,75)

$$P^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 1 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

(0,75)