L1_S2
2025/2026

Exam_Correction_ Signal theory

16/05/26

⊛ Exo1: Linearity Propriety of Fourier Transform:

1,5 pts

We have: $x(t) \xrightarrow{F.T} X(f) \Rightarrow X(f) = \int_{-\infty}^{+\infty} x(t) . e^{-i2\pi ft} . dt$

$\begin{cases} \text{and} \\ y(t) \xrightarrow{F.T} Y(f) \Rightarrow Y(f) = \int_{-\infty}^{+\infty} y(t) . e^{-i2\pi ft} . dt \end{cases}$

. For $\alpha x(t) \xrightarrow{F.T} \int_{-\infty}^{+\infty} \alpha . x(t) . e^{-i2\pi ft} dt = \alpha \int_{-\infty}^{+\infty} x(t) . dt . e^{-i2\pi ft} = \alpha X(f)$ (1) / α: real cte

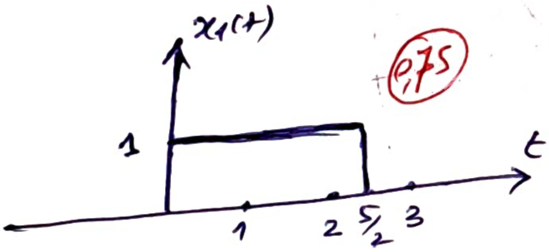
. For $\beta \in \mathbb{R}$: $\beta . y(t) \xrightarrow{F.T} \int_{-\infty}^{+\infty} \beta . y(t) . e^{-i2\pi ft} . dt = \beta \int_{-\infty}^{+\infty} y(t) . e^{-i2\pi ft} . dt = \beta Y(f)$ (2)

Now: (1) and (2): $\alpha x(t) + \beta y(t) \xrightarrow{F.T} \int_{-\infty}^{+\infty} (\alpha x(t) + \beta y(t)) . e^{-i2\pi ft} . dt$

$$= \underbrace{\int_{-\infty}^{+\infty} \alpha x(t) . e^{-i2\pi ft} . dt}_{\alpha X(f)} + \underbrace{\int_{-\infty}^{+\infty} \beta y(t) . dt . e^{-i2\pi ft}}_{\beta Y(f)} = \alpha X(f) + \beta Y(f)$$

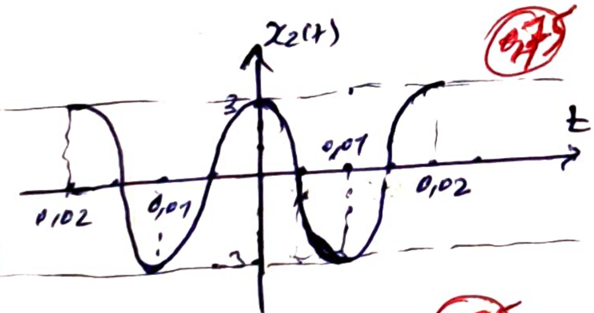
⊛ Exo2:

⊙ $x_1(t)$ =



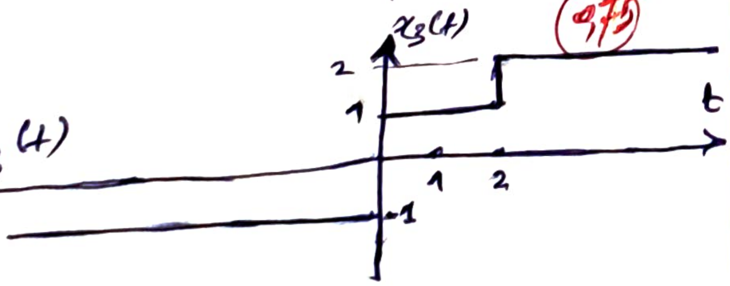
0,75

⊙ $x_2(t)$ =
$T = 0,025$



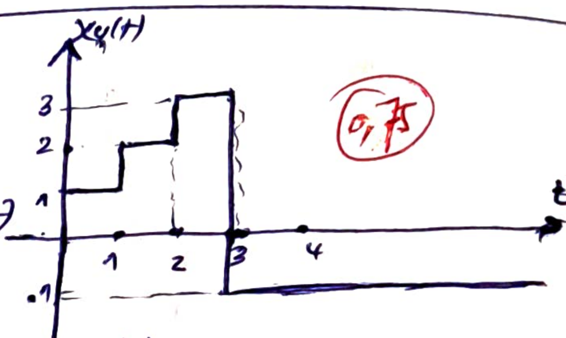
0,75

. $x_3(t)$



0,75

⊙ $x_4(t)$

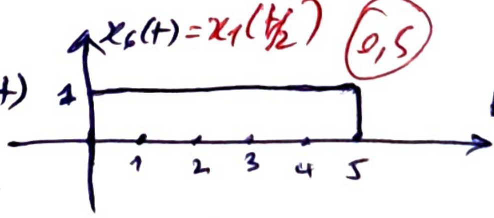


0,75

⊙ $x_5(t)$:

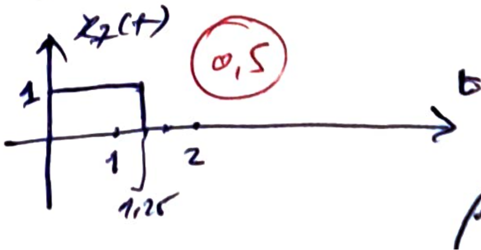
0,75

⊙ $x_6(t)$



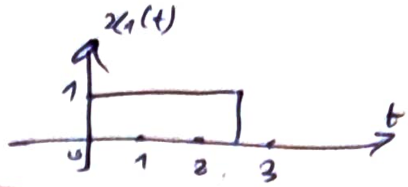
0,5

⊙ $x_7(t)$



0,5

2) $x_1(t)$: Energy or power signal?



$$E = \int_{-T}^{T} x_1^2(t)\,.dt = \int_0^{5/2} (1)^2.dt = t\Big|_0^{5/2} = \boxed{\frac{5}{2} < \infty}$$

$\Rightarrow x_1(t)$ is Energy signal , $\boxed{P = 0}$ 2 pts

3) Calculate : $\displaystyle\int_{-\infty}^{+\infty} x_1(t)\,.\,\delta(t+1)\,.dt = 0$ 1,5 pts

4) $FT(x_1(t)) = ?$ $\displaystyle X_1(f) = \int_{-\infty}^{+\infty} x_1(t).e^{-j2\pi f t}.dt = \int_0^{5/2} 1.e^{-j2\pi f t}.dt = \frac{-1}{j2\pi f}\left(e^{-j5\pi f} - 1\right)$

$$X_1(f) = \frac{5}{2}\,\frac{\sin\frac{5}{2}\pi f}{\frac{5}{2}\pi f}\,.\,e^{-j\frac{5}{2}\pi f} = \frac{5}{2}\,\mathrm{Sinc}\,\frac{5}{2}\pi f\,.\,e^{-j\frac{5}{2}\pi f}$$ 2 pts

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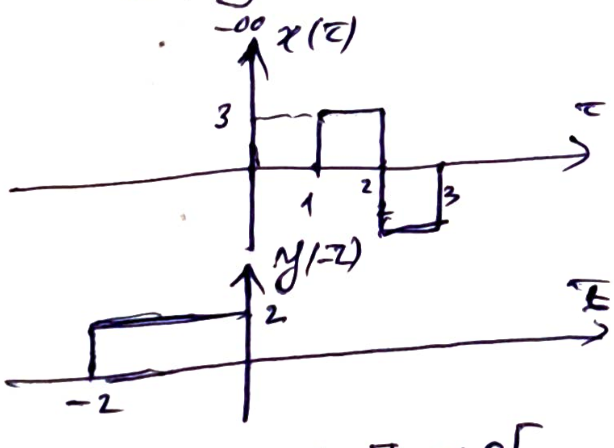
* **Exo 3** 1) A) Analytical expression of $x(t)$ and $y(t)$

$$x_1(t) = \begin{cases} 3, & 1 \le t \le 2 \\ -3, & 2 \le t \le 3 \\ 0, & \text{else} \end{cases} \quad 1\text{pt}, \qquad y(t) = \begin{cases} 2, & 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases} \quad 0{,}1\text{ pt}$$

1) B) Calculate the convolution $x(t) * y(t) = z(t)$

$$z(t) = x(t) * y(t) = \int_{-\infty}^{+\infty} x(\tau)\,y(t-\tau)\,.d\tau$$



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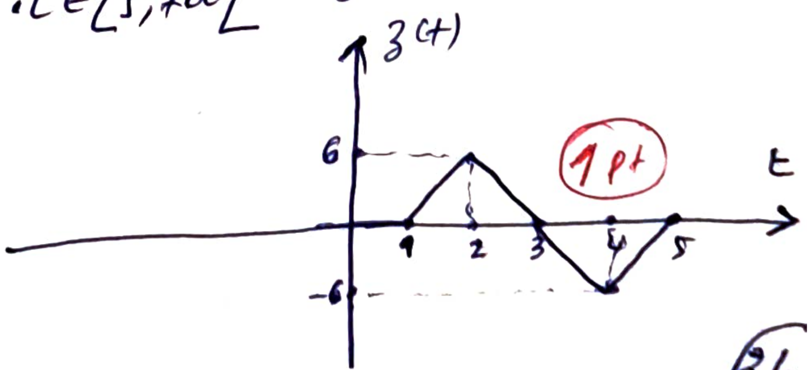
- when : $t < 0$, $t \in]-\infty, 0[$
  No intersection $\Rightarrow z(t) = 0$
- when : $t > 0 \Rightarrow$ we have 05 cases

- $t \in [0,1[\;\rightarrow z(t) = 0$ 0,5
- $t \in [1,2] \;\rightarrow z(t) = 6(t-1)$ 0,5
- $t \in [2,3] \;\rightarrow 6(3-t) = z(t)$ 1
- $t \in [3,4] \;\rightarrow z(t) = 6(3-t)$ 1
- $t \in [4,5] \;\rightarrow z(t) = 6(t-5)$ 0,5
- $t \in [5,+\infty[\;\rightarrow z(t) = 0$ 0,5



1 pt

2/2