solution of exam analyses 2

(a) $y = 1 - \frac{dy}{dx}\frac{x+1}{(x-1)(x+2)}$

$\Rightarrow \frac{dy}{dx}\frac{x+1}{(x-1)(x+2)} = 1-y$

$\Rightarrow \frac{1}{1-y}dy = \frac{(x-1)(x+2)}{x+1}dx$

$\Rightarrow \int \frac{1}{1-y}dy = \int \frac{x^2+x-2}{x+1}dx$

BY SUBSTITUTION $u = x+1$ OR BY ALGEBRAIC MANIPULATION

$\Rightarrow \int \frac{1}{1-y}dy = \int \frac{x(x+1)-2}{x+1}dx$

$\Rightarrow \int \frac{1}{1-y}dy = \int x - \frac{2}{x+1}dx$

1pt

2pt

(b) when $x=0$ $y=2$

$\ln 1 + 0 - 2\ln 1 = C$

$C = 0$

$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 12(x+e^x)$

START WITH THE AUXILIARY EQUATION

$\lambda^2 + 5\lambda + 6 = 0$

$(\lambda+2)(\lambda+3) = 0$

$\lambda = -2, -3$ 1pt

1pt

∴ COMPLEMENTARY FUNCTION: $y = Ae^{-2x} + Be^{-3x}$

FOR PARTICULAR INTEGRAL WE TRY $y = Px + Q + Re^x$

$\frac{dy}{dx} = P + Re^x$

$\frac{d^2y}{dx^2} = Re^x$

SUB INTO THE O.D.E

$(Re^x) + 5(P + Re^x) + 6(Px + Q + Re^x) \equiv 12x + 12e^x$

$6Px + (5P + 6Q) + e^x(R + 5R + 6R) \equiv 12x + 12e^x$

∴ $P = 2$ 0.5 $R = 1$ 0.5 & $5P + 6Q = 0$

$10 + 6Q = 0$

$Q = -\frac{5}{3}$ 0.5

HENCE THE GENERAL SOLUTION IS

$y = Ae^{-2x} + Be^{-3x} + e^x + 2x - \frac{5}{3}$ 0.5

Exercises 02 : $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3 y\cos(x)}{2x^6+y^3}$

X=0 $\lim\limits_{(x,y)\to(0,0)} \dfrac{0}{y^3} = 0$ 0.5

y=0 $\lim\limits_{(x,y)\to(0,0)} \dfrac{0}{2x^6} = 0$ 0.5

y=x² $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^5\cos(x)}{3y^6} = +\infty$ 0.5

$$\lim_{(x,y)\to(4,0)} \frac{\sqrt{x}-\sqrt{y+4}}{x-y-4} = \frac{x-y-4}{x-y-4\left(\sqrt{x}+\sqrt{y+4}\right)} = \frac{1}{\sqrt{x}+\sqrt{y+4}} = \frac{1}{2}$$

0.5 0.5 0.5

1 1

$$f'(x) = 3x^2y^4 - 2y^{\frac{3}{2}} + 6x^5 \quad f'(y) = 4x^3y^3 - 3xy^{\frac{1}{2}} - 3\cos(3y)$$

$$f''(x) = 6xy^4 + 30x^4 \quad f''(y) = 12x^3y^2 - \frac{3}{2}xy^{-\frac{1}{2}} + 9\sin(3y)$$

0.5 0.5

$$f'(x,y) = 12x^2y^3 - 3y^{\frac{1}{2}} \quad f''(y,x) = 12x^2y^3 - 3y^{\frac{1}{2}}$$

0.5 0.5

Exercises 03

$\int_0^3 \int_0^{2-\frac{2}{3}x} (6-2x-3y)\,dy\,dx$

$\int_0^{2-\frac{2}{3}x} (6-2x-3y)\,dy = -8x + \frac{4x^2}{3} - \frac{(-2x+6)^2}{6} + 12$ 2.5

$= \int_0^3 \left(-8x + \frac{4x^2}{3} - \frac{(-2x+6)^2}{6} + 12\right)dx$ 2.5

$\int_0^3 \left(-8x + \frac{4x^2}{3} - \frac{(-2x+6)^2}{6} + 12\right)dx = 6$ 1

$\frac{2}{3}x^2 - 4x + 6$