LARBI BEN M'HIDI OUM EL BOUAGHI UNIVERSITY
INSTITUTE OF APPLIED SCIENCES AND TECHNIQUES
NETWORKS AND TELECOMMUNICATIONS DEPARTMENT

Module: coding and compression — M1 RT S1

Solution type of First Exam

Course question (08 pts)

A. **Choose the correct answer(s)**

1. **In the process of transmitting information, noise is the disruptive element, in which part of a transmission chain?**
   - ☐ Source
   - ☐ Coder
   - ☐ User
   - ☑ Channel
   - ☐ Decoder

2. **In variable size source encoding (VLC), a symbol with minimum probability will have the fewest bits.**
   - ☐ TRUE
   - ☑ False

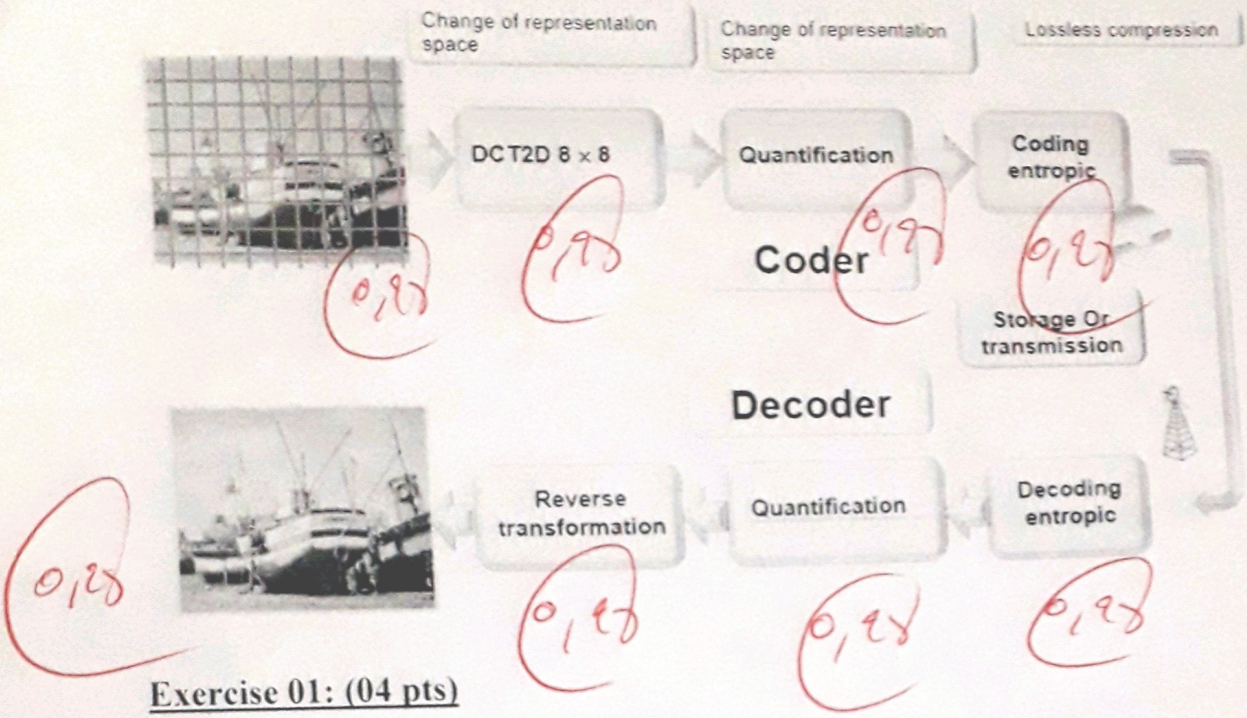
3. **A Huffman encoder takes a set of characters of fixed length and produces a set of characters of:**
   - ☑ Variable length,
   - ☐ Random length,
   - ☐ Fixed length,
   - ☐ Constant length

4. **For which value(s) of p is the binary entropy function H(p) minimized?**
   - ☑ 0
   - ☐ 0.5
   - ☑ 1
   - ☐ 0.8

5. **The human eye is less sensitive to:**
   - ☐ The average spatial frequency components
   - ☐ DC components
   - ☐ Lower spatial frequency components
   - ☑ Higher spatial frequency components

6. **Discrete cosine transform (DCT) is a Lossless compression**
   - ☐ True
   - ☑ False

B. **Give the block diagram of the different stages of JPEG data compression.**



Exercise 01: (04 pts)

Using arithmetic coding, code the word CALCUL

Base 9 :

CALCUL ⇒ C → 1/3, A → 1/6, L → 1/3, U → 1/6

C → [0,00 ; 0,33[
A → [0,33 ; 0,50[
L → [0,50 ; 0,83[
V → [0,83 ; 1,00[

$B_{sup} = B_{inf-1} + (B_{S-1} - B_{inf-1}) \cdot B_{sup\ caracter}$

$B_{inf} = B_{inf-1} + (B_{S-1} - B_{inf-1}) \cdot B_{I\ caracter}$

	[0,0000	1,0000 [
C	[0,0000	0,3333 [
A	[0,1111	0,1667 [
L	[0,1389	0,1574 [
C	[0,1389	0,1451 [
V	[0,1440	0,1451 [
L	[0,1449	0,1450

CALCUL → 0,1449

Exercise 02: (04 pts)

C(n,k) be a linear code whose generating matrix is given below

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

1. data k=3; the length of the code word n=6; number of parity bits r=3
2. Deduce the control matrix H

$$G = [I \quad P] \Rightarrow H = [P^t \quad I_{n-k}]$$

So we will have

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

4. Find the code words $C_D = D.G$

As the data has a size k=3 (see first question) then we have 8 data possibilities. Each of these 8 data vectors will be multiplied by the six columns of the matrix G to find a codeword of size n=6.

Data	Codeword
000	000000
001	001001
010	010101
011	011100
100	100111
101	101110
110	110010
111	111011

Exercise 03: (04 pts)

For the discrete channels described below,

1. Find the transition matrix

$$T = \begin{bmatrix} 0,9 & 0,03 & 0,07 \\ 0,1 & 0,8 & 0,1 \\ 0,15 & 0,05 & 0,8 \end{bmatrix}$$

2. What is its capacity? The capacity always assuming that the input X is equiprobable.

$$T = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0,9 & 0,03 & 0,07 \\ 0,1 & 0,5 & 0,1 \\ 0,15 & 0,05 & 0,8 \end{bmatrix} \end{array}$$

$p(x_i) = \frac{1}{3}$;

$C = \max I(X;Y)$

$I(X;Y) = H(Y) - H(Y|X)$

$H(Y) = \sum p_{y_i} \log_2 \left(\frac{1}{p_{y_i}}\right)$

$P(Y=0) = 0,9\,p(x=0) + 0,1\,(p(x=1)) + 0,15\,p(x=2) = 0,38$

$p(Y=1) = 0,03\,p(x=0) + 0,5\,p(x=1) + 0,05\,(p(x=2)) = 0,29$

$p(Y=2) = 0,07\,p(x=0) + 0,1\,p(x=1) + 0,8\,p(x=2) = 0,32$

$H(Y) = 0,38 \log_2 \left(\frac{1}{0,38}\right) + 0,29 \log_2 \left(\frac{1}{0,29}\right) + 0,32 \log_2 \left(\frac{1}{0,32}\right) = 1,57$ bits/symbol

$H(Y|X) = \sum p(x_i) \cdot H(Y|X_i) = \frac{1}{3}\left[H(Y|x=0) + H(Y|x=1) + H(Y|x=2)\right]$

$= \frac{1}{3}\left[0,9 \log_2 \frac{1}{0,9} + 0,03 \log_2 \frac{1}{0,03} + 0,07 \log_2 \frac{1}{0,07} + 2(0,1 \log_2 \frac{1}{0,1}) + 2 \cdot 0,8 \log_2 \frac{1}{0,8} + 0,15 \log_2 \frac{1}{0,15} + 0,05 \log_2 \frac{1}{0,05}\right]$

$= \frac{1}{3}\left[0,9(0,152) + 0,03(5,059) + 0,07(3,8365) + 0,2(3,322) + 1,6[0,3219] + 0,3(2,737) + 0,5(4,322)\right]$

$H(Y|X) = 0,7875$ bits/symbol.