# Final Exam Correction

**Exercise 1 : (6,5 Points)**

Let the signal, at finite energy, $x_\theta(t) = Rect_\theta(t)$ ;

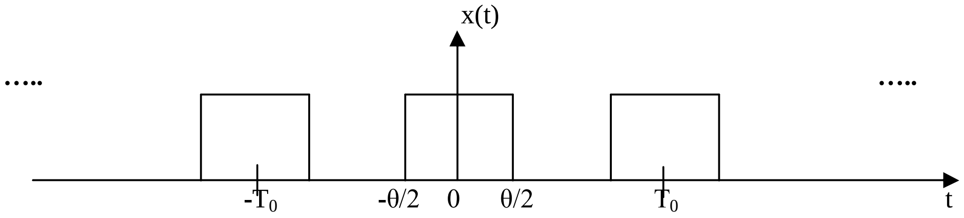
1. Demonstrate that the Fourier Transform of $Rect_\theta(t)$ ; $X_\theta(f) = \theta\, Sinc(f\,\theta)$

$$X_\theta(f) = \int_{-\infty}^{+\infty} Rect_\theta(t) e^{-j2\pi ft}\, dt = \int_{-\theta/2}^{\theta/2} e^{-j2\pi ft}\, dt = -\frac{1}{j2\pi f}\, e^{-j2\pi ft}\Big|_{-\theta/2}^{\theta/2} = -\frac{1}{j2\pi f}\left(e^{-j\pi f\theta} - e^{j\pi f\theta}\right)$$

$$= \frac{1}{\pi f}\frac{\left(e^{j\pi f\theta} - e^{-j\pi f\theta}\right)}{2j} = \frac{\sin(\pi f\theta)}{\pi f} = \theta\frac{\sin(\pi f\theta)}{\pi f\theta} = \theta\, Sinc(f\theta)$$

2. Let $x(t) = \sum_{k=-\infty}^{k=+\infty} x_\theta(t - kT_0)$ ;

   a. **Temporal representation of the signal x(t)**




   b. x(t) is it a finite mean power signal ($P_x < \infty$) ; x(t) is a periodized signal (unbounded support represented by the dots **…..** )

   c. **Calculation of X(f)**
   x(t) being a periodized signal; $X(f) = f_0 \sum_{n=-\infty}^{n=+\infty} X_\theta(nf_0)\, \delta(f - nf_0)$
   with $X_\theta(nf_0) = X_\theta(f)\big|_{f=nf0}$

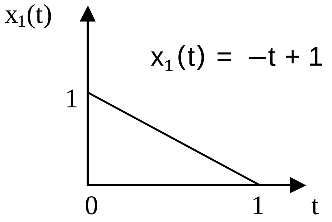
   $X_\theta(f) = \theta\, Sinc(f\,\theta)$ (Answer 1)
   $X_\theta(nf_0) = \theta\, Sinc(nf_0\,\theta)$

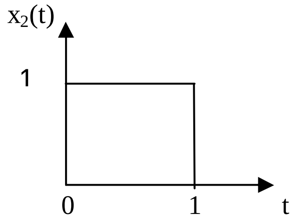
   And

$$X(f) = \theta\, f_0 \sum_{n=-\infty}^{n=+\infty} Sinc(nf_0\theta)\, \delta(f - nf_0)$$

**<u>Exercise 2</u> : (4 Points)**

***Calculation of $x_1(t)*x_2(t)$***



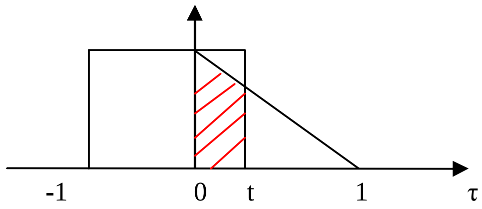




$$x_2(t) = \begin{cases} 1 & 0 \le t < 1 \\ 0 & \text{ailleurs} \end{cases}$$

$$x_1(t) * x_2(t) = \int_{-\infty}^{+\infty} x_1(\tau)x_2(t-\tau)d\tau$$

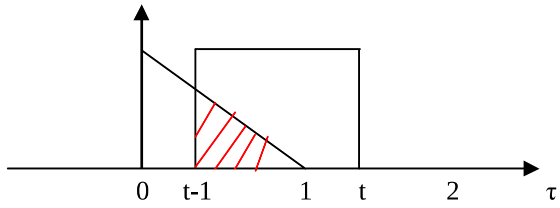
***Intervalle of  t : 0 < t < 2***

1. ***0 < t < 1***




$$x_1(t) * x_2(t) = \int_0^t (-t+1).1\ d\tau = \frac{-\tau^2}{2}\Big|_0^t + \tau|_0^t = \frac{-t^2}{2} + t$$

2. ***1 < t < 2***

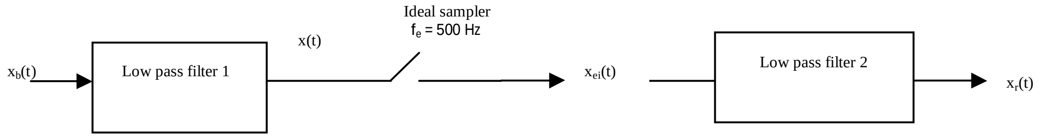



$$x_1(t) * x_2(t) = \int_{t-1}^1 (-t+1).1\ d\tau = \frac{-\tau^2}{2}\Big|_{t-1}^1 + \tau|_{t-1}^1 = \frac{t^2}{2} - 2t + 2$$

**2.** The convolution product expresses the amount of overlap of a signal $x_2(t)$ when moved to another signal $x_1(t)$

**Exercise 3 : (6,5 Points)**

Let the following synoptic diagram




Let $x_b(t)$ ; a noisy signal, $x_b(t) = \cos(400\pi t) + b(t)$ .

1. *Low pass filter 1 :*
   a. It's an anti-aliasing filter.
   b. $f_{c1}$ = 1KHz is a satisfactory value because $f_0$ =200 Hz is in the passband of the filter.

Let the signal $x(t) = \cos(400\pi t)$ $f_0=f_{max}=200Hz$

2. $x(t) = \cos(400\pi t)$

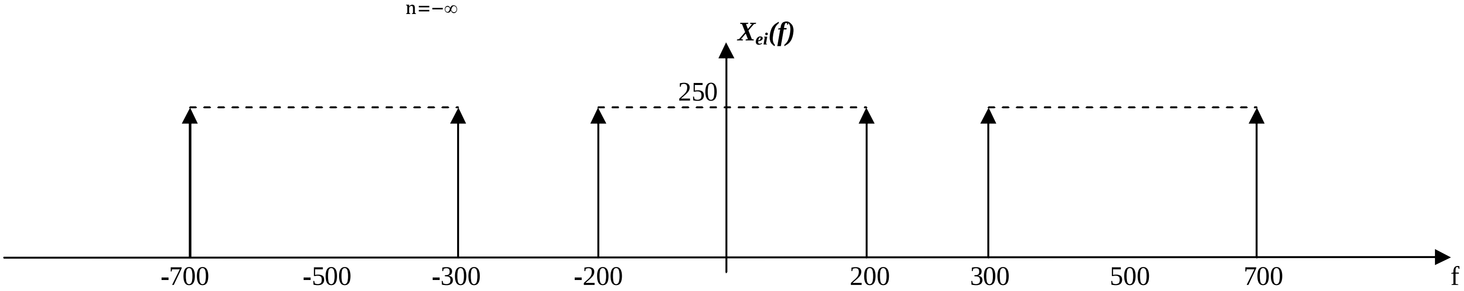
   a. **Calculation of Nyquist frequency $f_N$** : $f_N=2f_0=2.200 = 400Hz$

   b. This signal is ideally sampled at the frequency $f_e$ = 500Hz.
      i. The value fe = 500Hz is an appropriate choice ; $f_e = 500 > 400 = 2f_{max}$ (Shannon theorem respected)

      ii. **Calculation and plot *of $X_{ei}(f)$***

$$X_{ei}(f) = f_e \, Rep_{fe}[X(f)] = f_e \sum_{n=-\infty}^{n=+\infty} X(f - nf_e)$$

$$X_{ei}(f) = \frac{f_e}{2} \sum_{n=-\infty}^{n=+\infty} \{\delta[(f - nf_e) + 200] + \delta[(f - nf_e) - 200]\}$$




***Conclusion :*** The right choice of sampling frequency fe > 2fmax, Shannon's theorem respected, does not give a spectral folding which allows us to recover the signal

3. *Low pass filter 2*
   **a.** Signal recovery filter x(t).
   **b.** $f_{c2} \in ]f_0 , f_e\text{-}f_0[$ let $f_{c2} \in ]200 , 300[$ . So ; $f_{c2}$ = 250 Hz is a value that allows signal recovery.

**<u>Exercise 4</u> : ( 3 Points)**

Check the wrong answer with ( x ) and the correct answer with ( √ )

1. The analog modulation is:
   a. The adaptation of the signal to the transmission channel by varying the time according to the variations in the signal to be transmitted; ( x )
   b. The adaptation of the signal to the transmission channel by varying the spectrum depending on the variations of the signal to be transmitted; ( x )
   c. The adaptation of the signal to the transmission channel by varying the amplitude, frequency or phase depending on the variations of the signal to be transmitted. ( √ )

2. For an amplitude modulated signal (AM) given by its expression
   $s(t) = A_p [ 1 + m \cos (2\pi f_m t)] \cos (2\pi f_p t)$; m represents:
   a. The frequency of the modulating signal; ( x )
   b. The amplitude of the modulating signal; ( x )
   c. The modulation index. ( √ )