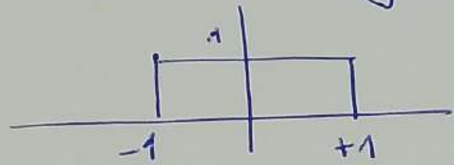
Ex 1 — Examen Signaux et Systeme.

$$x(t) \begin{cases} 1 & \text{si } |t| \leq 1 \\ 0 & \end{cases}$$



(1) $X(f) = \int_{-1}^{+1} 1 \cdot e^{-j2\pi f t}\, dt = -\frac{1}{j2\pi f}\left(e^{-j2\pi f t} \Big|_{-1}^{+1}\right.$

$$\boxed{X(f) = \frac{+1}{j2\pi f}\left(e^{j2\pi f t} - e^{-j2\pi f t} \right).}$$

$$\boxed{= \frac{1}{2j\pi f}\left(e^{j2\pi f} - e^{-j2\pi f} \right).}$$

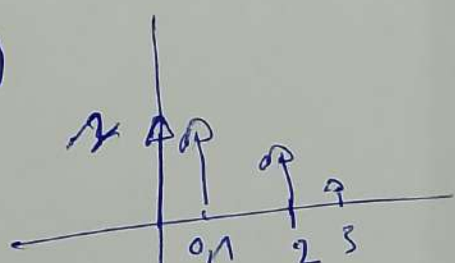
ok

(2) on a $e^{j2\pi f} = \cos(2\pi f) + j\sin(2\pi f)$

donc : $e^{j2\pi f} - e^{-j2\pi f} = \cos(2\pi f) + j\sin(2\pi f)$
$- \cos(2\pi f) + j\sin(2\pi f)$ ok
$= 2j \sin(2\pi f)$

donc $X(f) = \frac{1}{2j\pi f} \cdot 2j\sin(2\pi f) = \frac{\sin(w)}{w}$
$= \text{sinc}(w).$

(3)
$X(0) = 2$ ($\text{sinc}(0) = 1$)
$X(0,5) = 0$ ok
$X(1) = 0,45$
$X(1,5) = 0,047$



$E = \int_{-\infty}^{+\infty} x(t)^2\, dt$
$= \left(x(t) \Big|_{-1}^{+1} = 2 \right.$

$E = 2$ ok

(1)

exo 2 :

1) $\frac{dy(t)}{dt} + 2y(t) = 5u(t)$ $\mathcal{L}[u(t)] = \frac{1}{p}$ (01)

2) T.L. $\Rightarrow \mathcal{L}\left[\frac{dy(t)}{dt}\right] + 2\,\mathcal{L}[y(t)] = 5\,\mathcal{L}[u(t)]$.

$$p\,Y(p) + 2\,Y(p) = \frac{5}{p}$$ (01)

3) $H(p) = \frac{5}{p(p+2)}$

4) $H(p) = \frac{5}{2}\left(\frac{1}{p} - \frac{1}{p+2}\right)$. (01)

5) $y(t) = \frac{5}{2}\left(u(t) - e^{-2t}u(t)\right) = \frac{5}{2}\left(1 - e^{-2t}\right)$ (01)

exo 3 : 1) $H(p) = \frac{4}{2p+1} = \frac{2}{p+1/2}$.

F.T.B.F. $G(p) = \frac{H(p)}{1 + H(p)}$

$$= \frac{2/p+1/2}{1 + 2/p+1/2}$$

$$\boxed{G(p) = \frac{2}{p + 5/2}}$$ (01)

2) échelon unitaire $\Rightarrow Y_c(p) = 1/p$. (01)

donc $Y(p) = G(p) \cdot Y_c(p) = \frac{2}{p(p+5/2)}$ (01)

(2)

$$\frac{2}{p(p+3/2)} = \frac{4}{3}\cdot\left(\frac{1}{p} - \frac{1}{p+3/2}\right) \quad 0{,}1$$

$$\text{donc } y(t) = \frac{4}{3}\left(u(t) - e^{-\frac{3}{2}t}u(t)\right) = \frac{4}{3}\left(1 - e^{-3/2\, t}\right). \quad 0{,}1$$

3) $H_{BOZ} = \dfrac{(1-z^{-1})}{p}$

donc F.T.B.O. $H(p) = (1-z^{-1})\cdot\dfrac{2}{p(p+1/2)}$. 0,1

donc $H(z) = (1-z^{-1})\,\mathcal{Z}\left[\dfrac{2}{p(p+1/2)}\right]$. 0,1

$$= (1-z^{-1})\,\mathcal{Z}\left(4\left(\frac{1}{p} - \frac{1}{p+1/2}\right)\right).$$

$$= (1-z^{-1})\cdot\left(4\left(\frac{z}{z-1} - \frac{z}{z-e^{-\frac{1}{2}T}}\right)\right) \quad T=1$$

$$= 4\left(\frac{\cancel{z-1}}{\cancel{z}}\right)\left(\cancel{z}\left(\frac{\cancel{z} - e^{-1/2} - \cancel{z} + 1}{\cancel{(z-1)}(z-e^{-1/2})}\right)\right)$$

$$= 4\,\frac{1-e^{-1/2}}{(z-e^{-1/2})}. \quad (\exp(-0{,}5) = 0{,}6).$$

$$= \frac{1{,}6}{z-0{,}6} \quad 0{,}1$$

F.T.B.F. $F_{(z)} = \dfrac{1{,}6}{z+1}$

4) l'entrée échelon $U(z) = \dfrac{z}{z-1}$ 0,1

donc $Y(z) = \dfrac{1{,}6}{(z+1)}\cdot\dfrac{z}{(z-1)} \Rightarrow \dfrac{Y(z)}{z} = \dfrac{1{,}6}{2}\left(\dfrac{1}{z-1} - \dfrac{1}{z+1}\right)$

d

donc $Y(z) = \dfrac{1{,}6}{2}\left(\dfrac{z}{z-1} - \dfrac{z}{z+(1)}\right) \Rightarrow y(t) = 0{,}8\,(u(t) - e^{-t})$ 0,1