Exam correction

Exercise 1 : (8 Points)

$x(t) = 2\cos(2\pi f_0 t) + 3\sin(2\pi f_0 t) + 2\sin(4\pi f_0 t)$;

1. **f_0** : fundamental frequency

2. **a_0**$= 0$: Average value or continuous component.

3. **Number of harmonics** $n = 2$; from the general form $x(t) = a_0 + \sum_{n=1}^{\infty}[a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)]$

4. **Expression of x(t) complex form**

$x(t) = \sum_{n=-\infty}^{\infty} X_n\, e^{j2\pi n f_0 t}$ X_n : Fourier Coefficients

with $X_n = \frac{1}{2}(a_n - j\, b_n)$ for $n \geq 1$

$X_n = \frac{1}{2}(a_n + j\, b_n)$ for $n \leq -1$

$|X_n| = \frac{1}{2}\sqrt{a_n^2 + b_n^2}$ et $\varphi_n = \text{Arctg}\left(\frac{-b_n}{a_n}\right)$

n = 2

So ; $x(t) = \sum_{n=-2}^{2} X_n\, e^{j2\pi n f_0 t} = X_{-2}\, e^{-j4\pi f_0 t} + X_{-1}\, e^{-j2\pi f_0 t} + X_0 + X_1\, e^{j2\pi f_0 t} + X_2\, e^{j4\pi f_0 t}$

Determine a_n and b_n

$a_0 = 0$;

$a_1 = 2$; $b_1 = 3$;

$a_2 = 0$; $b_2 = 2$;

$X_0 = a_0 = 0$;

$X_{-1} = \frac{1}{2}(a_1 + j\, b_1) = 1 + j\,\frac{3}{2}$; $X_1 = \frac{1}{2}(a_1 - j\, b_1) = 1 - j\,\frac{3}{2}$

$X_{-2} = \frac{1}{2}(a_2 + j\, b_2) = j$; $X_2 = \frac{1}{2}(a_2 - j\, b_2) = -j$

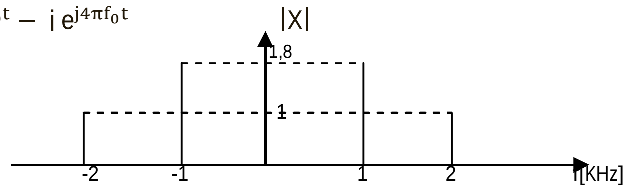
$\varphi_1 = \text{Arctg}\left(\frac{-b_1}{a_1}\right) = \text{Arctg}\left(\frac{-3}{2}\right) = -0{,}98$ $\varphi_2 = \text{Arctg}\left(\frac{-b_2}{a_2}\right) = \text{Arctg}\left(\frac{-2}{0}\right) = \text{Arctg}(-\infty) = -\frac{\pi}{2}$

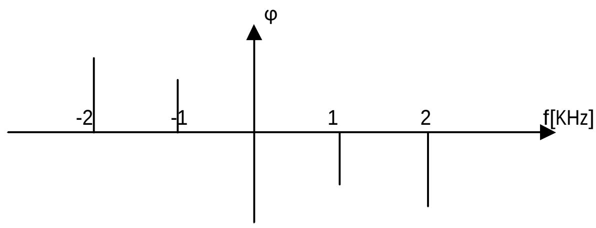
$x(t) = j\, e^{-j4\pi f_0 t} + \left(1 + j\,\frac{3}{2}\right) e^{-j2\pi f_0 t} + \left(1 - j\,\frac{3}{2}\right) e^{j2\pi f_0 t} - j\, e^{j4\pi f_0 t}$

5. **Represent the corresponding amplitude and phase spectra**

$|X_{-1}| = |X_1| = \frac{1}{2}\sqrt{a_1^2 + b_1^2} = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$

$|X_{-2}| = |X_2| = \frac{1}{2}\sqrt{a_2^2 + b_2^2} = \sqrt{1} = 1$





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<u>Exercise 2</u> : (7 Points)

Let f be a continuous function defined by :

$$f(x) = \begin{cases} 12x^2(1-x) & 0 \le x \le 1 \\ 0 & ailleurs \end{cases}$$

1. f(x) is a probability density function
   - ✓ f(x) continous
   - ✓ f(x) ≥ 0 ;
   - ✓ $\int_{-\infty}^{+\infty} \mathrm{f(x)dx} = 1$

$$\int_{-\infty}^{+\infty} f(x)dx = \int_0^1 12x^2(1-x)dx = \int_0^1 (12x^2 - 12x^3)dx = \int_0^1 12x^2\,dx - \int_0^1 12x^3\,dy$$
$$= \frac{12}{3}x^3 - \frac{12}{4}x^4\Big|_0^1 = (4x^3 - 3x^4)|_0^1 = 4 - 3 = 1$$

2. Calculation of E[X] et de Var[X]

$$\mathrm{E[X]} = \int_0^1 xf(x)dx = \int_0^1 (12x^3 - 12x^4)dx = \int_0^1 12x^3\,dx - \int_0^1 12x^4\,dx$$
$$= \frac{12}{4}x^4 - \frac{12}{5}x^5\Big|_0^1 = (3\,x^4 - \frac{12}{5}x^5)\Big|_0^1 = 3 - \frac{12}{5} = \frac{3}{5} = 0{,}6$$

Var[X] = E[X²] – (E[X])²

$$\mathrm{E[X^2]} = \int_0^1 x^2 f(x)dx = \int_0^1 (12x^4 - 12x^5)dx = \int_0^1 12x^4\,dx - \int_0^1 12x^5\,dx$$
$$= \frac{12}{5}x^5 - \frac{12}{6}x^6\Big|_0^1 = (\frac{12}{5}x^5 - 2x^6)|_0^1 = 2{,}4 - 2 = 0{,}4$$

$\mathrm{Var[X]} = 0{,}4 - (0{,}6)^2 = 0{,}04$

3. Distribution Fonction

$$F(x) = \int_{-\infty}^{x} f(y)dy$$

- ✓ x < 0 $\quad F(x) = \int_{-\infty}^{x} f(y)dy = 0$
- ✓ 0 ≤ x ≤ 1 $\quad F(x) = \int_{-\infty}^{x} f(y)dy = \cancelto{0}{\int_{-\infty}^{0} f(y)dy} + \int_0^x 12y^2(1-y)dy = 4y^3 - 3y^4|_0^x = 4x^3 - 3x^4$
- ✓ x > 1 $\quad F(x) = \int_{-\infty}^{x} f(y)dy = \int_{-\infty}^{0} \cancelto{0}{f(y)}dy + \int_0^1 f(y)dy + \int_1^x \cancelto{0}{f(y)}dy = 1$

$$F(x) = \begin{cases} 0 & x < 0 \\ 4x^3 - 3x^4 & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

4. P(0 ≤ X ≤ ½) = F(1/2) – F(0) $\tilde{-}$ 0,3

Exercise 3 : (5 Points)

Let the statistical table be:

Diameter in (X)	**Absolute frequency (n_i)**
18.50	1
19.00	2
20.00	3

1. The diameter of the parts (X) is a quantitative character, because it takes numerical values.

2. N=50 ; represents the population size

N=6 ; represents the sample size

3. Determination of central tendency measures

a. Mean

$$\bar{x} = \frac{\sum(x_i \mathrm{x} n_i)}{n} = \frac{(18.5\mathrm{x}1) + (19\mathrm{x}2) + (20\mathrm{x}3)}{6} = 19.42\ mm$$

The average diameter of the manufactured parts is 19.42 mm, which is very close to the target value of 20.00 mm. This result shows a good accuracy of the machine.

b. Mode

Mode is the value that appears most frequently in data. Here, the value that appears most often is 20.00 mm, with a frequency of 3.

The target diameter 20.00 mm is the value with the highest frequency; good manufacturing quality.

4. Determination of dispersion measures

a. Extent

X_{max}-X_{min}= 20 – 18.5 = 1.50 mm

b. Variance

$$\sigma_{ech}^2 = \frac{\sum(x_i - \bar{x})^2 \mathrm{x}\, n_i}{n-1} = \frac{(18.5 - 19.42)^2\mathrm{x}1 + (19 - 19.42)^2\mathrm{x}2(20 - 19.42)^2\mathrm{x}3}{5} = 0.44\ mm^2$$

The variance measures the dispersion of diameters compared to the mean. A low variance indicates that the values are close to the mean.

c. Standard deviation

$$\sigma_{ech} = \sqrt{\sigma_{ech}^2} = \sqrt{0.44} = 0.66\ mm$$

Standard deviation is the most commonly used indicator of dispersion, and it represents the average variation around the mean. A standard deviation of 0.66 mm means that most produced parts will have a diameter between 19.42 ± 0.66 mm, or between 18.82 mm and 20.08 mm. This shows that the parts are produced with very good consistency.

d. Coefficient of variation

$$CV_{ech}(\%) = \frac{\sigma_{ech}}{\bar{x}} \text{ x } 100 = \frac{0.66}{19.42} \text{x} 100 = 3.4\ \%$$

The coefficient of variation is therefore 3.4%.

This means that the dispersion of diameters compared to the average is relatively small. In other words, the values of diameters are quite close to the average, which is generally good for quality control.

5. Relative frequencies (f_i) and increasing cumulative headcount

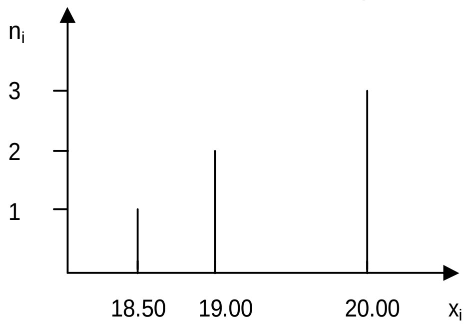
a. Relative frequencies $f_i = \frac{n_i}{K}$ avec K=n

- f_1 = 1/6 = 0.16 (16% des pièces ont un diamètre de 18.50 mm)
- f_2= 2/6 = 0.33 (33% des pièces ont un diamètre de 19.00 mm)
- f_3 = 3/6 = 0.5 (50% des pièces ont un diamètre de 20.00 mm)

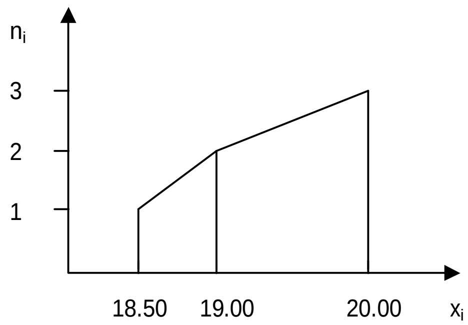
b. Increasing cumulative headcount $n_j^{c\uparrow} = n_1 + n_2 + \cdots + n_j$

$$n_1^{c\uparrow} = n_1 = 1$$
$$n_2^{c\uparrow} = n_1 + n_2 = 1 + 2 = 3$$
$$n_3^{c\uparrow} = n_1 + n_2 + n_3 = 1 + 2 + 3 = 6$$

6. Representation of the bar diagram and the polygon of headcounts.



Bar Diagram



Polygon des effectifs