11/05/2025

# * Exam Correction of : telecom 1 *

## Answers of the questions :

1) In analog modulation, the amplitude modulation is the process of varying the amplitude of the Cosine (Sine) signal (the Carrier Signal) according to the variation of our data signal (modulating signal) to get the modulated signal which is suitable for our transmission channel (medium)

(1 pts)

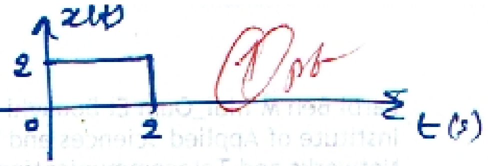
2) The analog modulation is necessary for the transmission due to:

1 – the channel caracteristics: for each channel (medium), we need a suitable signal caracteristics to ensure the transmission like: the frequency, Generated Power Pe, ... — (1 pt)

2 – (Heigh length of ~~the~~ antenna): to cover a long distance, we need a high frequency signal (ie: short wave lenght λ). For an antenna $\frac{\lambda}{4}$, it will be easy and practical ~~technique, that is~~ that's why for a low frequency signals, we should carried them by a high frequency signals (Carrier signals) (1 pt)

3 – Radiated Power: As we know, radiated Power is depending on the frequency parameter ($P = f(\lambda)$). to get a higher P, we need to employ a lower wave lenght ($\lambda$) => high frequency. That means, we need to carrier out our LF signal by a HF signal as a carrier signals to achieve a high generated radiated Power. (1 pt)

**Exercise 01:** I) $x(t) = 2u(t) - 2u(t-2) \Rightarrow$ a)




(1 pt)

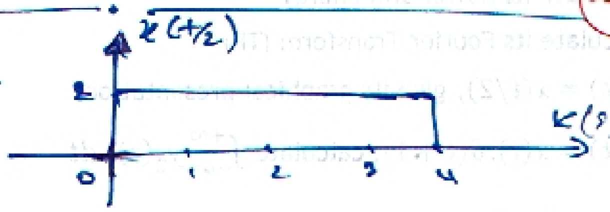
b) our signal is for a finite duration (borné) $\Rightarrow$ Energy signal (1 pt)

c) $E = \int_{-T}^{T} x^2(t)\, dt = \int_0^2 2^2 . dt = 8\,J \quad \left(P = 0 = \lim_{T \to \infty} \frac{E}{T} = 0\,W\right)$ (2 pts)

d) $FT(x(t)) = X(f) = \int_{-\infty}^{+\infty} x(t) . e^{-j2\pi f t} . dt = \int_0^2 2 . e^{-j2\pi f t} . dt = \frac{j}{\pi f}\left(e^{-j4\pi f} - 1\right)$

$= \frac{1}{\pi f} . e^{-j2\pi f}\left(e^{-j2\pi f} - e^{j2\pi f}\right) = \frac{2}{\pi f} . e^{-j2\pi f} . \sin 2\pi f = 4 . \mathrm{Sinc}\, 2\pi f . e^{-j2\pi f}$ (2 pts)

e) $y_1(t) = x(t/2) \rightarrow$




(1 pt)

f) $y_2(t) = x(t) . \delta(t+1) \rightarrow \int_{-\infty}^{+\infty} y_2(t) . dt = x(-1) = 0$ (1 pt)

**Exercise 02:** $f(t) = e^{-\pi t^2} \xrightarrow{F.T} F(f) = e^{-\pi f^2}$

a) $G(f) = FT(g(t)) = \int_{-\infty}^{+\infty} t . e^{-\pi t^2} . e^{-j2\pi f t} . dt$

but: $g(t) = \frac{-1}{2\pi} f'(t) = \frac{-1}{2\pi} \frac{\partial f(t)}{\partial t}$

$\Rightarrow G(f) = \frac{-1}{2\pi} . (j2\pi f) . F(f)$

(Linearity + Derivation Propriety)

$\Rightarrow G(f) = -jf . e^{-\pi f^2}$ (1,5 pt)

b) $H(f) = ?$ $h(t) = f(2t)$

$\Rightarrow H(f) = \frac{1}{2} F\left(\frac{f}{2}\right) = \frac{1}{2} e^{-\frac{\pi}{4} f^2}$ (1,5 pt)

(Domain Contraction Propriety)

c) $s(t) \xrightarrow{F.T} S(f)$

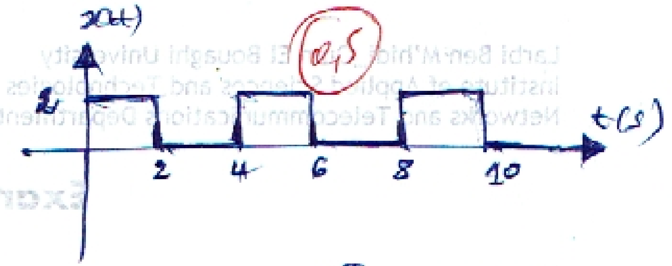
$s(t) = f(t-2) \Rightarrow S(f) = F(f) . e^{-j4\pi f}$

$S(f) = e^{-\pi f^2} . e^{-j4\pi f}$ (1,5 pt)

(temporal translation Propriety)

* **Exo 1 :** 03,5 pts

II) a) $x(t)$ periodic with $T_0 = 4s$ →




b) the FS of $x(t) = FS(x(t))$

$$x(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos 2\pi n f_0 t + b_n \sin 2\pi n f_0 t$$

Or
$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cos \frac{n\pi t}{L} + b_n \sin \frac{n\pi t}{L}$$

when : $L = \frac{T_0}{2}$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t).\cos 2\pi f_0 n t\,dt$$

$$b_n = \frac{1}{T_0}\int_{T_0} x(t).\sin 2\pi f_0 t\,dt$$

• $a_0 = \frac{1}{4}\int_0^4 2.\,dt = \frac{1}{2}\left(t\Big|_0^2\right) = 1$ 0,5 pt

• $a_n = \frac{1}{4}\int_0^4 2.\cos\frac{2\pi}{4}t\,dt = \frac{1}{2}\cdot\frac{2}{n\pi}\left(\sin n\pi - \sin 0\right) = 0$ 1 pts

• $b_n = \frac{1}{4}\int_0^4 2.\sin\frac{2\pi}{4}t\,dt = \frac{1}{2}\cdot\frac{(-2)}{n\pi}\left(\cos\frac{n\pi}{2}t\Big|_0^2\right) = \frac{-1}{n\pi}\left(\cos n\pi - \cos 0\right)$ 1 pts

$$b_n = \frac{1}{n\pi}\left(1 - \cos n\pi\right)$$

$$\Rightarrow x(t) = 1 + \sum_{n=1}^{+\infty} \frac{1}{n\pi}\left(1 - \cos n\pi\right)\left(\sin\frac{n\pi}{2}t\right)$$ 0,5 pts