Larbi Ben M'hidi University
Physical measurements 1st Year

Sciences and applied technics institute
Ain M'lila-Oum El Bouaghi

Ain M'lila 14/05/2025

Exam of mathematics 2 *(duration 1h30min)*

Exercise 1. *(7 pts)*

1. Use the numbers (called nodes) $x_0 = \mathbf{1.8}$, $x_1 = \mathbf{2.5}$ and $x_2 = \mathbf{3.4}$ to find the second Lagrange interpolating polynomial for $f(x) = \frac{1}{\sqrt{x}}$

2. Use this polynomial to approximate f(**2.9**), compare with the exact value

Exercise 2. *(6 pts)*

1. Construct the vector field of : $\vec{v}_1(x,y) = 2x\vec{i} + y\vec{j}$, $\vec{v}_2(x,y) = x\vec{i} - 2y\vec{j}$,

$$\vec{v}_3(x,y) = y\vec{i} + 0.5x\vec{j} \text{ , } \vec{v}_4(x,y) = 0.5y\vec{i} - x\vec{j}$$

Exercise 3. *(7 pts)*

1. Let have the matrix **A** as described below and calculate the value of x if the determinant is equal to **20.**

$$A = \begin{pmatrix} 2 & 3 & -5.5 \\ x & 4 & 1.5 \\ 1 & -2 & 2.5 \end{pmatrix}$$

2. Calculate the determinant of $B = \begin{pmatrix} 7 & 3 \\ 5 & 4 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 3 \end{pmatrix}$, can we perform the product BC and CB?

Exercise 01 (07 points / 07)

1. $f(x_0) \simeq 0,7454$; $f(x_1) = 0,6325$; $f(x_2) = 0,5423$ (0,5pt) (0,5pt) (0,5pt)

$P_2(x) = f(x_0).l_0(x) + f(x_1).l_1(x) + f(x_2).l_2(x)$ (0,5pt)

$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$ (0,5pt) and $l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$ (0,5pt)

$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$ (0,5pt) (*)

(0,5pt) $l_0(x) = \frac{(x-2,5)(x-3,4)}{1,12}$ ✓✓ $l_1(x) = \frac{(x-1,8)(x-3,4)}{-0,63}$ ✓ (0,5pt)

(*) $l_2(x) = \frac{(x-1,8)(x-2,5)}{(1,6)(0,9)} = \frac{(x-1,8)(x-3,5)}{1,44}$ ✓ (0,5pt)

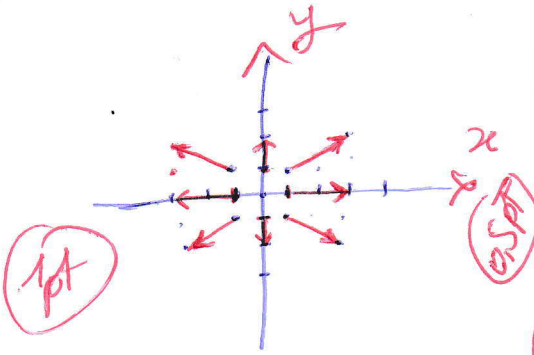
$P_2(x) \simeq 0,0382x^2 - 0,3256x + 1,2076$ (1pt)

2. $f(2,9) = \frac{1}{\sqrt{2,9}} = 0,5872$ (0,5pt) $f(2,9) \simeq 0,5848$ (0,5pt)

Absolute error is $= 0,0025$

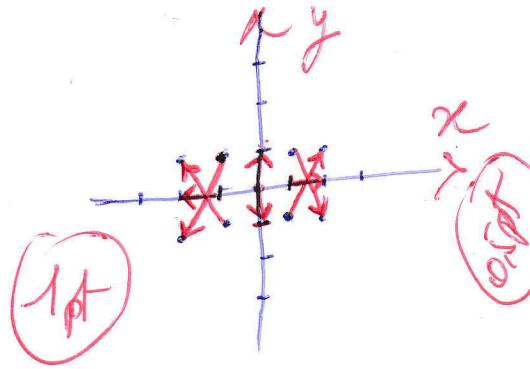
Exercice 02 (6pt/6)

$\vec{v_1}(x,y) = 2x\vec{i} + y\vec{j}$



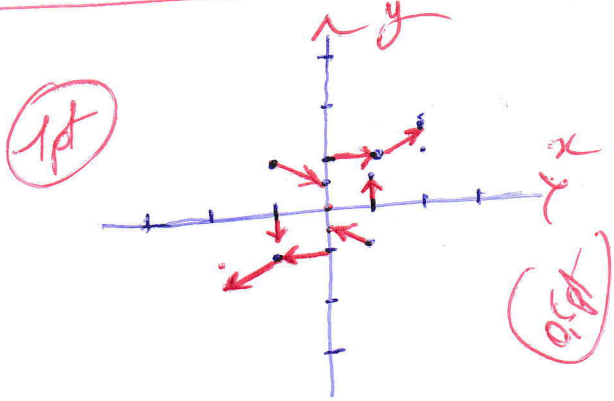
x	y	v_1
1	0	$2i$
0	1	j
-1	0	$-2i$
0	-1	$-j$
1	1	$2i+j$
-1	1	$-2i+j$
+1	-1	$2i-j$
-1	-1	$-2i-j$

$\vec{v_2}(x,y) = x\vec{i} - 2y\vec{j}$



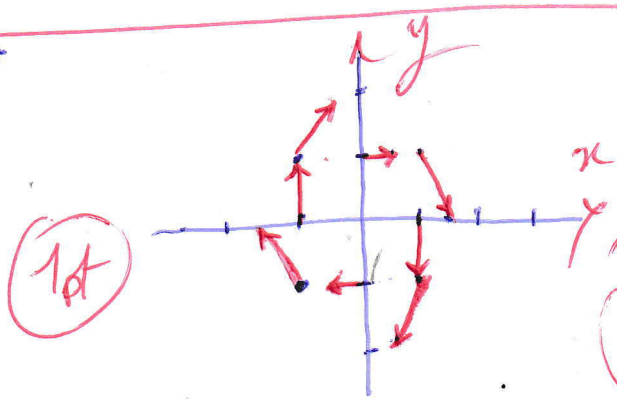
x	y	v_2
1	0	i
0	1	$-2j$
-1	0	$-i$
0	-1	$2j$
1	1	$i-2j$
-1	1	$-i-2j$
+1	-1	$i+2j$
-1	-1	$-i+2j$

$\vec{v_3}(x,y) = y\vec{i} + 0,5x\vec{j}$



x	y	v_3
1	0	$0,5j$
0	1	i
-1	0	$-0,5j$
0	-1	$-i$
1	1	$i+0,5j$
-1	1	$i-0,5j$
1	-1	$-i+0,5j$
-1	-1	$-i-0,5j$

$\vec{v_4}(x,y) = 0,5y\vec{i} - x\vec{j}$



x	y	v_4
1	0	$-j$
0	1	$0,5i$
-1	0	j
0	-1	$-0,5i$
1	1	$0,5i-j$
-1	1	$0,5i+j$
1	-1	$-0,5i-j$
-1	-1	$-0,5i+j$

Exercice 03 (7pt/7)

1. $A = \begin{pmatrix} 2 & 3 & -5,5 \\ x & 4 & 1,5 \\ 1 & -2 & 2,5 \end{pmatrix}$ $\det(A) = 20$

$\det(A) = 2(10 - (-3)) - 3(2,5x - 1,5) + (-5,5)(-2x - 4)$ (1pt)

$= 26 - (7,5x - 4,5) + (11x + 22)$

(1pt) $20 = 52,5 + 3,5x \Rightarrow x = \frac{-32,5}{3,5} = -\frac{65}{7}$ ✓

$x = -9,29$ ✓ (2pt)

2. $B = \begin{pmatrix} 7 & 3 \\ 5 & 4 \end{pmatrix}$ $\det(B) = 28 - 15 = 13$ ✓ (1,0pt)

$C = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 3 \end{pmatrix}$ $\det(C) = 1.\underbrace{\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}}_{(3-2)} - 2\underbrace{\begin{vmatrix} 2 & 2 \\ 3 & 3 \end{vmatrix}}_{(6-6)} + 1\underbrace{\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}}_{(2-3)}$ (1pt)

$\det(C) = 1 - 0 - 1 = 0$

$B \times C \rightarrow$ impossible (1pt)

$C \times B \rightarrow$ impossible (1pt)