Corrigé type session normale2024/2025 : Math 02

$$C= \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 5 & 1 \end{pmatrix}$$

$$C^t=\begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix}$$

02pts

Exercice 01

$$\begin{bmatrix} 4 & 8 & 12 \\ 3 & 8 & 13 \\ 2 & 9 & 18 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 11 \end{bmatrix}$$

1) calculer le déterminant de la matrice

$$\det\begin{bmatrix} 4 & 8 & 12 \\ 3 & 8 & 13 \\ 2 & 9 & 18 \end{bmatrix} = 16$$

01pts

$$x1 = \frac{\det\begin{bmatrix} 4 & 8 & 12 \\ 5 & 8 & 13 \\ 11 & 9 & 8 \end{bmatrix}}{16} = \frac{16}{16} = 1$$

02pts

$$x2 = \frac{\det\begin{bmatrix} 4 & 4 & 12 \\ 3 & 5 & 3 \\ 2 & 11 & 8 \end{bmatrix}}{16} \frac{-48}{16} = -3$$

02pts

$$x3 = \frac{\det\begin{bmatrix} 4 & 8 & 4 \\ 3 & 8 & 5 \\ 2 & 9 & 11 \end{bmatrix}}{16} \frac{32}{16} = 2$$

02pts

Exercice 02 :

01pts 01pts 01pts 01pts

$$\frac{4x^3 + 11x^2 + 13x - 6}{(x^2 - 1)(x^2 + 4x + 6)} = \frac{c_1}{x-2} + \frac{c_2}{x+2} + \frac{c_3 x + c_4}{x^2 - 4x + 5}$$

$$\frac{1}{2(x+2)} - \frac{1}{2(x-2)} + \frac{11x - 19}{x^2 - 4x + 5}$$

$$I = \int \frac{11x^3 - 21x^2 - 36x + 66}{(x^2-4)(x^2-4x+5)} dx$$

$$= \frac{1}{2}\ln(x+2) - \frac{1}{2}\ln(x-2) + \frac{11}{2}\ln\left|x^2 - 4x + 5\right| + 3\arctan(x-2)$$

01pts 01pts 1.5pts 1.5pts

Exercice 04 :

$$I_{n+2} = \int_0^{\frac{\pi}{2}} \sin^{n+1} x \cdot \sin x \, dx.$$

En posant $u(x) = \sin^{n+1} x$ et $v'(x) = \sin x$ et en intégrant par parties nous obtenons

$$\begin{aligned} I_{n+2} &= \Big[-\cos x \sin^{n+1} x\Big]_0^{\frac{\pi}{2}} + (n+1)\int_0^{\frac{\pi}{2}} \cos^2 x \sin^n x \, dx \\ &= 0 + (n+1)\int_0^{\frac{\pi}{2}} (1-\sin^2 x)\sin^n x \, dx \\ &= (n+1)I_n - (n+1)I_{n+2}. \end{aligned}$$

Donc $(n+2)I_{n+2} = (n+1)I_n$. Conclusion

$$I_{n+2} = \frac{n+1}{n+2} I_n.$$

02pts