

Exam - Correction

Exo 1 : I) QPSK modulation $\Rightarrow$ 4 levels $\Rightarrow L = 4$

$\Rightarrow \eta = \log_2 L = 2$ (symbol length) $\eta = 2$ bits

The bit rate: $D = 4 \text{ Gbps} = 4 \cdot 10^9 \text{ bps}$

a) The band rate: $R = \frac{D}{\eta} = \frac{4}{2} = 2 \text{ Gband}$ $2 \cdot 10^9 \text{ band}$

b) $T_b = \frac{1}{D} = \frac{1}{4 \cdot 10^9} = 0,25 \cdot 10^{-9} \text{ s} = \text{0,25 nanos}$

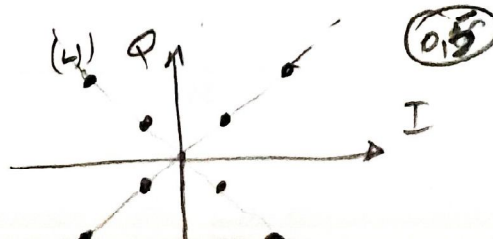
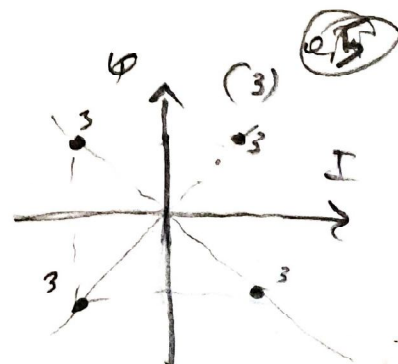
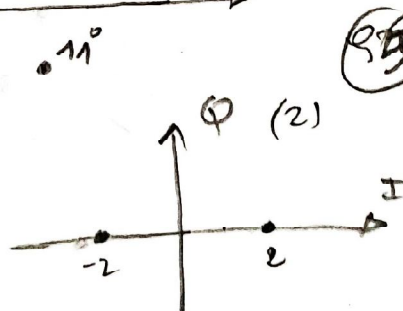
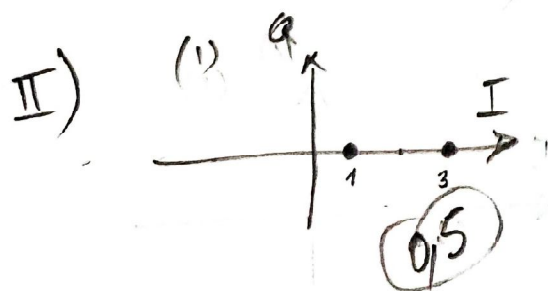
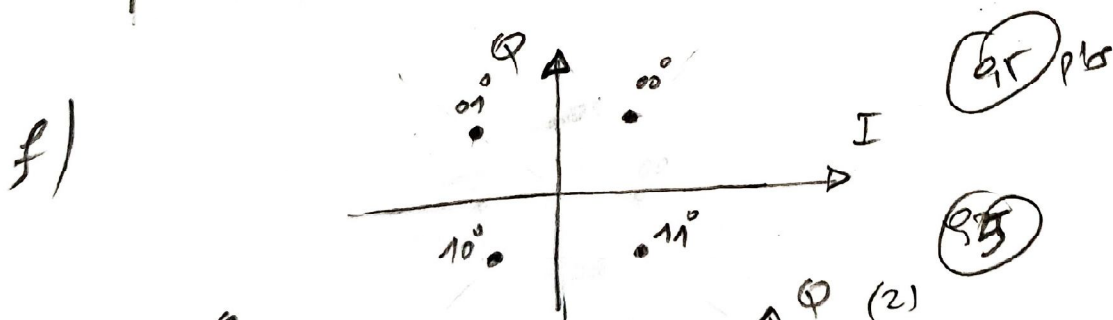
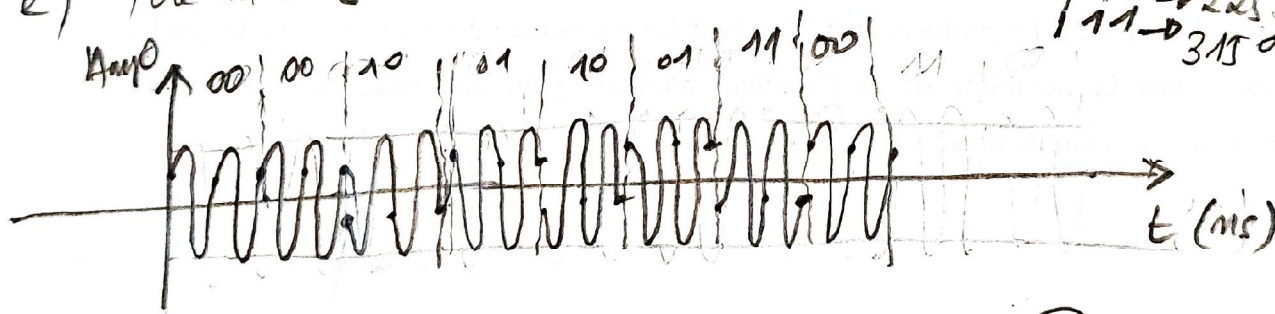
c) $T_s = \frac{1}{R} = \frac{1}{2 \cdot 10^9} = 0,5 \cdot 10^{-9} \text{ s} = \text{0,5 nanos}$

d) $B_{vr} = ?$ $B_{vr} = (1+d) \cdot R$ / $d=0$ (ideal system)

$B_{vr} = (1+0) \cdot 2 \cdot 10^9 = \text{2. GHz}$

e) The message is: 0000 1001 1001 1100

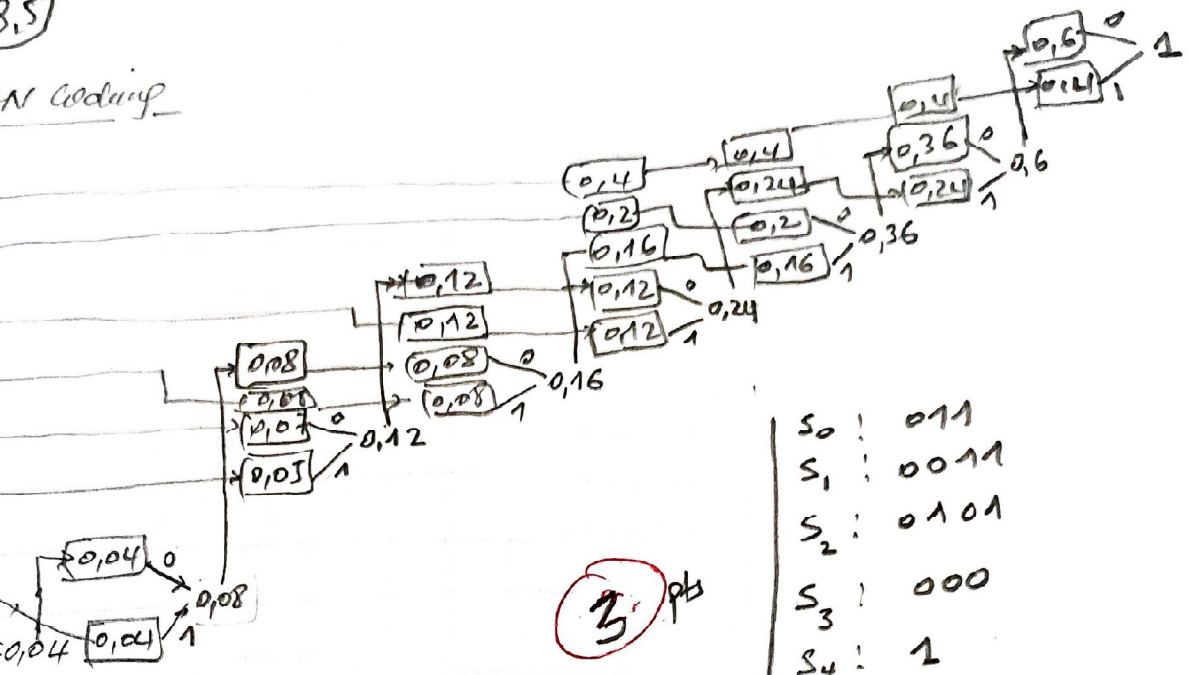
00 $\rightarrow 45^\circ$
01 $\rightarrow 135^\circ$
10 $\rightarrow 225^\circ$
11 $\rightarrow 315^\circ$



Exo 3 (8,5)

1) HUFFMAN coding

- $S_4: 0,40$
- $S_3: 0,2$
- $S_0: 0,12$
- $S_1: 0,08$
- $S_5: 0,07$
- $S_2: 0,05$
- $S_6: 0,04$
- $S_8: 0,03$
- $S_7: 0,01$



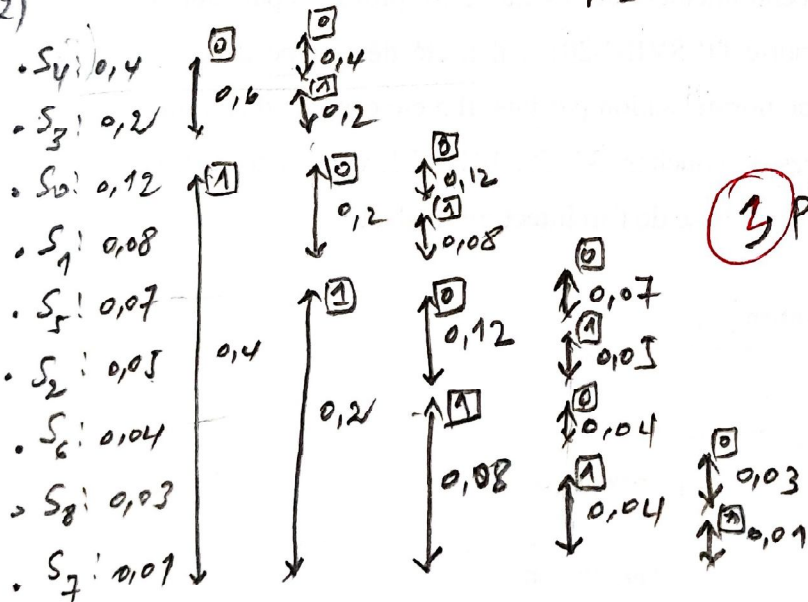
$S_4: 1 / S_3: 000 / S_0: 011 / S_1: 0011 / S_5: 0100$

$S_6: 00101 / S_8: 001000 / S_7: 001001$

$S_2: 0100 / S_2: 0101$

- $S_0: 011$
- $S_1: 0011$
- $S_2: 0101$
- $S_3: 000$
- $S_4: 1$
- $S_5: 0100$
- $S_6: 00101$
- $S_7: 001001$
- $S_8: 001000$

2)



- $S_4: 00$
- $S_3: 10$
- $S_0: 001$
- $S_1: 101$
- $S_5: 0011$
- $S_2: 1101$
- $S_6: 1011$
- $S_8: 0111$
- $S_7: 1111$

- $S_0: 100$
- $S_1: 101$
- $S_2: 1101$
- $S_3: 10$
- $S_4: 00$
- $S_5: 0011$
- $S_6: 0111$
- $S_7: 1111$
- $S_8: 1110$

3) HUFFMAN is prefix, but Shannon Fano is not prefix

$$4) H(x) = \sum_i P_i(x) \cdot \lg \frac{1}{P_i(x)} = - \sum_{i=0}^8 P_i(x) \cdot \lg_2 P_i(x)$$

$$= - [0,22 \lg_2 0,22 + 0,08 \lg_2 0,08 + 0,05 \lg_2 0,05 + 0,2 \lg_2 0,2 + 0,4 \lg_2 0,4 + 0,07 \lg_2 0,07 + 0,04 \lg_2 0,04 + 0,09 \lg_2 0,09 + 0,03 \lg_2 0,03] = 2,54 \text{ bits}$$

$H(x) = 2,54 \text{ bits}$

(1) pt

$$5) \eta = \frac{H(x)}{\bar{L}} \quad \bar{L} = \sum_{i=1}^n L_i P_i(x)$$

• Huffman: $\bar{L}_H = \sum_{i=0}^8 L_i P_i = [3 \times 0,12 + 4 \times 0,08 + 4 \times 0,05 + 3 \times 0,2 + 1 \times 0,4 + 4 \times 0,07 + 5 \times 0,04 + 6 \times 0,01 + 6 \times 0,03]$

$$\boxed{\bar{L}_H = 2,60} \text{ bits}$$

• Shannon-Fano: $\bar{L}_S = [3 \times 0,12 + 3 \times 0,08 + 4 \times 0,05 + 2 \times 0,2 + 2 \times 0,4 + 4 \times 0,07 + 4 \times 0,04 + 5 \times 0,01 + 5 \times 0,03]$

$$\boxed{\bar{L}_S = 2,64} \text{ bits}$$

$$\Rightarrow \eta_H = \frac{H(x)}{\bar{L}_H} = \frac{2,54}{2,60} = 0,977\% \rightarrow \boxed{\eta_H = 97,7\%} \quad (1) \text{ pt}$$

$$\eta_S = \frac{H(x)}{\bar{L}_S} = \frac{2,54}{2,64} = 0,962\% \rightarrow \boxed{\eta_S = 96,2\%} \quad (1) \text{ pt}$$

6) Remark: Huffman coding is most effective than Shannon-Fano (1) pt

Exo 2 (6 pts): The received message v' is 101110
 Coder: Hamming (7,4) $\rightarrow v' = 0101110$ | $k=3, n=7$
 = Syndrome method

* control bits method

1) $C_2' = P(v_1', v_3', v_5', v_7') = P(0, 1, 0, 0) = 2 \neq 0$
 $\Rightarrow v'$ is wrong.

$C_1' = P(v_2', v_3', v_6', v_7') = P(1, 1, 1, 0) = 1$ (3 pts)

$C_2' = P(v_4', v_5', v_6', v_7') = P(1, 0, 1, 0) = 0$

2) The bit having error is (1011) $\rightarrow 3^{\text{rd}}$ bit
 $v_3' = 0101110$ (1 pt)

3) The correct $v' \rightarrow v_c' = 0101010$ (1 pt)

4) $m = 0100$ (2 pts)

1) $v' \times H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $v' \times H^T = 001 \neq 000$
 $\Rightarrow v'$ is wrong.

2) Position 1 $\rightarrow 3^{\text{rd}}$ ligne
 $\rightarrow 3^{\text{rd}}$ Position 0101110

3) $v_c' \rightarrow 0101010$

4) $m = 0100$ (not included)