

Last name & First nameG:.....

written type FINAL EXAM

Course questions: (08 points)

1. Give the expression for the finite energy?

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad (0.5)$$

Give the expression for the finite average power?

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt \quad (1)$$

2. Consider a communication system made up of:

- A transmitter,
- A transmitting antenna,
- A receiving antenna distant d from the transmitting antenna,
- A receiver.

Give the expression for the link budget by giving the expression for P_r (power received by the receiver, including Losses)

$$P_R = P_t + G_t + G_R - \frac{L}{R} - \frac{L}{C} - L_{FS} \quad (1)$$

3. Give the three forms of the Fourier series expansion; with all the details.

• *general form*: $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$ (0.1)
 $a_0 = \frac{1}{T} \int_0^T x(t) dt$, $a_n = \frac{2}{T} \int_0^T x(t) \cos(n\omega_0 t) dt$, $b_n = \frac{2}{T} \int_0^T x(t) \sin(n\omega_0 t) dt$ (0.7)

• *cosine form*: $x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \varphi_n)$ (0.1)

$c_0 = a_0$; $c_n = \frac{a_n + jb_n}{2}$; $\varphi_n = -\arctan\left(\frac{b_n}{a_n}\right)$ (0.1)

• *complex form*: $x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$ (0.1)
 $X_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega_0 t} dt$; $X_0 = a_0$; $X_{-n} = \frac{1}{2} (a_n - jb_n)$; $X_n = \frac{1}{2} (a_n + jb_n)$; $\varphi_n = -\arctan\left(\frac{b_n}{a_n}\right)$ (0.8)

4. Let $m(t)$ be the baseband information signal. The carrier is written as follows:

$$x_p(t) = A_p \sin(2\pi f_p t + \theta_p)$$

a. What is the modified characteristic for AM amplitude modulation?

$$A_p$$

b. What is the modified characteristic for PM phase modulation?

$$\theta_p$$

c. What is the modified characteristic for frequency modulation FM?

$$f_p$$

GOOD LUCK

Ex 3

$$X(f) = \int_{-\infty}^{+\infty} \text{rect}\left(\frac{t}{T}\right) e^{-j2\pi f t} dt = \int_{-T/2}^{T/2} e^{-j2\pi f t} dt$$

$$= -\frac{1}{j2\pi f} \left[e^{-j2\pi f t} \right]_{-T/2}^{T/2} = -\frac{1}{j2\pi f} \left[e^{-j2\pi f T/2} - e^{j2\pi f T/2} \right]$$

$$= \frac{1}{j2\pi f} \left[e^{j\pi f T} - e^{-j\pi f T} \right] = \frac{2j}{2j\pi f} \left[\sin \pi f T \right]$$

$$= \frac{T}{\pi f} \sin(\pi f T) = T \text{sinc}(fT)$$

2/ $x(t) = \sin(2\pi f_0 t)$

$$X(f) = \int_{-\infty}^{+\infty} \sin(2\pi f_0 t) e^{-j2\pi f t} dt$$

$$= \frac{1}{2j} \int_{-\infty}^{+\infty} \left(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t} \right) e^{-j2\pi f t} dt$$

$$= \frac{1}{2j} \int_{-\infty}^{+\infty} \left(e^{-j2\pi(f-f_0)t} - e^{-j2\pi(f+f_0)t} \right) dt$$

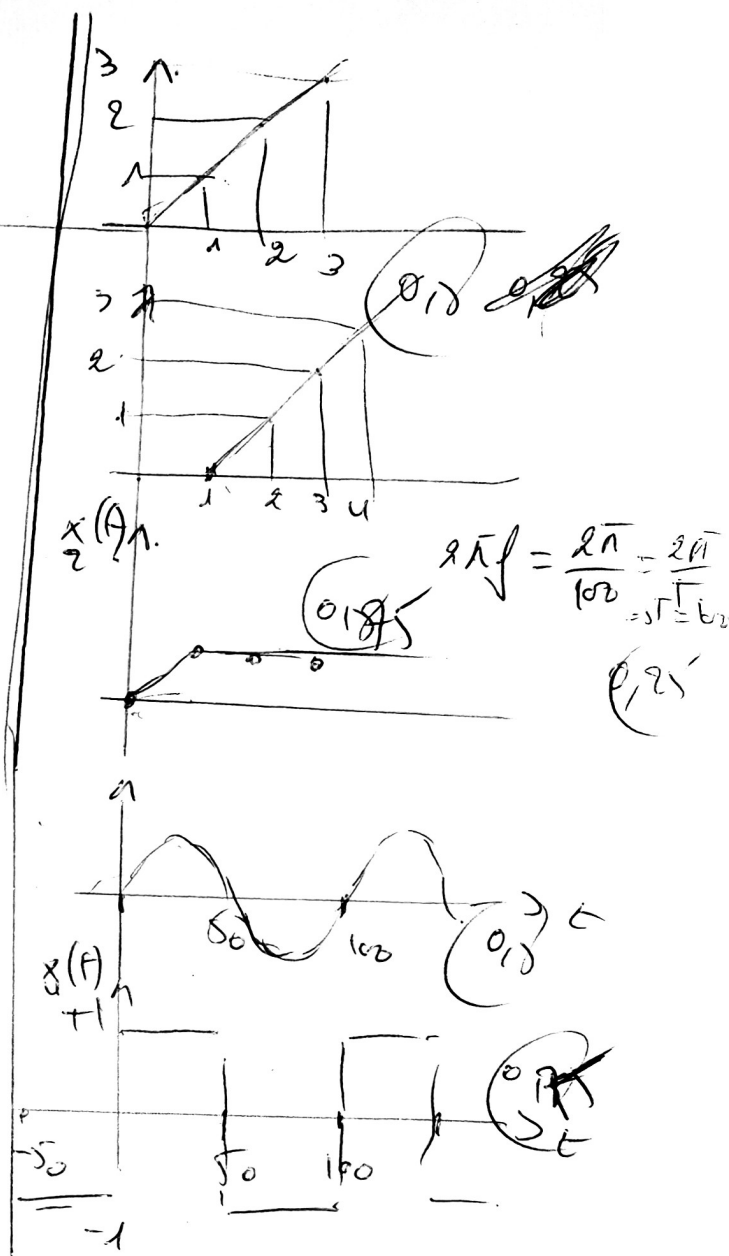
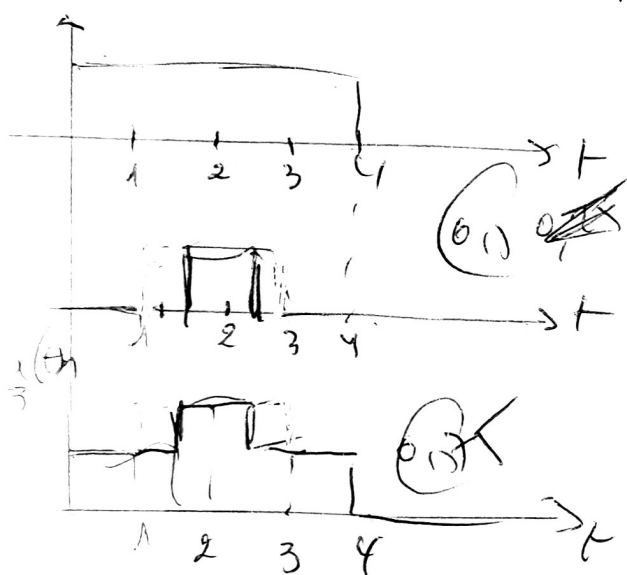
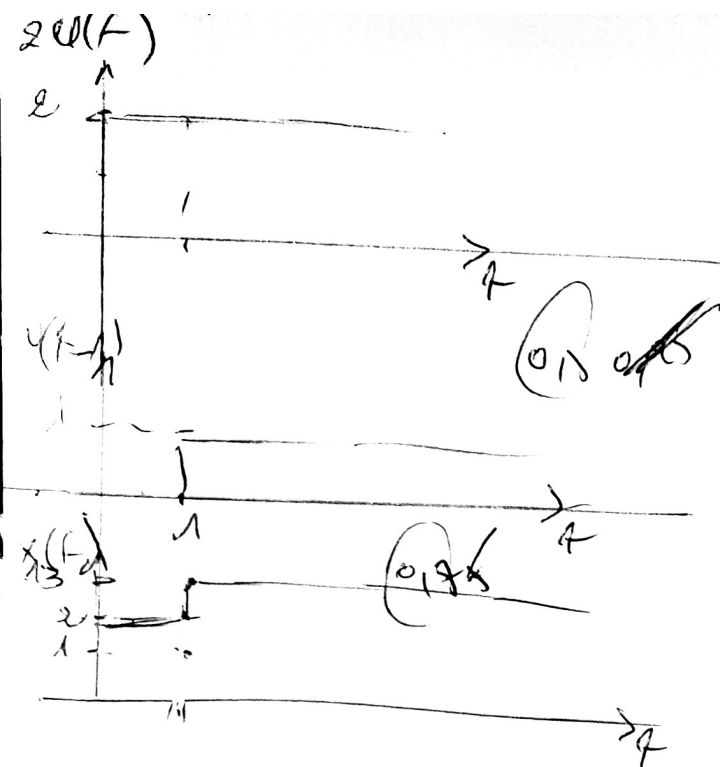
$$= \frac{1}{2j} \left(\delta(f-f_0) - \delta(f+f_0) \right)$$

$$X(f) = \frac{1}{2j} \left(\delta(f-f_0) - \delta(f+f_0) \right) \quad (u)$$

$$3/ \quad x(t) = e^{-\alpha t} u(t); \quad \alpha > 0.$$

$$\begin{aligned} X(f) &= \int_{-\infty}^{+\infty} e^{-\alpha t} u(t) e^{-j2\pi f t} dt \\ &= \int_0^{+\infty} e^{-\alpha t} e^{-j2\pi f t} dt \\ &= \int_0^{+\infty} e^{-(\alpha + j2\pi f)t} dt \\ &= -\frac{1}{\alpha + j2\pi f} e^{-(\alpha + j2\pi f)t} \Big|_0^{+\infty} \end{aligned}$$

$$X(f) = \frac{1}{\alpha + j2\pi f} [0 - 1] = \frac{1}{\alpha + j2\pi f} \quad (1)$$

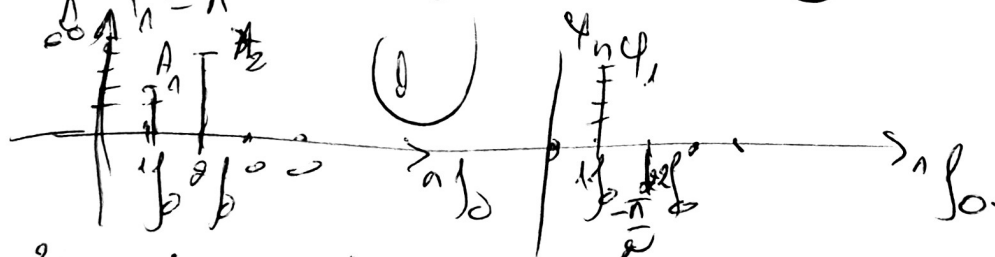


Ex 3

$$x(t) = 6 - 2 \cos(2\pi f_0 t) + 3 \cos(4\pi f_0 t - \frac{\pi}{2}) \quad (1)$$

$$A_0 = 6, A_1 = 2; A_2 = 3; A_3 = 0, A_4 = 0 \quad (0.15)$$

$$\varphi_0 = 0; \varphi_1 = \pi; \varphi_2 = -\frac{\pi}{2}; \varphi_3 = 0 \quad (0.15)$$



$$P_x = \frac{1}{2} \sum (a_n^2 + b_n^2) = 6^2 + \frac{1}{2} (4 + 9) = 36 + 6.5 = 42.5 \quad (1)$$