

Module: coding and compression

First & Last Name :

First Exam

Exercise 01 (Course question): (08 points)

A. Choose the correct answer(s)

- What are the properties of entropy?
 - Positive
 - Negative
 - All the possibilities
- In the process of transmitting information, the noise is the disruptive element. In which part of a transmission chain?
 - Source
 - Code
 - User
 - Channel
 - Decode
- For which value the entropy will be maximum.
 - $p_i = 1/n$
 - $p_i = 1$
 - $p_i = 0$
 - $p_i = 1 + \epsilon$
 - $p_i = 1 - \epsilon$
- In variable size source encoding (VLC), a symbol with minimum probability will have the fewest bits.
 - TRUE
 - False
- A Huffman encoder takes a set of characters of fixed length and produces a set of characters of:
 - Variable length,
 - Random length,
 - Fixed length,
 - Constant length
- For which value(s) of p is the binary entropy function $H(p)$ maximized?
 - 0
 - 0.5
 - 1
 - 0.8

B. Give a block diagram of the different stages of Jpeg data compression



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Exercise 02: 04 points

Consider a linear code $C(n, k)$ whose generating matrix is given below

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

1. Find the length of the message (source) blocks, the length of the codewords and the number of parity bits
2. Deduce the control matrix H
3. find the code words
4. Deduce the minimum distance $d_{\min}$, the number of errors (e) that the code can detect and can correct.

Exercise 03: 04 points

A discrete channel has as input the alphabet $X = \{a, b, c\}$ and the output alphabet $Y = \{r, s, t, u\}$. Its transition matrix is as follows:

$$Q = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

What is its capacity?

Exercise 04: 04 points

Using arithmetic coding, code the word CALCUL

Note: take 4 digits after the decimal point

GOOD LUCK

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Exercise 01 (Course question): (08 points)

A. Choose the correct answer(s)

1. What are the properties of entropy?

- ☒ a. Positive
- b. Negative
- d. All the possibilities

2. In the process of transmitting information, the noise is the disruptive element. In which part of a transmission chain?

- a. Source
- b. Code
- c. User
- ☒ d. Channel
- e. Decode

3. For which value the entropy will be maximum.

- ☒ a. $p_i = 1/n$
- b. $p_i = 1$
- c. $p_i = 0$
- d. $p_i = 1 + \epsilon$
- e. $p_i = 1 - \epsilon$

4. In variable size source encoding (VLC), a symbol with minimum probability will have the fewest bits.

- a. TRUE
- ☒ b. False

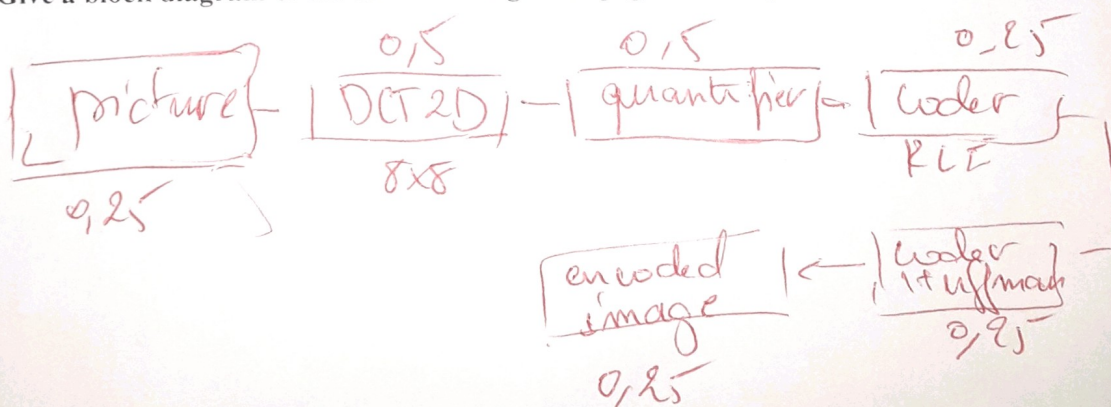
5. A Huffman encoder takes a set of characters of fixed length and produces a set of characters of:

- ☒ a) Variable length,
- b) Random length,
- c) Fixed length,
- d) Constant length

6. For which value(s) of p is the binary entropy function $H(p)$ maximized?

- a. 0
- ☒ b. 0.5
- c. 1
- d. 0.8

B. Give a block diagram of the different stages of Jpeg data compression



Exercice 02 :

$$G_k = \begin{bmatrix} \overbrace{1 \ 0 \ 0}^n & 1 \ 1 \ 1 \\ \overbrace{0 \ 1 \ 0}^n & 1 \ 0 \ 1 \\ \overbrace{0 \ 0 \ 1}^n & 0 \ 0 \ 1 \end{bmatrix} = \begin{bmatrix} I & P \end{bmatrix}$$

1/ $C(n, k) \Rightarrow n=6, k=3 \Rightarrow R=6-3=3$ (0,75)

2/ $H = \begin{bmatrix} P^T & I_{n-k} \end{bmatrix} = \begin{bmatrix} 1 \ 1 \ 0 & 1 \ 0 \ 0 \\ 1 \ 0 \ 1 & 0 \ 1 \ 0 \\ 1 \ 1 \ 1 & 0 \ 0 \ 1 \end{bmatrix}$
(0,5) (0,5)

3/ $C_D = D \cdot G$ (0,25)

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \quad (1)$$

4/ $d_{\min} = 2$; $e = d_{\min} - 1 = 1$ error.
can't correct any error.

correction = detection

Exercise 03:

$$Q = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

$$H(Q) = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix} \text{ ; the matrix is symmetrical. } \textcircled{0,15}$$

$$P(a) = P(b) = P(c) = \frac{1}{3} \textcircled{0,25}$$

$$C = \max I(X; Y) \textcircled{0,5}$$

$$I(X; Y) = H(Y) - H(Y|X) \textcircled{0,25}$$

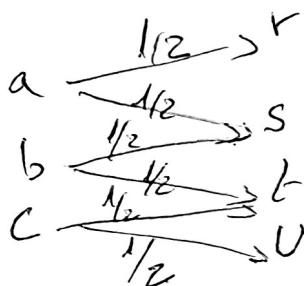
$$H(Y) = - \sum_i P_{y_i} \log_2 \left(\frac{1}{P_{y_i}} \right) \textcircled{0,25}$$

$$P_{y_s} = \frac{1}{2} P(x_a) = \frac{1}{6} \textcircled{0,25}$$

$$P_{y_s} = \frac{1}{2} (P_{x_a} + P_{x_b}) = \frac{1}{3} \textcircled{0,25}$$

$$P_{y_c} = \frac{1}{2} (P_{x_b} + P_{x_c}) = \frac{1}{3} \textcircled{0,25}$$

$$P_U = \frac{1}{2} P(x_c) = \frac{1}{6} \textcircled{0,25}$$



$$H(Y) = 2 \cdot \frac{1}{6} \log_2(6) + 2 \cdot \frac{1}{3} \log_2(3) = \frac{1}{3} [\log_2 6 + 2 \log_2 3] = 1,92 \textcircled{0,25}$$

$$H(Y|X) = \sum P(x_i) H(Y|x_i) \textcircled{0,25}$$

$$= P(x_a) H(Y|x_a) + P(x_b) H(Y|x_b) + P(x_c) H(Y|x_c)$$

$$H(Y|x) = \frac{1}{3} [6 \cdot \frac{1}{2} \log_2(2)] = 1 \textcircled{0,25}$$

$$C = 1,92 - 1 = 0,92 \textcircled{0,15}$$

Exercise 4;

$$CALCUL \rightarrow C \xrightarrow{0,25} \frac{1}{3}, A \xrightarrow{0,25} \frac{1}{6}, L \xrightarrow{0,25} \frac{1}{3}, U \xrightarrow{0,25} \frac{1}{6}$$

$$C \rightarrow [0,00 ; 0,33] \quad 0,25$$

$$A \rightarrow [0,33 ; 0,50] \quad 0,25$$

$$L \rightarrow [0,50 ; 0,83] \quad 0,25$$

$$U \rightarrow [0,83 ; 1,00] \quad 0,25$$

$$B_{sup} = B_{inf-1} + (B_{S-1} - B_{inf-1}) \cdot B_{sup \text{ caractere.}} \quad (0,25)$$

$$B_{inf} = B_{inf-1} + (B_{S-1} - B_{inf-1}) \cdot B_{I \text{ caractere.}} \quad (0,25)$$

$$[0,0000 \quad 1,0000] \quad \text{over}$$

$$C \quad [0,0000 \quad 0,3333] \quad 0,25$$

$$A \quad [0,1111 \quad 0,1667] \quad 0,25$$

$$L \quad [0,1389 \quad 0,1574] \quad 0,25$$

$$C \quad [0,1389 \quad 0,1451] \quad 0,25$$

$$U \quad [0,1440 \quad 0,1451] \quad 0,25$$

$$L \quad [0,1451 \quad \underline{0,1449}] \quad 0,25$$

$$CALCUL \rightarrow 0,1449$$