وزارة التعليم العالي و البحث العلمي
جامعة العربي بن مهيدي - أم البواقي

Nom : .... :الاسم
Prénom : .... :اللقب
N° d'Inscription: .... :رقم التسجيل
Faculté/ Institut : Corrigé de l'examen .... :كلية / معهد
Département: .... :قسم
امتحان رقم : .... Algèbre 01 .... :مقياس
Année: .... :السنة Date: .... :بتاريخ
.... 2025 - 2026. .... :القسم و الفوج

خاص بالإدارة

Exercise 01 : (06 pts)

$f : \underset{A}{[-1,1]} \longrightarrow \underset{B}{[-1,1]}$

$x \longmapsto f(x) = \dfrac{2x}{x^2+1}$

01 - a - Calculate $f^{-1}(\{1\})$

$f^{-1}(B_0) = \{x \in A ; f(x) \in B_0\}$ (0,25)

$= \{x \in [-1,1] ; f(x) \in \{1\}\}$

$f(x) \in \{1\} \Leftrightarrow f(x) = 1 \Leftrightarrow \dfrac{2x}{x^2+1} = 1$ (0,25)

$\Leftrightarrow x^2+1-2x = 0 \Leftrightarrow (x-1)^2 = 0 \Leftrightarrow x = 1$ (0,25)

So $f^{-1}(\{1\}) = \{1\}$ (0,25)

b - Show that f is an injective function:

f is injective : $\forall x_1, x_2 \in A$: $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$

Let $x_1, x_2 \in [-1,1]$ st $f(x_1) = f(x_2)$, then:

$\dfrac{2x_1}{x_1^2+1} = \dfrac{2x_2}{x_2^2+1} \Leftrightarrow x_1 - x_2 + x_1 x_2^2 - x_2 x_1^2 = 0$ (1,25)

$\Leftrightarrow (x_1 - x_2)(1 - x_1 x_2) = 0$

So $x_1 - x_2 = 0$ or $1 - x_1 x_2 = 0$

So $x_1 = x_2$ or $x_1 = \dfrac{1}{x_2}$; since $x_1, x_2 \in [-1,1]$ then x_1 must equal to x_2 and equals to 1 or -1.

Conclusion : f is injective

2nd Method

($x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$)

Let $x_1, x_2 \in [-1,1]$ st $x_1 \neq x_2$ then $x_1^2 \neq x_2^2 \Rightarrow x_1^2+1 \neq x_2^2+1$

$\Rightarrow \dfrac{2x_1}{x_1^2+1} \neq \dfrac{2x_2}{x_2^2+1}$

02 - $\forall x, y \in \mathbb{R}^*$; $x \mathcal{R} y \Leftrightarrow \frac{x}{y} = \frac{y}{x}$.

- $\mathcal{R}$ is an equivalence relation over $\mathbb{R}^*$.

• $\mathcal{R}$ is reflexive : $\forall x \in \mathbb{R}^*$: $x \mathcal{R} x$? 0,25

let $x \in \mathbb{R}^*$; we have $\frac{x}{x} = 1$ so $\frac{x}{x} = \frac{x}{x} = 1 \Leftrightarrow x \mathcal{R} x$.

$x \mathcal{R} x \Leftrightarrow \frac{x}{x} = \frac{x}{x}$ 0,75

• $\mathcal{R}$ is symmetric : $\forall x, y \in \mathbb{R}^*$: $x \mathcal{R} y \Leftrightarrow y \mathcal{R} x$? 0,25

let x and $y \in \mathbb{R}^*$ st $x \mathcal{R} y$ so $\frac{x}{y} = \frac{y}{x}$. 0,5

Thus $\frac{y}{x} = \frac{x}{y} \Leftrightarrow y \mathcal{R} x$.

$y \mathcal{R} x \Leftrightarrow \frac{y}{x} = \frac{x}{y}$.

• $\mathcal{R}$ is transitive : $\forall x, y, z \in \mathbb{R}^*$: $x \mathcal{R} y \wedge y \mathcal{R} z \Rightarrow x \mathcal{R} z$? 0,25

let $x, y, z \in \mathbb{R}^*$ st $x \mathcal{R} y \Leftrightarrow \frac{x}{y} = \frac{y}{x}$
$\wedge$ $y \mathcal{R} z \Leftrightarrow \frac{y}{z} = \frac{z}{y}$ 1,5

$x \mathcal{R} z \Leftrightarrow \frac{x}{z} = \frac{z}{x}$.

Now $\frac{x}{z} = \frac{x}{y} \cdot \frac{y}{z} = \frac{y}{x} \cdot \frac{z}{y} = \frac{z}{x} \Leftrightarrow x \mathcal{R} z$.

Conclusion : $\mathcal{R}$ is an equivalence relation over $\mathbb{R}^*$.

Exercise 02 : (09 pts) $z_1 = \frac{3+6i}{3-i}$; $z_2 = \frac{2+5i}{-i+1} + \frac{2-5i}{i+1}$.

01- Write z_1 in the algebraic form ($z_1 = a + bi$)

$z_1 = \frac{3+6i}{3-i} = \frac{(3+6i)(3+i)}{(3-i)(3+i)} = \frac{9+18i+3i-6}{3^2+1^2} = \frac{3+21i}{10} = \frac{3}{10} + \frac{21}{10} i$ 0,25 1 0,25

02- Write z_2 in the trigonometric form : $z_2 = r(\cos\theta + i \sin\theta)$

$z_2 = \underbrace{\frac{2+5i}{1-i}}_{w_1} + \underbrace{\frac{2-5i}{1+i}}_{w_2}$; $w_1 = \frac{2+5i}{1-i} = \frac{(2+5i)(1+i)}{(1-i)(1+i)} = \frac{2+2i+5i-5}{1^2+1^2} = \frac{-3}{2} + \frac{7}{2} i$ 0,25 0,25 0,75

$w_2 = \frac{2-5i}{1+i} = \frac{(2-5i)(1-i)}{(1+i)(1-i)} = \frac{2-5i-2i-5}{1^2+1^2} = \frac{-3}{2} - \frac{7}{2} i$ 0,25 0,75

1

So $z_2 = w_1 + w_2 = -\frac{3}{2} + \cancel{\frac{7}{2}i} + \frac{-3}{2} - \cancel{\frac{7}{2}i} = -\frac{6}{2} = -3 = \underset{a}{-3} + \underset{b}{0}i$.

• $r = \sqrt{a^2+b^2} = \sqrt{(-3)^2+0} = 3$. 0,25 0,25

$\theta = ?$ $\begin{cases} \cos\theta = \frac{a}{r} = \frac{-3}{3} = -1 \\ \sin\theta = \frac{b}{r} = \frac{0}{3} = 0 \end{cases}$ 0,25 0,25 0,25 $\Rightarrow \theta = \pi$ 0,25

$\Rightarrow z_2 = 3(\cos\pi + i \sin\pi)$ 0,25 0,25

Scanned By Camera Scanner

• find Z_2^4 :

Method 01 : Using De Moivre's formula : Since $Z_2 = 3(\cos\pi + i\sin\pi)$

Then $Z_2^4 = (3(\cos\pi + i\sin\pi))^4 = 3^4(\cos 4\pi + i\sin 4\pi) =$

$= 3^4(1 + i0) = (3^2)^2 = (9)^2 = 81$ 1

Method 02 : $Z_2 = -3 \Leftrightarrow Z_2^4 = (-3)^4 = ((-3)^2)^2 = 9^2 = 81$

02 - Solve : $\underbrace{1}_{a}z^2 \underbrace{-(1+2i)}_{b} z + \underbrace{i-1}_{c} = 0$ --- $P(z)$

$\Delta = b^2 - 4ac = (-(1+2i))^2 - 4(1)(i-1)$ 0,75

$= 1 - 4 + 4i - 4i + 4 = \underline{1} > 0$

So $P(z)$ has two distinct roots z and z':

$z = \dfrac{-b - \sqrt{\Delta}}{2a} = \dfrac{-(-(1+2i)) - \sqrt{1}}{2} = \dfrac{1+2i-1}{2} = i$. 0,5

$z' = \dfrac{-b+\sqrt{\Delta}}{2a} = \dfrac{-(-(+1+2i)) + \sqrt{1}}{2} = \dfrac{1+2i+1}{2} = 1+i$ 0,5

$S = \{ i , 1+i \}$.

Exercise 03 : (05 pts):

01- $E = \{(x,y,z) \in \mathbb{R}^3, x = -y + 3z\}$.

• Show that E is a subspace of $\mathbb{R}^3$:
$\begin{cases} E \neq \phi \ (\text{or } 0_{\mathbb{R}^3} \in E) \\ \forall u,v \in E : u+v \in E. \\ \forall u \in E ; \forall \lambda \in \mathbb{R} : \lambda u \in E. \end{cases}$

• $0_{\mathbb{R}^3} \in E$? 0,25

$0_{\mathbb{R}^3} = (\underset{x}{0}, \underset{y}{0}, \underset{z}{0})$; $0_{\mathbb{R}^3} \in E$ iff $x = -y + 3z$: 0,5

Since $0 = -0 + 3\cdot 0 \Leftrightarrow 0_{\mathbb{R}^3} \in E$.

• $\forall u,v \in E : u+v \in E$? 0,25

let $u = (x_1, y_1, z_1) \in E$ and $v = (x_2, y_2, z_2) \in E$; So $x_1 = -y_1 + 3z_1$ --- (1) 0,25

$\wedge \ x_2 = -y_2 + 3z_2$. --- (2) 0,25

Now $u+v = (\overbrace{x_1+x_2}^{x}, \overbrace{y_1+y_2}^{y}, \overbrace{z_1+z_2}^{z})$ 0,25

$u+v \in E \Leftrightarrow x = -y + z$:

In fact $x = x_1 + x_2 = -y_1 + 3z_1 - y_2 + 3z_2$ 0,5

Using (1) and (2) $= -(y_1 + y_2) + 3(z_1 + z_2)$

(1) + (2) $\Leftrightarrow x = -y + 3z$.

Thus $u+v \in E$.

- $\forall u \in E, \forall \lambda \in \mathbb{R} : \lambda u \in E$? 0,25

let $u=(x,y,z) \in E$, and let $\lambda \in \mathbb{R}$, $u \in E \Leftrightarrow x = -y + 3z$ (3) 0,25

we have: $\lambda u = (\lambda x, \lambda y, \lambda z)$.

From (3): $x = -y + 3z \xRightarrow{\times \lambda} \lambda x = -\lambda y + 3\lambda z \Leftrightarrow \lambda u \in E$ 0,25

Conclusion: E is a subspace of $\mathbb{R}^3$

02: $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$(x,y,z) \longmapsto f(x,y,z) = \begin{pmatrix} 2y-z \\ -x+3y-z \\ -2x+4y-z \end{pmatrix}$; f is a linear application

- Determine $\ker f$:

$\ker f = \{X \in \mathbb{R}^3 ; f(X) = 0_{\mathbb{R}^3}\} = \{(x,y,z) \in \mathbb{R}^3, f(x) = 0_{\mathbb{R}^3}\}$ 0,25

$f(x) = 0_{\mathbb{R}^3} \Leftrightarrow \begin{pmatrix} 2y-z \\ -x+3y-z \\ -2x+4y-z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} 2y-z=0 \\ -x+3y-z=0 \\ -2x+4y-z=0 \end{cases}$ 0,25

$\Leftrightarrow \begin{cases} z=2y \\ -x+3y-2y=0 \Leftrightarrow x=y. \\ -2y+4y-2y=0 \Leftrightarrow 0=0 \checkmark \end{cases}$ 0,25

So $\ker f = \{(y,y,2y);\ y \in \mathbb{R}\} \Leftrightarrow \ker f = [\underbrace{(1,1,2)}_{v_1}]$ 0,25

. Since $\ker f$ is generated by one vector then:

Nullity f = $\dim \ker f = 1$ 0,25

- Deduce Rank f:

By Rank-Nullity theorem: Rank f + Nullity f = $\dim \mathbb{R}^3$ 0,25

$\Rightarrow$ Rank f = $\dim \mathbb{R}^3$ − Nullity f

Now: we know that $\dim \mathbb{R}^3 = 3$. 0,25

Then Rank f = $3 - 1 = 2$.

Rank $f = 2$ 0,25

Faculty of Sciences and Applied Sciences Department of Technology - 1st year	**Sample solutions** **Regular Session Exam (Computer Science 1)**

Question 01 (2.5m): Choose the correct characteristic(s) (MCQ) اختر الخاصية أو الخصائص الصحيحة

Keyboard	☐ Processing	☐ Output	0.5 ■ Input	☐ Storage
Screen	■ Output 0.5	☐ Volatile	☐ Storage	☐ Input
CPU	☐ Input	0.25 ■ Processing	☐ Software	0.25 ■ Frequency
RAM	☐ Permanent	0.25 ■ Capacity	0.25 ■ Volatile	☐ Software
Operation system	■ Software 0.25	☐ Hardware	☐ Printer	0.25 ■ Version

Question 02 (8m): Perform the following operations: أنجِز العمليات التالية:

1) $(10010)_2 = (\,?\,)_{10}$ $= \mathbf{0 . 2^0 + 1 . 2^1 + 0 . 2^2 + 0 . 2^3 + 1 . 2^4 = 2 + 16 =}$ $\boxed{\mathbf{18}}$ 1.0

2) $(1100111011101)_2 = (\,?\,)_8$ $= \mathbf{(001\ \ 100\ \ 111\ \ 011\ \ 101)_2}$ $= \boxed{\mathbf{(14735)_8}}$ 1.0

3) $(4E0B3)_{16} = (\,?\,)_2$ $= \mathbf{(0100\ \ 1110\ \ 0000\ \ 1011\ \ 0011)_{16}}$ $= \boxed{\mathbf{(10011100000101100 11)_{16}}}$ 1.0

4) $(512)_8 = (\,?\,)_{16}$ $= \mathbf{(101\ \ 001\ \ 010)_2 = (1\ \ 0100\ \ 1010)_2}$ $= \boxed{\mathbf{(14A)_{16}}}$ 1.0

5) 8 Mb = ? KB = **8 x 1024 Kb = 8 x 1024 / 8 KB** = **1024 KB** 1.0

6) The size in **Bytes** of the text **Hello** encoded in ASCII: **1 char → 1 byte, 5 chars → size = 5 bytes** 1.0

7) The size (in KB) of a **1024 x 800 black and white** image: **size = 1024 . 800 / 8 bytes = 100 KB** 1.0

8) The binary operation: 101110 + 10110 = ? = $\boxed{\mathbf{(1000100)_2}}$ 1.0

```
  1 ¹0 ¹1 ¹1 1 0
+    1 0 1 1 0
----------------------
= 1 0 0 0h 1 0 0
```

1.0

Question 03 (4.5m): Give the value of each expression, or state: (**invalid**) :أعطِ قيمة كل عبارة، أوأكتب

Declaration

`int a = 5;`
`float x = 9.0;`
`const int C = 5;`

Exp	Value	
`2x + 1`	**invalid**	0.5
`a + 2 * C`	**15**	0.5
`2 x a + 1`	**invalid**	0.5

Exp	Value	
`x \ 2`	**invalid**	0.5
`a % 3`	**2**	0.5
`x % 2`	**invalid**	0.5

Exp	Value	
`a / 2`	**2**	0.5
`x / 2`	**4.5**	0.5
`C = a`	**invalid**	0.5

Question 04 (5m): The following program calculates the area and perimeter of a circle but contains 10 errors. Correct it. البرنامج التالي يحسب مساحة ومحيط دائرة لكنه يحتوي على 10 أخطاء. أعد كتابته بشكل صحيح.

```
\\ Program: Circle area calculation
#include <studio.h>
int main {
  constante float PI = 3.14;
  float radius;
  int area;
  printf(Enter the radius:);
  scanf("%d" &radius);
  area == Pi * radius * radius;
  printf("Area =\n", area);
  return 0;
}
```

```
// Program: Circle area calculation        0.5
#include <stdio.h>        0.5
int main() {        0.5
  const float PI = 3.14;        0.5
  float radius;
  float area;        0.5
  printf("Enter the radius:");        0.5
  scanf("%d", &radius);        0.25
  area = PI * radius * radius;
  printf("Area = %f \n", area);        0.25
  return 0;        0.5
}
```

Université L'Arbi Ben Mhidi
Département de Sciences & Technologies
1ère Année -ING, ST- 2025/2026

Correction du contrôle de Métiers de l'ingénieur

«Chaque bonne réponse vaut 1 pt»

1- Le savoir-faire désigne les compétences à la fois techniques et pratiques. **V**

2- La technologie est l'action de transformer des matières pour fabriquer des objets. **F**

3- Le béton est caractérisé par sa résistance à la compression. **V**

4- Les sciences de base sont composées par la physique, les mathématiques et l'informatiques. **F**

5- Le port des équipements de protection individuelle est obligatoire lorsqu'un risque est identifié. **V**

6- Le génie minier concerne la production d'énergie solaire. **F**

7- Les ingénieurs doivent maitriser le travail en équipe. **V**

8- L'ingénieur électromécanicien élabore et industrialise les composants électroniques. **F**

9- Une liaison de télécommunication ne se limite pas uniquement à un émetteur et un récepteur. **V**

10- Une étude géotechnique sert à évaluer la résistance des matériaux de surface. **F**

11- Un microsystème est une puce électronique contenant des parties non électroniques. **V**

12- L'hydrostatique est la branche de l'hydraulique qui étudie les propriétés des fluides au repos. **V**

13- Les produits de la pétrochimie sont des composés naturels utilisées dans la vie courante. **F**

14- La modélisation des systèmes dynamiques parmi les rôles d'un automaticien. **V**

15- Le traitement de l'énergie recouvre les techniques de transmission des signaux électriques. **F**

16- Les principaux domaines de génie biomédical sont les biomatériaux. **V**

17- L'ingénieur installations pétrolières est responsable du traitement de l'eau et du transport du gaz. **F**

18- Le génie pétrolier est une activité pluridisciplinaire, associant la géologie et la géophysique. **V**

19- Le métier HSE nécessite des connaissances à la fois techniques et administratives. **F**

20- La mécanique est la science qui s'intéresse à l'étude des forces et du mouvement pour la matière solide. **F**

Correction of the "Engineering Professions" Test

"Each correct answer is worth 1 point"

1. Know-how refers to both technical and practical skills. **T**
2. Technology is the act of transforming materials to manufacture objects. **F**
3. Concrete is characterized by its compressive strength. **T**
4. The basic sciences include physics, mathematics, and computer science. **F**.
5. Wearing personal protective equipment is mandatory when a risk is identified. **T**
6. Mining engineering deals with the production of solar energy. **F**
7. Engineers must master teamwork. **T**
8. The electromechanical engineer designs and industrializes electronic components. **F**
9. A telecommunication link is not limited to only a transmitter and a receiver. **T**
10. A geotechnical study is used to assess the strength of surface materials. **F**
11. A microsystem is an electronic chip containing non-electronic parts. **T**
12. Hydrostatics is the branch of hydraulics that studies the properties of fluids at rest. **T**
13. Petrochemical products are natural compounds used in everyday life. **F**
14. Modeling dynamic systems is one of the roles of an automation engineer. **T**
15. Power processing covers techniques for transmitting electrical signals. **F**
16. The main fields of biomedical engineering include biomaterials. **T**
17. The oil facilities engineer is responsible for water treatment and gas transport. **F**
18. Petroleum engineering is a multidisciplinary field combining geology and geophysics. **T**
19. The HSE profession requires both technical and administrative knowledge. **F**
20. Mechanics is the science that studies forces and motion in solid matter. **F**

Exam-Solution (structure of matter)

Exercise 01: (6 points)

I.

12 % (by weight) $\Longrightarrow$ 100 g of solution contains 12 g of KOH (0.25)

Density = 1.1 g/ml $\Longrightarrow$ 1 L contains 1100 g of solution. (0.25)

The mass of KOH is $\frac{12 \times 1100}{100} = 132$ g of KOH (0.5)

- Molarity:

$$C = \frac{n_{KOH}}{V_{solution}} = \frac{\frac{m_{KOH}}{M_{KOH}}}{V_{solution}} = \frac{\frac{132}{56.109}}{1l} = \frac{2.35}{1l} = 2.35\ moles/l \quad (1.00)$$

- Molality:

$$M = \frac{n_{KOH}}{m_{solvant}} = \frac{2.35}{(1100-132)\times10^{-3}} = 2.43\ moles/kg \quad (1.00)$$

1- The nature of X :

${}^{7}_{3}X + {}^{1}_{1}H \longrightarrow 2\,{}^{4}_{2}He$ (0.5) $\Longrightarrow {}^{7}_{3}X \equiv {}^{7}_{3}Li$ (Lithium) (0.5)

2- The corresponding energy :

$\Delta E = \Delta m\, C^2 = 0{,}0186$ u.m.a x $1{,}66\ 10^{-27}$kg x $(3\ 10^{8})^2 = 2.77\ 10^{-12}$ J (1.00)

- Δm is positive, the energy is also positive $\Longrightarrow$ energy absorbed.

(0.5) (0.5)

Exercise 02: **(06 points)**

1- The difference:

The frequency (ν) is the number of waves traveled per unit time, while the wave number ($\bar{\upsilon}$) is the number of waves traveled per unit of length. (1.00)

2- Paschen series corresponds to n = 3. (0.25)

The first line of Paschen series: $4 \longrightarrow 3$ (0.25)

- The wavelength of the first line:

Using Ritz-Rydberg formula: $\frac{1}{\lambda} = R_H \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)$ (0.5)

$\Rightarrow \frac{1}{\lambda} = 1.097\ 10^7 \left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 0.0533 \times 10^7$ (0.25)

$\Rightarrow \lambda = \frac{1}{0.0533 \times 10^7} = 18.76\ 10^{-7}\ m = 1876\ nm$ (0.25)

- The frequency (υ):

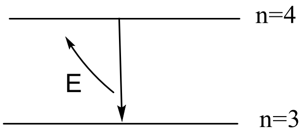
$\nu = \frac{c}{\lambda}$ (0.5) ; $\nu = \frac{3\times10^8}{18.76 \times 10^{-7}}$ (0.25) $\nu = 0.16 \times 10^{15} s^{-1}$ (0.25)

- The energy (ΔE)

$\Delta E = h.\nu = \frac{hc}{\lambda}$ (0.5)

$\Delta E = \frac{6.626\times10^{-34}\times3.10^8}{18.76\ 10^{-7}} = 1.06 \times 10^{-19} J = 0.66\ ev$ (0.5)

3- The first line of Paschen series corresponds to an emission. (0.25)



(0.25)

Emission spectrum

4- The ionization energy from the level corresponding to Paschen series :

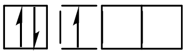
$IE = E_f - E_i = E_\infty - E_3$ (0.5)

$IE = 0 - E_3 = -\frac{-13.6}{3^2} = 1.51\ eV$ (0.5)

Exercise 03: (8points)

1-

- ${}_{5}B$: $[{}_{2}He]\ 2s^2\ 2p^1$ (0.25)


(0.25)

Period: 2; (0.25) group: IIIA (0.25)

- ${}_{8}O$: $[{}_{2}He]\ 2s^2\ 2p^4$ (0.25)


(0.25)

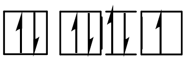
Period: 2 ; (0.25) group: VIA (0.25)

- ${}_{34}Se$: $[{}_{18}Ar]\ 3d^{10}\ 4s^2\ 4p^4$ (0.25)


(0.25)

Period: 4 ; (0.25) group: VIA (0.25)

- ${}_{35}Br$: $[{}_{18}Ar]\ 3d^{10}\ 4s^2\ 4p^5$ (0.25)


(0.25)

Period: 4 ; (0.25) group: VIIA (0.25)

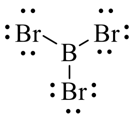
2- The radii classification:

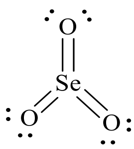
Same period Z↑ ⇒ r↓ ; same group n↑ ⇒ r↑ (0.5)

${}_{5}B$ and ${}_{8}O$ have the same period. ${}_{8}O$ and ${}_{34}Se$ have the same group. ${}_{34}Se$ and ${}_{35}Br$ have the same and highest period.

$$\Rightarrow \qquad r_O < r_B < r_{Br} < r_{Se} \qquad (1.00)$$

3- Lewis model :


(0.5)


(0.5)

Geometry :

$BBr_3 \equiv AX_3$ (0.25) ⇒ the geometry is **trigonal planar**. (0.5)

$SeO_3 \equiv AX_3$ (0.25) ⇒ the geometry is **trigonal planar**. (0.5)

Exo-01: (5 pts)

1) Elements between Cartesian-R and Cylindrical-R

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = z \end{cases} \textbf{ or } \begin{cases} \rho = \sqrt{x^2+y^2} \\ \theta = \text{arctg}(y/x) \\ z = z \end{cases}$$ 0.25 0.25 0.25

2) Unit vectors

$$\begin{cases} \vec{u_\rho} = \cos\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath} & 0.5 \\ \vec{u_\theta} = -\sin\theta\,\vec{\imath} + \cos\theta\,\vec{\jmath} & 0.5 \\ \vec{k} = \vec{k} & 0.5 \end{cases}$$

3) Temporal derivative of unit vectors

$$\begin{cases} \dfrac{d\vec{u_\rho}}{dt} = \dfrac{d\vec{u_\rho}}{d\theta}\dfrac{d\theta}{dt} = \dfrac{d\theta}{dt}\vec{u_\theta} & 0.5 \\ \dfrac{d\vec{u_\theta}}{dt} = \dfrac{d\vec{u_\theta}}{d\theta}\dfrac{d\theta}{dt} = -\dfrac{d\theta}{dt}\vec{u_\rho} & 0.5 \end{cases}$$

4) Position vector $\overrightarrow{OM}$

$$\overrightarrow{OM} = \overrightarrow{OM'} + \overrightarrow{M'M}$$

$$\overrightarrow{OM} = \rho\,\vec{u_\rho} + z\,\vec{k}$$ 0.5

5) Velocity vector $\vec{V}$

$$\vec{V} = \frac{d\overrightarrow{OM}}{dt}$$ 0.25

$$= \frac{d\rho}{dt}\vec{u_\rho} + \rho\frac{d\vec{u_\rho}}{dt} + \frac{dz}{dt}\vec{k}$$ 0.5

$$\vec{V} = \frac{d\rho}{dt}\vec{u_\rho} + \rho\frac{d\theta}{dt}\vec{u_\theta} + \frac{dz}{dt}\vec{k}$$ 0.5

Exo-02: (7 pts)

1) The position vectors with respect to R_1 and R_2

$$\overrightarrow{OM}\Big|_{R_1} = (t^3-5)\vec{\imath} + (3t^3-2t+1)\vec{\jmath} + (t^4+4)\vec{k}$$ 0.5

$$\overrightarrow{O'M}\Big|_{R_2} = (t^3-7t)\vec{\imath} + (3t^3+7)\vec{\jmath} + (t^4-t+1)\vec{k}$$ 0.5

2) The velocity vectors with respect to R_1 and R_2

$$\vec{V_a} = \frac{d\overrightarrow{OM}}{dt}\Big|_{R_1} = (3t^2)\vec{\imath} + (9t^2-2)\vec{\jmath} + 4t^3\,\vec{k}$$ 0.5 0.5

$$\vec{V_r} = \frac{d\overrightarrow{O'M}}{dt}\Big|_{R_2} = (3t^2-7)\vec{\imath} + (9t^2)\vec{\jmath} + (4t^3-1)\vec{k}$$ 0.5

3) The training velocity of R_2 with respect to R_1

Using the law of composition of velocities: 0.5

$$\vec{V_e} = \vec{V_a} - \vec{V_r}$$

$$\vec{V_e} = 7\,\vec{\imath} - 2\,\vec{\jmath} + \vec{k}$$ 0.5

4) The acceleration vectors with respect to R_1 and R_2

$$\vec{\gamma_a} = \frac{d\vec{V_a}}{dt}\Big|_{R_1} = (6t)\vec{\imath} + (18t)\vec{\jmath} + (12t^2)\,\vec{k}$$ 0.5 0.5

$$\vec{\gamma_r} = \frac{d\vec{V_r}}{dt}\Big|_{R_2} = (6t)\vec{\imath} + (18t)\vec{\jmath} + (12t^2)\,\vec{k}$$ 0.5

5) Coriolis acceleration and training acceleration

Using the law of composition of accelerations:

$$\vec{\gamma_a} = \vec{\gamma_r} + \vec{\gamma_c} + \vec{\gamma_e}$$ 0.5

Coriolis acceleration $\vec{\gamma_c}$ 0.5

$$\vec{\gamma_c} = 2\,\vec{\omega}\wedge\vec{V_r} = \vec{0},$$

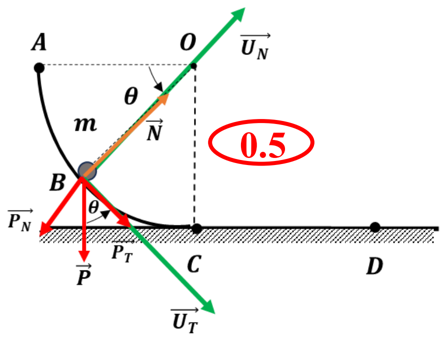
The mvt of R_2 with respect to R_1 is a translational mvt, i.e. the angular velocity of rotation is zero ($\vec{\omega} = \vec{0}$)

$$\boldsymbol{\vec{\gamma_c} = \vec{0}}$$ 0.5

Training acceleration $\vec{\gamma_e}$ 0.5

$$\vec{\gamma_e} = \vec{\gamma_a} - \vec{\gamma_r} - \vec{\gamma_c},\ \Rightarrow \boldsymbol{\vec{\gamma_e} = \vec{0}}$$

Exo-03: (8 pts)



0.5

1/ Forces

$$\vec{P}\begin{pmatrix} mg\cos\theta \\ -mg\sin\theta \end{pmatrix};\ \ \vec{N}\begin{pmatrix} 0 \\ N \end{pmatrix}.$$ 0.5 0.5

2/ Equations of movement

$$\sum \vec{F}_{ext} = m\vec{\gamma} \Longrightarrow \vec{P} + \vec{N} = m\vec{\gamma}$$ 0.5

knowing that : $\begin{cases} \gamma_T = \dfrac{dV}{dt} & 0.25 \\ \gamma_N = \dfrac{V^2}{R} & 0.25 \end{cases}$

The equations of movement will be as follows:⇒

$$\begin{cases} \boldsymbol{mg\cos\theta + 0 = m\frac{dV}{dt}} \dots\dots\dots (1) & 0.5 \\ \boldsymbol{-mg\sin\theta + N = m\frac{V^2}{R}} \dots\dots (2) & 0.5 \end{cases}$$

3/ Expression of the velocity $V(B)$

According to eq (1), we have: $mg\cos\theta = m\dfrac{dV}{dt}$

$$\Rightarrow g\cos\theta = \frac{dV}{d\theta}\frac{d\theta}{dt} = \frac{dV}{d\theta}\frac{V}{R} \text{ and } \boldsymbol{\frac{d\theta}{dt} = \frac{V}{R}}$$

$$\Rightarrow V.dV = g\,R\cos\theta\ d\theta$$

$$\Rightarrow \int_0^V V.dV = g\,R\int_0^\theta \cos\theta\ d\theta$$ 0.5

$$\Rightarrow \boldsymbol{V(B) = \sqrt{2Rg\sin\theta}}$$ 0.5

4/ Expression of the velocity $V(B)$

using kinetic energy theorem:

$$Ec(B) - Ec(A) = W(\vec{P}) + W(\vec{N})$$ 0.25

$$-\frac{1}{2}m\,V_B^2 = mgh; \quad h = R\sin\theta \quad 0.25$$

$$\Rightarrow \boldsymbol{V(B) = \sqrt{2Rg\sin\theta}} \quad 0.5$$

5/ <u>Expression of the contact force *N*</u>

According to eq (2), we have:

$$-mg\sin\theta + N = m\frac{V^2}{R}$$

$$\boldsymbol{N = 3\,mg\sin\theta} \quad 1$$

6/ <u>Expression of the velocity $V(C)$</u>

At point C : $\theta = 90°$

$$\Rightarrow \boldsymbol{V(C) = \sqrt{2Rg}} \quad 0.5$$

7/ <u>Maximum height on the incline</u>

Using **mechanical energy theorem**:

$$E_m\,(C) = E_m\,(D)$$

$$\frac{1}{2}m\,V_C^2 = mgh_{max} \quad 0.5$$

$$\Rightarrow \boldsymbol{h_{max} = R} \quad 0.5$$

جامعة الشهيد العربي بن مهيدي ام البواقي

ام البواقي امتحان السداسي الاول

مقياس : الحوكمة واخلاقيات المهنة

السنة الجامعية 2026/2025

نموذج التصحيح

السؤال الاول :اذكرمصادرالاخلاق واشرحها في سطر واحد :

1.الدين : نصوص الكتب المقدسة كالقران .... (1ن)

2.الضمير: ما يخبرنا به الضمير بأنه خير ....(1ن)

3.الشعور بالواجب ....(1ن)

4.السبب وهو المعنى الفلسفي ....(1ن)

5.معاني الاحترام يقصد بها العلاقات الشخصية المبنية على الاحترام..................................(1ن)

6.العدالة: نولد جميعا متساوون في الحقوق ويوجد قانون واحد يطبق على الجميع ويحترم من طرف الجميع (1ن)

7.الفضيلة :الفضيلة متأصلة في الإنسان فالشخص الفضيل ينجز أعمالا جيدة.................................. (1ن)

السؤال الثاني:ترجم مايلي الى العربية:

1- Morality : what society considers good

الاخلاق هي مايعتبره المجتمع جيد... (1ن)

2- Etics : what i consider good.

الاخلاق ما اعتبره جيد ....(1ن)

3- Professional coduct :waht the profession requires of me .

الاخلاقيات المهنية ماتطلبه المهنة مني ....(1ن)

4- What the law defenise as permited or forbiden.

القانون : مايعرفه القانون بانه مسموح به او ممنوع ....(1ن)

السؤال الثالث: في مايكمن الفرق بين الأخلاق المهنية والأخلاق (l'éthique et la morale) في الجانب الديني ؟

الاخلاق المهنية :لها دلالة علمانية .... (2ن)

الاخلاق:لها دلالة دينية .... (2ن)

السؤال الربع:cite les acteurs principaux de la structure universitaire :

1 :le recteur de l'université et le personnel d'encadrement ....(1ن)

2 :le conseil d'administration ....(1ن)

3 :le conseil d'éthique et de déontologie...(1ن)

4 :le conseil de discipline ....(1ن)

5 :commissions paritaires..(1ن)

بالتوفيق للجميع

LARBI BEN MHIDI-OUM EL BOUAGHI UNIVERSITY

(1^{st}year, (st & Eng) *Analysis 01-Final Exam* ***17/01/2026**(***(Duration : 1.30 min)**

Exercise1 : (6pts) :

a) ***Let A be the set defined by :*** $A = \left\{3 + \frac{1}{n} \ , \ \ \forall n \in \mathbb{N}^*\right\}$

1-Prove that A is <u>bounded</u>,(محدودة)

2-Prove that $sup(A) = 4$ ***and*** $inf(A) = 3$, ***then determine <u>max(A)</u> and <u>min(A)</u> if they exist.***

b)-<u>Solve</u> the following absolute value inequality: $for \ \ x \in \mathbb{R}$

$$|x - 1| \leq 2x - 1$$

Exercise 2(5pts) : ***let*** $(u_n)_{n\in\mathbb{N}}$ ***be the sequence defined*** $\forall n \in \mathbb{N}$ ***by :***

$$u_n = \frac{n}{n+1} \ sin\left(\frac{n\pi}{2}\right)$$

1-Compute u_{4n} ***and*** u_{4n+1}.

2-Deduce that $(u_n)_{n\in\mathbb{N}}$ ***is a <u>divergent</u> sequence. (متتالية متباعدة)***

Exercise 3(6pts) : ***Let f be the function defined by:***

$$f(x) = \begin{cases} \dfrac{sin(x-1)}{x^2-1} & if \ x < 1 \\ \sqrt{x+1} & if \ \ x \geq 1 \end{cases} \quad ,$$

1- ***Evaluate*** $lim_{x\to 1}\frac{sin(x-1)}{x^2-1}$ ($using \ equivalent \ functions$)(***الدوال المكافئة***)
2- ***Study the <u>continuity</u>(الاستمرار) and <u>differentiability</u> (قابلية الاشتقاق) of the function f at*** $x_0 = 1$
3- ***Prove that :*** $\forall x > 0, \ \ \sqrt{x+1} < \frac{x}{2} + 1$

(Using <u>Mean</u> Value Theorem)(التزايدات المنتهية)

Exercise4 (Course): (3pts) : <u>Justify</u> the following results:(برر النتائج التالية)

a- $[x^n] \neq [x]^n$, $for \ all \ \ x \in \mathbb{R}$

b- $\forall x \in \mathbb{R}/\mathbb{Z}$, $[x] + [-x] = -1$.

c- $\mathrm{v_n} = \frac{3n}{3n+1}$, $\forall n \in \mathbb{N}$ **is a subsequence (جزئية او مستخرجة)of** $\mathrm{u_n} = \frac{n}{n+1}$, $\forall n \in \mathbb{N}$

d- if $\lim_{n\to+\infty} u_n = L$(u_n is CV) , then u_n is **bounded** (محدودة)

e- $\int_0^2 \frac{1}{x-1} dx \neq [ln|x-1|]_0^2$

<u>Good luck</u>

Final Exam Solutions (Analysis 01)	Marks

Exercise 1 : ***(6p) :***

a) **Let:** $A = \left\{3 + \frac{1}{n} \ , \ \ \forall n \in \mathbb{N}^*\right\}$

We have ; $n \geq 1 \Rightarrow 0 < \frac{1}{n} \leq 1$

$$\Rightarrow 3 < 3 + \frac{1}{n} \leq 4$$

1

$\Rightarrow 4$ *is an upper bound and* 3 *is a lower bound*

So A is bounded, $\sup A$ *and* $\inf A$ exist

$4 \in A$(*because* If $n = 1 , 3 + \frac{1}{n} = 4$)

0.5

So: $\max A = \sup A = 4$

1

But: $3 \notin A$(*because*: If $3 + \frac{1}{n} = 3$, *then* $\frac{1}{n} = 0$(*It's a contradiction*)

0.5

We must prove that : $\inf A = 3.$

By definition we have:

$$3 = \inf A \Leftrightarrow \begin{cases} \forall a \in A, \ \ a \geq 3 \\ \wedge \\ \forall \varepsilon > 0, \exists a \in A, \ \ a < 3 + \varepsilon \end{cases}$$

$$\Leftrightarrow \begin{cases} \forall n \in \mathbb{N}^*, \ \ 3 + \dfrac{1}{n} > 3 (proved) \\ \forall \varepsilon > 0, \exists n \in \mathbb{N}^*, \ \ 3 + \dfrac{1}{n} < 3 + \varepsilon \end{cases}$$

0.25

Let : $\varepsilon > 0$

$$3 + \frac{1}{n} < 3 + \varepsilon \Leftrightarrow n > \frac{1}{\varepsilon}$$

By taking: $n = \left[\frac{1}{\varepsilon}\right] + 1 \in \mathbb{N}^*$

0.5

We obtain: $\forall \varepsilon > 0, \exists n = \left[\frac{1}{\varepsilon}\right] + 1 \in \mathbb{N}^*, \ \ n > \frac{1}{\varepsilon} \Leftrightarrow 3 + \frac{1}{n} < 3 + \varepsilon$

Therefore: $\inf A = 3.$

But $\min A$ does not exist because $3 \notin A$

0.25

b)

$$|x - 1| \leq 2x - 1$$

$$\Leftrightarrow -2x + 1 \leq x - 1 \leq 2x - 1$$

$$\Leftrightarrow \begin{cases} x-1 \le 2x-1 \\ \wedge \\ -2x+1 \le x-1 \end{cases} \Leftrightarrow \begin{cases} x \ge 0 \\ \wedge \\ x \ge \dfrac{2}{3} \end{cases}$$ **2**

So: $S = \left[\frac{2}{3}, +\infty\right[$

Exercise 2: <u>(5p)</u>

$$let: \quad u_n = \frac{n}{n+1} \sin\left(\frac{n\pi}{2}\right)$$

$$u_{4n} = \frac{4n}{4n+1} \sin\left(\frac{4n\pi}{2}\right) = \frac{4n}{4n+1} \sin(2n\pi) = 0$$ **1**

$$u_{4n+1} = \frac{4n+1}{4n+2} \sin\left(\frac{(4n+1)\pi}{2}\right) = \frac{4n+1}{4n+2} \sin\left(2n\pi + \frac{\pi}{2}\right) = \frac{4n+1}{4n+2}$$ **1**

- $\lim_{n\to+\infty} u_{4n} = 0$ **1**
- $\lim_{n\to+\infty} u_{4n+1} = \lim_{n\to+\infty} \frac{4n+1}{4n+2} = 1$ **1**

since the **two subsequences of (u_n) Converge to different limits,** **1**
(u_n) a divergent sequence .

Exercise 3 <u>(6p)</u> :

$$f(x) = \begin{cases} \dfrac{\sin(x-1)}{x^2-1} & if\ x < 1 \\ \sqrt{x+1} & if\ x \ge 1 \end{cases}$$

1) $\sin(x-1) \sim x-1$ *as* $x \curvearrowright 1$ **because :** $\lim\limits_{x\to1} \frac{\sin(x-1)}{x-1} = \lim\limits_{t\to0} \frac{\sin(t)}{t} = 1$ **1.5**

Then : $\lim\limits_{x\to1} \frac{\sin(x-1)}{x^2-1} = \lim\limits_{x\to1} \frac{x-1}{x^2-1} = \lim\limits_{x\to1} \frac{x-1}{(x-1)(x+1)} = \frac{1}{2}$

2)

Continuity *at* $x_0 = 1$

$$\lim_{x\to>1} f(x) = \lim_{x\to>1} \sqrt{x+1} = \sqrt{2} = f(1)$$ **1**

$$\lim_{x\to<1} f(x) = \lim_{x\to<1} \frac{\sin(x-1)}{x^2-1} = \frac{1}{2} \neq f(1)$$

Then f is not continuous at $x_0 = 1$

Differentiability *at* $x_0 = 1$ **1**

- *If f is differentiable at x_0 then f is continuous at x_0*
- *If f is not continuous at $x_0 = 1$ then f is not differentiable at $x_0 = 1$*
-

3) By taking: $\forall x > 0$

$$\begin{cases} g(x) = \sqrt{x+1} \\ [a,b] = [0,x] \end{cases}$$ **0.5**

1. $g(x))$ is continuous on $[-1,+\infty[\ \Rightarrow\ g(x))$ is continuous on $[0,x]$ **0.5**

2. $g(x)$is differentiable on $]-1,+\infty[\ \Rightarrow\ g(x)$is differentiable on $]0,x[$ **0.5**

Using <u>Mean</u> Value Theorem:

$$\exists c \in\ ]a,b[: g(b)-g(a)=g'(c)(b-a)$$ **0.5**

$$\exists c \in\ ]0,x[: g(x)-g(0)=xg'(c)$$

First we obtain:

$$\exists c \in\ ]0,x[: \sqrt{x+1}=1+\frac{x}{2\sqrt{C+1}}$$

And:

$$0<C<x \Rightarrow 1<\sqrt{C+1}<\sqrt{x+1} \Rightarrow 1+\frac{x}{2\sqrt{C+1}}<\frac{x}{2}+1$$ **0.5**

Therefore: $\forall x>0,\ \sqrt{x+1}<\frac{x}{2}+1$

<u>Exercise4 (Course): (3pts)</u> **:**

- $[x^n] \neq [x]^n$, *for all* $x \in \mathbb{R}$. **0.5**
  Example: if $x=\sqrt{3}$, $n=2$
  $[x^n]=3 \neq [x]^n=1$
- $\forall x \in \mathbb{R}/\mathbb{Z}$, $[x]+[-x]=-1.$ **0.5**

  Let: $[x]=k \Rightarrow k<x<k+1 \Rightarrow -k-1<-x<-k \Rightarrow [-x]=-k-1$

  So: $[x]+[-x]=k-k-1=-1$

- $v_n=\frac{3n}{3n+1}$ is a subsequences (جزئية او مستخرجة)from $u_n=\frac{n}{n+1}$ **1**
  Let: $\varphi(n)=3n$ ($\varphi: \mathbb{N} \to \mathbb{N}$, *and φ is increasing*)
  So: $u_{\varphi(n)}=u_{3n}=\frac{3n}{3n+1}=v_n$ is a subsequences of(u_n)
- *if* $\lim_{n\to+\infty} u_n=L$ then $u_n$is bounded (محدودة) **0.5**

  $\lim_{n\to+\infty} u_n=L \Leftrightarrow (\forall \varepsilon>0, \exists N(\varepsilon) \in \mathbb{N}, \forall n \in \mathbb{N}: n \geq N \Rightarrow |u_n-L|<\varepsilon)$

  $\Rightarrow -\varepsilon<u_n-L<\varepsilon \Rightarrow L-\varepsilon<u_n<L+\varepsilon \Rightarrow u_n$is bounded $(L\pm\varepsilon \in \mathbb{R})$

- $\int_0^2 \frac{1}{x-1}dx \neq [ln|x-1|]_0^2$ **0.5**
  Because $f(x)=\frac{1}{x-1}$ is not continuous on $[0,2]$