
Final exam of Algebra 01 (Duration: 1h' 30)

The use of calculator is strictly forbidden.

Exercise 01: (05 points)

- Let P , Q , and R be three assertions.
 - Write the table of truth of : $(\overline{P} \Leftrightarrow Q) \wedge R$.
 - Write the table of truth of : $Q \Rightarrow (\overline{Q} \vee P)$.
- Use the proof by induction to demonstrate: " $\forall n \in \mathbb{N}, (1+x)^n \geq 1+nx, x > -1$."

Exercise 02: (03 points)

Let

$$\begin{aligned} f: \mathbb{N}^* &\longrightarrow \mathbb{Q} \\ n &\longmapsto f(n) = \frac{1}{n^2} \end{aligned}$$

- Determine $\text{Im}(f)$.
- Calculate $f([1, 4])$, and $f^{-1}(\{\frac{1}{2}, 9\})$
- Is f bijective?

Exercise 03: (07 points)

Let $\alpha \in \mathbb{R}^*$. We provide $G = \mathbb{R} - \{\frac{1}{\alpha}\}$ by the internal composition law $*$ defined by

$$x * y = x + y - \alpha xy, \quad \forall x, y \in G.$$

- Show that $(G, *)$ is an abelian (commutative) group.
- We define:

$$\begin{aligned} f: (G, *) &\longrightarrow (\mathbb{R}, +) \\ x &\longmapsto f(x) = -\alpha x + 1. \end{aligned}$$

- Show that f is a homomorphism of groups.

Exercise 04: (05 points)

Let $z_1 = \frac{1+i}{i}$, $z_2 = \frac{1}{-i+1}$, and $z_3 = \frac{-2+i}{2+i}$.

- Simplify $\overline{z_2}$.
- Calculate $[-\sqrt{3}(4z_1 + 5z_3) + i]^3$.

Final exam of Algebra 01 - Session normale -

Exercise 01: (05,50 pts).

01- let P, Q , and R be three assertions.

a- The table of truth of: $(\bar{P} \Leftrightarrow Q) \wedge R$.

P	Q	R	$\bar{P}$	$(\bar{P} \Leftrightarrow Q)$	$(\bar{P} \Leftrightarrow Q) \wedge R$
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	F
F	F	F	T	F	F

b)- The table of truth of: $Q \Rightarrow (\bar{Q} \vee P)$

P	Q	$\bar{P}$	$\bar{Q} \vee P$	$\overline{(\bar{Q} \vee P)}$	$Q \Rightarrow (\bar{Q} \vee P)$
T	T	F	T	F	F
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	T	F	T

02- Proof by "induction": " $\forall n \in \mathbb{N}; (1+x)^n \geq 1+nx; x > -1$ ".

$P(n) = \forall n \in \mathbb{N}; (1+x)^n \geq 1+nx, x > -1$ " (2)

Step 01: Prove for n_0 ; $P(n_0)$; $n_0 = 0$:

$$P(0) = (1+x)^0 \geq 1+0 \cdot x?$$

$$\text{Since } (1+x)^0 = 1 \quad \wedge \quad 1+0x = 1 \quad \text{so } 1 \geq 1$$

Then $P(0)$ is true.

Step 02: Suppose that $P(n)$ is true; i.e.:
 $\forall n \in \mathbb{N} : (1+x)^n \geq 1 + nx \dots (*)$ (0,25)

Step 03: Try to prove: $P(n+1)$ is true; i.e.:
 $(1+x)^{n+1} \geq 1 + (n+1)x, x > -1$?

We have: $(1+x)^{n+1} = (1+x)^n (1+x)$ (0,25)

$$1 + (n+1)x = 1 + nx + x$$

Using (*); we find: $(1+x)^{n+1} = (1+x)^n (1+x) \geq (1+nx)(1+x)$
 and multiplying by $(1+x)$

Now since $x > -1 \Rightarrow 1+x > 0$. (0,25)

$$\begin{aligned} \text{So } (1+x)^{n+1} &\geq 1 + nx + x + nx^2 & n \geq 1, x^2 \geq 0 \\ &\geq 1 + (n+1)x + nx^2 & nx^2 \geq 0. \\ &\geq 1 + (n+1)x. \end{aligned} \quad (0,25)$$

Thus $P(n+1): (1+x)^{n+1} \geq 1 + (n+1)x$ is true.

$\therefore P(n)$ is true.

Exercise 02: (03 pts): $f: \mathbb{N}^* \rightarrow \mathbb{Q}$
 $n \mapsto f(n) = \frac{1}{n^2}$

01- Determine $\text{Im}(f) \cdot (\text{D}f)$:

let $n \in \mathbb{N}^*$; $0 < n \leq 1 \Rightarrow n^2 \geq 1$

$$\Rightarrow \frac{1}{n^2} \leq 1 \text{ and } \frac{1}{n^2} > 0 \quad (0,5)$$

$$\text{So } \text{Im}f =]0, 1]$$

02- Calculate: $f([1, 4])$ and $f^{-1}(\{\frac{1}{2}, 9\})$ (0,25)

$$\begin{aligned} f([1, 4]) &= \{f(n); n \in [1, 4]\} = \{\frac{1}{n^2}; 1 \leq n \leq 4\} \\ &= [f(1), f(4)] = [\frac{1}{16}, 1] \end{aligned} \quad (0,25)$$

$$f\left(\frac{1}{n^2}, 9\right) = \{n \in \mathbb{N}^*, f(n) \in \{\frac{1}{2}, 9\}\}$$

$$f(n) = \frac{1}{n^2} = \frac{1}{2} \Rightarrow n = \pm \sqrt{2} \notin \mathbb{N}^* \quad (0.25)$$

$$f(n) = \frac{1}{n^2} = 9 \Rightarrow n = \pm \frac{1}{3} \in \mathbb{N}^* \quad (0.25)$$

$$\text{So } f^{-1}\left(\left\{\frac{1}{2}, 9\right\}\right) = \emptyset \quad (0.25)$$

031- Is f bijective?

f is bijective = f is injective + f is surjective (0.25)

From question 01, we get: $\text{Im} f =]0, 1] \neq \mathbb{Q}$; which (0.25)

means that f is not surjective.

Thus f is not bijective (0.25).

Exercise 03: (67 pts): $G = \mathbb{R} - \{\frac{1}{\alpha}\}$; $\alpha \in \mathbb{R}^*$.

$$\forall x, y \in G: x * y = x + y - \alpha xy.$$

01- $(G, *)$ is an abelian group:

- $*$ is I.C.L.
- $*$ is associative
- $*$ has an identity element in G
- Every element in G has an inverse in G .
- $*$ is commutative

• $*$ is I.C.L (Already given in the exercise). (0.5)

• $*$ is associative: $\forall x, y, z \in G: (x * y) * z = x * (y * z)$

let x, y and $z \in G$. we have:

$$\begin{aligned} (x * y) * z &= (x + y - \alpha xy) * z = \\ &= x + y - \alpha xy + z - \alpha (x + y - \alpha xy)z \quad (0.5) \\ &= x + y - \alpha xy + z - \alpha xz - \alpha yz + \alpha^2 xyz \quad (1) \end{aligned}$$

$$\begin{aligned} x * (y * z) &= x * (y + z - \alpha yz) = \\ &= x + y + z - \alpha yz - \alpha x(y + z - \alpha yz) \quad (0.5) \\ &= x + y + z - \alpha yz - \alpha xy - \alpha xz + \alpha^2 xyz \quad (2) \end{aligned}$$

So $*$ is associative because (1) = (2) (0,24)
 has an identity element in G ? $\exists e \in G: x * e = x \quad \forall x \in G$. (0,24)
 $*$ let x in G : such that $x * e = x$ (0,24)
 $x + e - \alpha e x = x \Rightarrow e(1 - \alpha x) = 0$.

So $e = 0$ or $1 - \alpha x = 0$
 (0,24) $\Rightarrow x = 1/\alpha \in G = \mathbb{R} - \{1/\alpha\}$ (0,25)

So $e = 0 \in G$ is the identity element of $*$. (0,5)

Every element in G has an inverse in G : $\forall x \in G, \exists x^{-1} \in G: x * x^{-1} = e$ (0,217)

let $x \in G$ and x^{-1} st: $x * x^{-1} = e \Rightarrow x + x^{-1} - \alpha x x^{-1} = 0$ (0,24)
 $\Rightarrow x^{-1}(1 - \alpha x) = -x$.

$\Rightarrow x^{-1} = \frac{-x}{1 - \alpha x}$

So $\exists x^{-1} = \frac{-x}{1 - \alpha x} \in G$, and x^{-1} is the inverse of x . (0,15)

So $(G, *)$ is a group.

$(G, *)$ is an abelian (commutative) group because $\forall x, y \in G$:
 $x * y = y * x$ (0,24)

let $x, y \in G$ st $x * y = x + y - \alpha x y = y + x - \alpha y x$ (0,24)
 $= y * x$ (0,25)

$\therefore (G, *)$ is an abelian group.

62. $f: (G, *) \rightarrow (\mathbb{R}, \cdot)$
 $x \mapsto f(x) = -\alpha x + 1$. (0,24)

f is a homomorphism of groups? $\forall x, y \in G: f(x * y) = f(x) \cdot f(y)$ (0,24)

let $x, y \in G$; then $f(x * y) = f(x + y - \alpha x y) = -\alpha(x + y - \alpha x y) + 1$ (0,25)
 $= -\alpha x - \alpha y + \alpha^2 x y + 1 \dots (1)$

$f(x) \cdot f(y) = (-\alpha x + 1)(-\alpha y + 1) = \alpha x - \alpha y + \alpha^2 x y + 1 \dots (2)$ (0,24)

Since (1) = (2), then f is a homomorphism of groups. (0,24)

1. Mettez au féminin, selon le modèle

Mon frère est petit. Ma sœur est petite

Le professeur est souriant et gentil. **La professeure est souriante et gentille** **1pt.....**

Le serveur est bavard et compétent. **La serveuse bavarde et compétente** **1pt....**

L'épicier est matinal et bavard. **L'épicière est matinale et bavarde** **1pt...**

2. Mettez au pluriel .

Un tableau original **des tableaux originaux** **1pt....**

Un plat local. **des plats locaux** **1pt...**

3. Complétez avec du, de la, de l', des

Dans le jardin, il y a ..**des..** roses et **des....** iris. **1pt**

Dans ce film, il y a ...**de l'** action(f) et**du..** suspense.(m) **1pt**

4. Indique le type de chaque phrase : déclarative, interrogative, exclamative ou impérative

→ Quelle belle journée ensoleillée ! **exclamative.....** **1pt**

→ Avait-il peur de sortir ?**interrogative.....** **1pt**

5. Cherchez le groupe nominal dans les phrases suivantes, ce qui vous permettra ensuite de conjuguer le verbe entre parenthèses au présent de l'indicatif :

La voiture de mon père est chez le garagiste, elle (être) **est..**en panne. **1pt**

Chloé et ses camarades (avoir) ...**ont..**rendez-vous avec le directeur **1pt**

Sur le carrelage du séjour (se reposer) ...**se repose....**la chienne **1pt**

6.

Je n'ai pas examiné ton travail. **1pt**

ton travail :

Est complément d'objet direct.

☒

Est complément circonstanciel

☐

Est complément d'objet indirect

☐

7.

L'enfant caresse le petit chat. 2pts

le petit chat :

est sujet

est complément d'objet direct

8.

- Il a rencontré des amis sur la plage

des amis : 1pt

Est sujet

Complément circonstanciel de lieu

Complément d'objet direct

Sur la plage : 1pt

Est sujet

Complément circonstanciel de lieu

Complément d'objet direct

9.

Remplacez ces complément d'objet indirect (souligné) par le pronom personnel adéquat (lui, elle, eux, elles)

Exemple : Il s'intéresse à ses élèves réponse : il s'intéresse à eux

Il se souvient de sa première femme réponse : il se souvient d'elle... 1pt

Il s'occupe de ses enfants réponse : il s'occupe d'eux..... 1pt

10.

La maîtresse parle à cet enfant : 1pt

La maîtresse :

Est sujet :

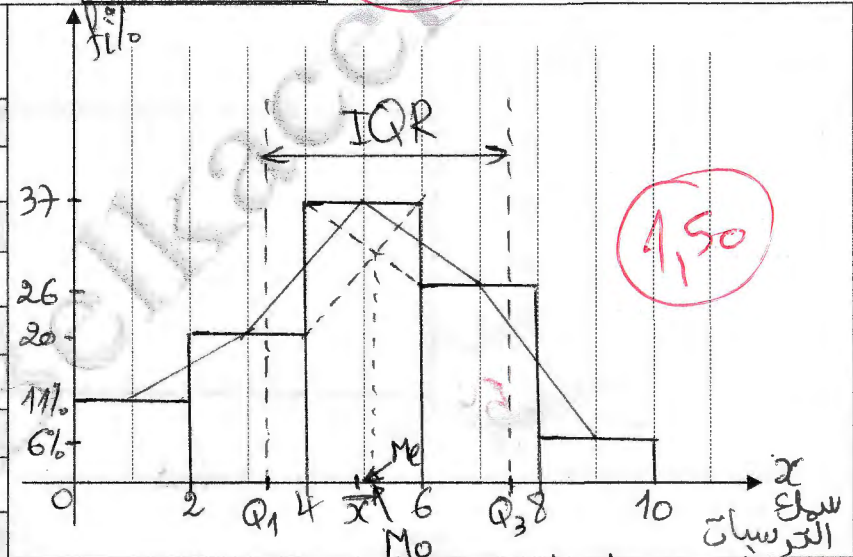
Est complément d'objet indirect

Est complément d'objet direct

Aim :

Population (Universe) $\Omega = \dots$ $\Omega = \dots$ Statistical Variable $x = \dots$ Exercise01:
Continuous
UnivariateNames : Correction Examen Normal
E00/10 Regular

0,50	2	4	6	8	10
0,50	c_1	c_2	c_3	c_4	c_5
0,50	1	3	5	7	9
0,50	c_1^2	c_2^2	c_3^2	c_4^2	c_5^2
0,50	1	9	25	49	81
0,50	n_1	n_2	n_3	n_4	n_5
0,50	4	7	13	9	2
0,50	$\tilde{n}_1$	$\tilde{n}_2$	$\tilde{n}_3$	$\tilde{n}_4$	$\tilde{n}_5$
0,50	4	11	24	33	35
0,50	f_1	f_2	f_3	f_4	f_5
0,50	0,111	0,200	0,377	0,266	0,106
0,75	$\bar{x} = 0,11 \times 1 + 0,20 \times 3 + 0,37 \times 5 + 0,26 \times 7 + 0,10 \times 9 = 0,492$				
0,75	$\alpha = 0,50 \Rightarrow R = 17,5 \Rightarrow Me = 4 + \frac{(17,5 - 11) \cdot (6 - 4)}{24 - 11} = 5$				
0,75	$\alpha = 0,25 \Rightarrow R = 8,75 \Rightarrow Q_1 = 2 + \frac{(8,75 - 4) \cdot (4 - 2)}{11 - 4} = 3,36$				
0,75	$\alpha = 0,75 \Rightarrow R = 26,25 \Rightarrow Q_3 = 6 + \frac{(26,25 - 24) \cdot (8 - 6)}{33 - 24} = 6,50$				
0,75	$Mo = 4 + \frac{(13 - 7) \cdot (6 - 4)}{2 \times 13 - 7 - 9} = 5,20$				



- In most of cases, أغلب البطاريات has شمارات الترسبات = 05,20.
- As $\bar{x}$ & Mo are متقاربة $\Rightarrow$ choice of sample is successful moderately accepted, Bad.
- 50% of البطاريات has شمارات الترسبات from 03,36 to 06,50.
- The البطاريات has شمارات الترسبات = 04,92 $\pm$ 02,13.

$$V(x) = 0,455 \Rightarrow S_x = 0,213 \Rightarrow RSD = 43\%$$

Exercise 02: Bivariate Statistics

Names: Correction Examen Normal
"Regular"

X \ Y	[b _{j-1} , b _j]		[... 11, ... 40]		[... 40, ... 69]		[... 69, ... 98]		[... 98, ... 127]		[... 127, ... 156]		n _j * & f _j
	[a _{i-1} , a _i]	x _i	y _j										
				25,50		54,50		83,50		112,50		141,50	
[... 17, ... 36]	26,50	7	0,47	2	0,13	0							9 0,60
[... 36, ... 55]	45,50	0		3	0,20	0	2						3 0,20
[... 55, ... 74]	64,50	0		0		0							0 0,00
[... 74, ... 93]	83,50	0		1	0,07	2	0,13						3 0,20
[... 93, ... 112]	102,50	0		0		0							0 0,00
[... 112, ... 131]	121,50	0		0		0							0 0,00
[... 131, ... 150]	140,50	0		0		0							0 0,00
[... 150, ... 169]	159,50	0		0		0							0 0,00
[... 169, ... 188]	178,50	0		0		0							0 0,00
[... 188, ... 207]	197,50	0		0		0							0 0,00
[... 207, ... 226]	216,50	0		0		0							0 0,00
[... 226, ... 245]	235,50	0		0		0							0 0,00
[... 245, ... 264]	254,50	0		0		0							0 0,00
[... 264, ... 283]	273,50	0		0		0							0 0,00
[... 283, ... 302]	292,50	0		0		0							0 0,00
[... 302, ... 321]	311,50	0		0		0							0 0,00
[... 321, ... 340]	330,50	0		0		0							0 0,00
[... 340, ... 359]	349,50	0		0		0							0 0,00
[... 359, ... 378]	368,50	0		0		0							0 0,00
[... 378, ... 397]	387,50	0		0		0							0 0,00
[... 397, ... 416]	396,50	0		0		0							0 0,00
[... 416, ... 435]	405,50	0		0		0							0 0,00
[... 435, ... 454]	414,50	0		0		0							0 0,00
[... 454, ... 473]	423,50	0		0		0							0 0,00
[... 473, ... 492]	432,50	0		0		0							0 0,00
[... 492, ... 511]	441,50	0		0		0							0 0,00
[... 511, ... 530]	450,50	0		0		0							0 0,00
[... 530, ... 549]	459,50	0		0		0							0 0,00
[... 549, ... 568]	468,50	0		0		0							0 0,00
[... 568, ... 587]	477,50	0		0		0							0 0,00
[... 587, ... 606]	486,50	0		0		0							0 0,00
[... 606, ... 625]	495,50	0		0		0							0 0,00
[... 625, ... 644]	504,50	0		0		0							0 0,00
[... 644, ... 663]	513,50	0		0		0							0 0,00
[... 663, ... 682]	522,50	0		0		0							0 0,00
[... 682, ... 701]	531,50	0		0		0							0 0,00
[... 701, ... 720]	540,50	0		0		0							0 0,00
[... 720, ... 739]	549,50	0		0		0							0 0,00
[... 739, ... 758]	558,50	0		0		0							0 0,00
[... 758, ... 777]	567,50	0		0		0							0 0,00
[... 777, ... 796]	576,50	0		0		0							0 0,00
[... 796, ... 815]	585,50	0		0		0							0 0,00
[... 815, ... 834]	594,50	0		0		0							0 0,00
[... 834, ... 853]	603,50	0		0		0							0 0,00
[... 853, ... 872]	612,50	0		0		0							0 0,00
[... 872, ... 891]	621,50	0		0		0							0 0,00
[... 891, ... 910]	630,50	0		0		0							0 0,00
[... 910, ... 929]	639,50	0		0		0							0 0,00
[... 929, ... 948]	648,50	0		0		0							0 0,00
[... 948, ... 967]	657,50	0		0		0							0 0,00
[... 967, ... 986]	666,50	0		0		0							0 0,00
[... 986, ... 1005]	675,50	0		0		0							0 0,00
[... 1005, ... 1024]	684,50	0		0		0							0 0,00
[... 1024, ... 1043]	693,50	0		0		0							0 0,00
[... 1043, ... 1062]	702,50	0		0		0							0 0,00
[... 1062, ... 1081]	711,50	0		0		0							0 0,00
[... 1081, ... 1100]	720,50	0		0		0							0 0,00
[... 1100, ... 1119]	729,50	0		0		0							0 0,00
[... 1119, ... 1138]	738,50	0		0		0							0 0,00
[... 1138, ... 1157]	747,50	0		0		0							0 0,00
[... 1157, ... 1176]	756,50	0		0		0							0 0,00
[... 1176, ... 1195]	765,50	0		0		0							0 0,00
[... 1195, ... 1214]	774,50	0		0		0							0 0,00
[... 1214, ... 1233]	783,50	0		0		0							0 0,00
[... 1233, ... 1252]	792,50	0		0		0							0 0,00
[... 1252, ... 1271]	801,50	0		0		0							0 0,00
[... 1271, ... 1290]	810,50	0		0		0							0 0,00
[... 1290, ... 1309]	819,50	0		0		0							0 0,00
[... 1309, ... 1328]	828,50	0		0		0							0 0,00
[... 1328, ... 1347]	837,50	0		0		0							0 0,00
[... 1347, ... 1366]	846,50	0		0		0							0 0,00
[... 1366, ... 1385]	855,50	0		0		0							0 0,00
[... 1385, ... 1404]	864,50	0		0		0							0 0,00
[... 1404, ... 1423]	873,50	0		0		0							0 0,00
[... 1423, ... 1442]	882,50	0		0		0							0 0,00
[... 1442, ... 1461]	891,50	0		0		0							0 0,00
[... 1461, ... 1480]	900,50	0		0		0							0 0,00
[... 1480, ... 1500]	909,50	0		0		0							0 0,00
[... 1500, ... 1519]	918,50	0		0		0							0 0,00
[... 1519, ... 1538]	927,50	0		0		0							0 0,00
[... 1538, ... 1557]	936,50	0		0		0							0 0,00
[... 1557, ... 1576]	945,50	0		0		0							0 0,00
[... 1576, ... 1595]	954,50	0		0		0							0 0,00
[... 1595, ... 1614]	963,50	0		0		0							0 0,00
[... 1614, ... 1633]	972,50	0		0		0							0 0,00
[... 1633, ... 1652]	981,50	0		0		0							0 0,00
[... 1652, ... 1671]	990,50	0		0		0							0 0,00
[... 1671, ... 1690]	999,50	0		0		0							0 0,00
[... 1690, ... 1709]	1008,50	0		0		0							0 0,00
[... 1709, ... 1728]	1017,50	0		0		0							0 0,00
[... 1728, ... 1747]	1026,50	0		0		0							0 0,00
[... 1747, ... 1766]	1035,50	0		0		0							0 0,00
[... 1766, ... 1785]	1044,50	0		0		0							0 0,00
[... 1785, ... 1804]	1053,50	0		0		0							0 0,00
[... 1804, ... 1823]	1062,50	0		0		0							0 0,00
[... 1823, ... 1842]	1071,50	0		0		0							0 0,00
[... 1842, ... 1861]	1080,50	0		0		0							0 0,00
[... 1861, ... 1880]	1089,50	0		0		0							0 0,00
[... 1880, ... 1900]	1098,50	0		0		0							0 0,00
[... 1900, ... 1919]	1107,50	0		0		0							0 0,00
[... 1919, ... 1938]	1116,50	0		0		0							0 0,00
[... 1938, ... 1957]	1125,50	0		0		0							0 0,00
[... 1957, ... 1976]	1134,50	0		0		0							0 0,00
[... 1976, ... 1995]	1143,50	0		0		0							0 0,00
[... 1995, ... 2014]	1152,50	0		0		0							0 0,00
[... 2014, ... 2033]	1161,50	0		0		0							0 0,00
[... 2033, ... 2052]	1170,50	0		0		0							0 0,00
[... 2052, ... 2071]	1179,50	0		0		0							0 0,00
[... 2071, ... 2090]	1188,50	0		0		0							0 0,00
[... 2090, ... 2109]	1197,50	0		0		0							0 0,00
[... 2109, ... 2128]	1206,50	0		0		0							0 0,00
[... 2128, ... 2147]	1215,50	0		0		0							0 0,00
[... 2147, ... 2166]	1224,50	0		0		0							0 0,00
[... 2166, ... 2185]	1233,50	0		0		0							0 0,00
[... 2185, ... 2204]	1242,50	0		0		0							0 0,00
[... 2204, ... 2223]	1251,50	0		0		0							0 0,00
[... 2223, ... 2242]	1260,50	0		0		0							0 0,00
[... 2242, ... 2261]	1269,50	0		0		0							0 0,00
[... 2261, ... 2280]	1278,50	0		0		0							0 0,00
[... 2280, ... 2300]													

Analysis 1: solutions of the Final Exam:

Exercise 01: (08pts)

Let: $A = \left\{ \frac{n+2}{n-1}, \forall n \in \mathbb{N} : n \geq 2 \right\}$.

a) We have: $\frac{n+2}{n-1} = \frac{n-1+3}{n-1} = 1 + \frac{3}{n-1}$.

and: $n \geq 2 \Rightarrow n-1 \geq 1 \Rightarrow \frac{1}{n-1} \leq 1 \Rightarrow \frac{3}{n-1} \leq 3$.

$$\Rightarrow 1 + \frac{3}{n-1} \leq 4 \rightarrow \textcircled{1}$$

Also we have: $n-1 \geq 0 \Rightarrow \frac{3}{n-1} > 0 \Rightarrow \frac{3}{n-1} + 1 > 1 \rightarrow \textcircled{2}$

from $\textcircled{1}$ and $\textcircled{2}$ we obtain:

$$\forall n \geq 2: 1 < \frac{n+2}{n-1} \leq 4 \Rightarrow A \text{ is bounded.}$$

• 4 is an upper bound, and $4 \in A$

because: if $\frac{n+2}{n-1} = 4 \Leftrightarrow n+2 = 4n-4 \Leftrightarrow 3n = 6 \Leftrightarrow n = 2$

so: $\sup A = \max A = 4$.

• 1 is a lower bound, but: $1 \notin A$.

because: if $\frac{n+2}{n-1} = 1 \Leftrightarrow n+2 = n-1 \Leftrightarrow 2 = -1$ contradiction.

so we must prove that: $\inf A = 1$.

$$\inf A = 1 \Leftrightarrow \begin{cases} \forall n \geq 2: \frac{n+2}{n-1} > 1 \text{ (proved)} \\ \forall \varepsilon > 0: \exists n \geq 2: \frac{n+2}{n-1} < 1 + \varepsilon. \end{cases}$$

so: Let: $\varepsilon > 0$: $1 + \frac{3}{n-1} < 1 + \varepsilon \Leftrightarrow \frac{3}{n-1} < \varepsilon$.

$$\Leftrightarrow n-1 > \frac{3}{\varepsilon}$$

$$\Leftrightarrow n > \frac{3}{\varepsilon} + 1$$

by taking: $n = \left\lceil \frac{3}{\varepsilon} + 1 \right\rceil + 1 \geq 2$.

so: $\forall \varepsilon > 0: \exists n = \left\lceil \frac{3}{\varepsilon} + 1 \right\rceil + 1 \geq 2: \frac{n+2}{n-1} < 1 + \varepsilon$. (its the definition)

$$\Rightarrow \inf A = 1$$

but: $\min A$ not exist because $1 \notin A$.

• solution of: $\left| \frac{1}{2}x + 2 \right| + 6 \leq 15 \rightarrow \textcircled{2}$

1st method

$$\Rightarrow \left| \frac{1}{2}x + 2 \right| < 9 \Rightarrow -9 < \frac{1}{2}x + 2 < 9$$

$$\Rightarrow -11 < \frac{1}{2}x < 7 \Rightarrow -22 < x < 14$$

$$\Rightarrow S =]-22, 14[$$

11

2nd method:
$$\left| \frac{1}{2}x + 2 \right| = \begin{cases} \frac{1}{2}x + 2 & \text{if } \frac{1}{2}x + 2 \geq 0 \\ -\frac{1}{2}x - 2 & \text{if } \frac{1}{2}x + 2 \leq 0 \end{cases}$$
$$= \begin{cases} \frac{1}{2}x + 2 & \text{if } x \geq -4 \\ -\frac{1}{2}x - 2 & \text{if } x \leq -4 \end{cases}$$

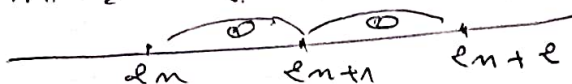
So: (2) $\Leftrightarrow \begin{cases} \frac{1}{2}x + 2 + 6 \leq 15 & \text{if } x \geq -4 \\ \vee \\ -\frac{1}{2}x - 2 + 6 \leq 15 & \text{if } x \leq -4 \end{cases}$

$\Leftrightarrow \begin{cases} x < 14 & \wedge x \geq -4 \Rightarrow x \in [-4, 14[\\ \vee \\ x > -22 & \wedge x \leq -4 \Rightarrow x \in]-22, -4] \end{cases}$

So: $S =]-22, -4] \cup [-4, 14[=]-22, 14[$.

• solution of: $[2x] + 2[x] = 13 \rightarrow ①$

we put: $[x] = n \Rightarrow n \leq x < n+1 \Rightarrow 2n \leq 2x < 2n+2$



if: $2n \leq 2x < 2n+1$, then: $[2x] = 2n$.

So: ① $\Leftrightarrow 2n + 2n = 13 \Leftrightarrow 4n = 13$

$\Leftrightarrow n = \frac{13}{4} \notin \mathbb{N} \Leftrightarrow S_1 = \emptyset$.

if: $2n+1 \leq 2x < 2n+2$

then: $[2x] = 2n+1$

So: ① $\Leftrightarrow 2n+1 + 2n = 13 \Rightarrow 4n = 11 \Rightarrow n = \frac{11}{4} \notin \mathbb{N}$

$[x] = 3 \Rightarrow 3 \leq x < 4 \Rightarrow S_2 = [3, 4[$

$\Rightarrow S = S_1 \cup S_2 = [3, 4[$

Exercise 02: (4 pts): $u_n = \sum_{k=1}^n \frac{1}{k^2}$ and $v_n = u_n + \frac{1}{n}$

• $u_{n+1} - u_n = \sum_{k=1}^{n+1} \frac{1}{k^2} - \sum_{k=1}^n \frac{1}{k^2} = \frac{1}{(n+1)^2} > 0 \Rightarrow u_n$ is increasing $\rightarrow ①$

• $v_{n+1} - v_n = u_{n+1} + \frac{1}{n+1} - u_n - \frac{1}{n} = u_{n+1} - u_n + \frac{1}{n(n+1)} - \frac{1}{n}$
$$= \frac{1}{(n+1)^2} + \frac{-1}{n(n+1)} = \frac{n - (n+1)}{n(n+1)^2}$$
$$= \frac{-1}{n(n+1)^2} < 0 \Rightarrow v_n$$
 is decreasing $\rightarrow ②$

• $\lim_{n \rightarrow +\infty} (u_n - v_n) = \lim_{n \rightarrow +\infty} \frac{1}{n} = 0 \rightarrow ③$

from ①, ② and ③: u_n and v_n are adjacent sequences.

②

1.1

1.1

1.1

1

Exercise 03: (8pts):

- $g(x) = x^3 + 3x - e$ is a continuous function on $[0, 1]$.

$$g(0) = -e \text{ and } g(1) = e.$$

$$\text{and } g(0) \cdot g(1) < 0$$

by intermediate value theorem we obtain:

$$\exists c \in]0, 1[: g(c) = 0.$$

$$\text{but: } g'(x) = 3x^2 + 3 > 0 \Rightarrow g \text{ is monotonic.}$$

$$\text{So: } \exists! c \in]0, 1[: g(c) = 0. \quad (\text{there is only one solution})$$

- $\lim_{x \rightarrow 0} \left(\frac{x^3 + x^2}{\ln(x^2 + 1)} \sqrt{x^2 + 3} \right) = \frac{0}{0} \text{ (IF)}$

$$\ln(x^2 + 1) \sim x^2 \text{ because: } \lim_{x \rightarrow 0} \frac{\ln(x^2 + 1)}{x^2} \stackrel{x^2 = t}{=} \lim_{t \rightarrow 0} \frac{\ln(t + 1)}{t} = 1$$

$$\text{So: } \lim_{x \rightarrow 0} \frac{(x^3 + x^2) \sqrt{x^2 + 3}}{\ln(x^2 + 1)} = \lim_{x \rightarrow 0} \frac{x^2(x + 1) \sqrt{x^2 + 3}}{x^2} = \sqrt{3}.$$

- we have: $D_f = \mathbb{R}^*$ so f is continuous on $\mathbb{R}^*$.

yes f admits a continuous extension on $\mathbb{R}$, because: $\lim_{x \rightarrow 0} f(x) = \sqrt{3}$ (exists and unique).

$$\text{So: } \tilde{f}(x) = \begin{cases} f(x) & \text{if } x \in \mathbb{R}^* \\ \sqrt{3} & \text{if } x = 0 \end{cases}$$

- $h(x) = \begin{cases} ax^2 + 1 & \text{if } x > 1 \\ bx - 3 & \text{if } x < 1 \end{cases}$ $\left\{ \begin{array}{l} ax^2 + 1 \text{ is continuous on } (x > 1) \\ bx - 3 \text{ is continuous on } (x < 1) \end{array} \right.$

$h(x)$ is continuous on $\mathbb{R} \Rightarrow h(x)$ is continuous at $x_0 = 1$.

$$\Rightarrow \lim_{x \rightarrow 1} h(x) = \lim_{x \rightarrow 1} h(x) = h(1) = a + 1.$$

$$\text{we have: } \lim_{x \rightarrow 1} h(x) = a + 1 \text{ and } \lim_{x \rightarrow 1} h(x) = b - 3.$$

$$\text{So: } h(x) \text{ is continuous on } \mathbb{R} \Rightarrow a + 1 = b - 3 \rightarrow \textcircled{1}$$

- Also we have: $ax^2 + 1$ is differentiable on $(x > 1)$ and: $bx - 3$ is " " " " on $(x < 1)$

So: $h(x)$ is differentiable on $\mathbb{R} \Rightarrow h(x)$ is differentiable at $x_0 = 1$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{h(x) - h(1)}{x - 1} = L \text{ (exists and unique)}$$

Remark: if: $a + 1 \neq b - 3 \Rightarrow h(x)$ is not continuous at $x_0 = 1$

So: $h(x)$ is differentiable at $x_0 = 1$

$$\Rightarrow a + 1 = b - 3$$

$$\lim_{x \rightarrow 1} \frac{h(x) - h(1)}{x-1} = \lim_{x \rightarrow 1} \frac{ax^2 + 1 - a - 1}{x-1} = \lim_{x \rightarrow 1} \frac{a(x^2-1)}{x-1} =$$

$$= \lim_{x \rightarrow 1} a(x+1) = 2a.$$

$$\text{anal: } \lim_{x \rightarrow 1} \frac{h(x) - h(1)}{x-1} = \lim_{x \rightarrow 1} \frac{bx-3-(a+1)}{x-1}$$

$$= \lim_{x \rightarrow 1} \frac{bx-3-(b-3)}{x-1} \quad (\text{because } a+1=b-3)$$

$$= \lim_{x \rightarrow 1} \frac{bx-b}{x-1} = b$$

$h(x)$ is differentiable on $\mathbb{R} \Rightarrow$ diff at $x_0 = 1$.

$$\Rightarrow 2a = b \rightarrow \textcircled{2}$$

So: $h(x)$ is continuous and differentiable on $\mathbb{R}$

$$\Rightarrow \begin{cases} a+1=b-3 \rightarrow \textcircled{1} \\ 2a=b \rightarrow \textcircled{2} \end{cases}$$

$$\textcircled{2} \text{ in } \textcircled{1}: a+1=2a-3 \Rightarrow a=4$$

$$\text{from } \textcircled{2}: b=2a \Rightarrow b=8.$$

0,17

0,17

محضر تقديم الأعمال الموجهة/.....

الرقم	المجموعة	اللقب	الاسم	29/9../2024	3../10../2024	6../10../2024	10/10../2024	13/10../2024	17../10../2024	20../10../2024	24../10../2024	27../10../2024	31../10../2024	3../11/2024	7../11../2024	10/11../2024	14/11../2024	17../11../2024	21../11/2024	24../11../2024	28../11/2024	1/12../2024	5/12../2024	8/12../2024	12../12../2024	15/12../2024	19/12../2024	نقطة الحضور (05 ن)	نقطة المشاركة / العمل الفردي (05 ن)	نقطة امتحان التطبيق (05 + 05 ن)	علامة التقييم المستمر		
	01	ALLOUANE	MOUHAMED	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	2.5	2.5	10		
1		ABDENOURI	SALSABIL	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1.66	1.5	0.25	3.41		
2		AMARA	ISLEM	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.16	1.5	1.5	7.16		
3		BAHRI	LOUAI	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0		
4		BAKLI	NEDJM EDDINE AYOUB	0	0.5	0	0	1	1	1	1	1	0.5	1	1	1	1	1	1	1	1	0.5	1	1	1	1	1	1	4.37	4	5.5	13.87	
5		BECHARA	ADEM	0	1	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2.81	2.5	4.75	10.06		
6		BELALOUI	HANADI	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.37	5	3.75	13.12		
7		BELGHOUL	ABDELOUAKIL	0.5	0.5	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.54	2.5	2	8.04		
8		BOUDIS	BADEREDDINE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
9		BOUKHERS	ISLAM	1	1	1	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2.6	2.25	2.5	7.35		
10		BOUMEDJOU	MILISSA FATIMA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2.08	1.5	0	3.58		
11		BRADJA	MILOUD	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1.87	1	0	2.87		
12		BRIZA	SANAA	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5	1	1	1	1	1	1	1	1	1	3.85	4	3.75	11.6		
13		CHAIB	HADJER	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0
14		CHELOUACHE	LOUAY ABDERRAHMANE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.5	9.25	18.75		
15		DIR	IBTISSAM	1	1	1	1	0	1	0	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1.25	1.5	0	2.75		
16		DJAAFRI	ABDELBASSET	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0
17		FERRAG	HADIL	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.37	4.5	5.25	14.12		
18		FIZI	KHAWLA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.25	7.75	17		
19		GAHMOUS	CHAHD AMANI	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	5	5.5	15.5		
20		HADJERES	WAFI	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	0	1	1	0	1	0	1	0	1.25	1	0	2.25		
21		HAMAIZIA	AOUATEF	0	0	1	1	0	1	1	0	1	1	0	1	1	1	0	1	0	1	1	1	1	1	1	1	3.95	4.5	5.25	13.7		
22		HAZIL	MOHAMED ISLEM DJIHANE	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0
23		KHETTAB	AYAT ALLAH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.75	3.75	2.5	10		
24		LAOUADI	AICHA	1	1	1	1	1	1	1	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.47	4	7	15.47		
25		LOGBI	ADEM ABD EL HAMID	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.95	2.25	3.25	9.45		
26		MALKI	NASREDDINE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	3	2.25	10.25		
27		MANSOURI	MOHAMMED	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2.29	1	0	3.29		
28		MEROUANI	CHAHD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29		MEROUANI	YOUCEF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30		MESSAOUD	MALAK ERRAHMANE	1	1	1	1	1	1	1	1	1	1	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1	4.68	2.75	5	11.68		
31		MESSAOUDEN	SALAH EDDINE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.25	5	14.25		
32		MEZIANE	MOHAMED NADJIB	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5	1	0.5	1	1	1	1	1	1	4.68	4.25	2.75	11.68		
33		NAILI	MADJDA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	5	9.75	19.75		
34		OUCHENE	RIHEB	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1.25	1	0	2.25		
35		OUTTAR	BILAL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36		RAHALI	YAKOUB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37		REKKAB	DJIHANE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4	4.75	13.75		
38		REZAIGUIA	MARWA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	5	9	19		
39		SAHEB	FADHILA	1	1	1	1	1	1	1	1	1	1	1	1																		

ملاحظات: - توضع العلامة المخصصة لكل معيار حسب ما ورد في الجدول (بناء على ما تم الاتفاق عليه بين رئيس القسم وفريق ميدان التكوين بالرجوع
يقدم هذا المحضر من قبل الأستاذ المكلف بالأعمال الموجهة أو التطبيقية إلى رئيس القسم عند تقديم محضر علامات الامتحان النهائي (ويكون ممضيا من طرفه ومن طرف الأستاذ مسؤول المادة)

علامة الحضور = (عدد الحصص التي حضر فيها الطالب (ة) فعلا ÷ العدد الفعلي للحصص التي قدمها الأستاذ) **5X**
تخصص نصف علامة الحضور للحصة الواحدة للغياب المبرر على أن لا تتجاوز الغيابات المبررة **05** غيابات في التقييم

محضر تقييم الأعمال الموجبة/

الرقم	المجموعة	الفوج	اللقب	الاسم	29./9./2024	2../10./2024	6../10./2024	9../10./2024	..13/10../2024	16../10../2024	20../10../2024	23../10./2024	..27/..10./2024	..30/..10./2024	..3../11./2024	6../..11/2024	..10../11../2024	13../11../2024	..17../11../2024	20../11../2024	..24../..11/2024	27../11../2024	..1../..12/2024	4../12/2024	8../12../2024	..11/12/2024	15../12/2024	18../12../2024	نقطة الحضور (05 ن)	نقطة المشاركة /العمل الفردي (05 ن)	نقطة امتحان التطبيق (05 + 05 ن)	علامة التقييم المستمر	
1	01	GE	ABDAOUI	ABDESSATTAR	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	2.91	3.25	3.5	9.66	
2			AHMED CHAWQI	BOUDJEMAA	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	3.54	1.5	2	7.04	
3			AYAD	NOR HANE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
4			AYAICHIA	CHAHD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.5	10	19.5	
5			BELABED	ALAA ARAHMEN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6			BENYARBAH	ISHAQ	0	1	1	1	1	1	1	1	0	1	1	0.5	1	1	1	1	1	1	1	0	1	1	1	1	1	4.06	3	1.5	8.56
7			BERCHEN	AYA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.5	8.25	17.75	
8			BOUAKAZ	KAOUTHAR	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.37	3.75	6.25	14.37
9			BOULEGREB	ILHAM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10			CHENIKHAR	SABAH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.5	8	17.5	
11			GHOUL	ASMA NOUR EL HOUDA	1	1	1	1	0	1	1	1	1	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	4.47	3	4	11.47
12			HADJ AISSA	AMINA	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0.5	0.5	1	1	0	1	1	1	1	0	0	3.75	3	2.25	9
13			HAZEM	NAHLA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14			KHELIFI TOUHAMI	ABDERRAHMANE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15			KHEMMAR	CHIRAZ	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0.5	1	1	4.47	3.25	3.5	11.22
16			LADOUALI	ABDELWADOUD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	3.5	6	14.5	
17			LAMRI	NOURELHOUDA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18			LAMRI	YAAKOUB FATH ELLAH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19			LAMRI	EZAKARYA MOHAMMED	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.25	7.75	17
20			MELLAL	IBTIHAL	1	1	1	1	1	1	1	0.5	1	1	1	1	1	1	1	1	1	1	1	0.5	1	1	1	1	1	4.79	4	7	15.79
21			MERABET	RASIM	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.41	0	0	0.41
22			MEZIANI	DHIYA EDDINE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	4.37	3	0	7.37
23			NEMER	INES	1	1	1	1	0	1	1	1	0.5	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	3.85	4	6.25	14.1
24			OMAICHE	SOUNDOUS	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	5	4.75	10	19.75	
25			SAADI	ROUMAISSA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26			SERARMA	ANFEL	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.5	7.25	16.75	
27			SOULA	KHADIDJA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28			TAMRABET	ZAKARYA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.25	6.5	15.75	
29			ZAIDI	YACER SALAH EDDINE	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	4.16	4.25	6.25	14.66	
30			ZAIDI	MANAR	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	0	1	1	2.7	3	2.25	7.95
31			BENMECHRI	ABDELRRAHIM	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	5	4.75	10	19.75	
32			BEN HAMLA	MOSLIM								1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	1.94	2	3.75	7.69

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محضر تقييم الأعمال الموجهة/

الرقم	المجموعة	الفوج	اللقب	الاسم	29/9./2024	2../10./2024	6../10../2024	9../10./2024	13/10../2024	16../10../2024	20../10../2024	23../10./2024	27../10./2024	30../10./2024	3../11./2024	6../11./2024	10../11./2024	13/11../2024	17../11./2024	20../11../2024	24../11./2024	27../11./2024	1../12/2024	4/12/2024	8../12../2024	11/12/2024	15../12/2024	18../12../2024	نقطة الحضور (05 ن)	نقطة المشاركة /العمل الفردي (05 ن)	نقطة امتحان التطبيق (05 + 05 ن)	علامة التقييم المستمر		
1	01	GM	AISSAOUI	MESSAOUDA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	2.91	4	4	10.91		
2			ATMANI	WESIM MOHAED RIDA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	4.58	4	2	10.58		
3			AZZAB	ABDELMALEK	0	1	0	1	0	1	0	1	0	1	0	1	1	1	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	
4			BELFETNI	FADI	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5	1	0	1	0	0	1	1	4.47	2.5	1	7.97		
5			BENHAMLA	HIDAIA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.75	10	19.75		
6			BOUCHACHOUA	HAMZA	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2.29	1	0	3.29		
7			BOUGUERRA	BEDREDDINE	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.16	1	1.5	6.66		
8			BOUREKHOUM	ISLEM NOUR ELYAKINE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.37	2	5.25	11.62		
9			BOUZID	WAJDI ISLEM	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.12	2	3.5	8.62		
10			BRAKNIA	AHLEM	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.95	3	0	6.95		
11			BRIK	MOHAMMED AMINE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.58	4.5	6	15.08		
12			FERHOUNE	Selsabil	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.79	4.25	3	12.04		
13			GAOUAS	FAHD	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0
14			GASMI	TAMER	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.33	1.5	1.75	6.58		
15			HABRI	IKRAM	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.37	3.5	2.75	10.62		
16			HAFSI	Seif eddine	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.5	6.5	16		
17			HAMAIZIA	ABDERRAOUF	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	5	8	18		
18			HAMANA	LOQMANE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4.5	3	12.5		
19			HAMLAOUI	DOUAA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	4	5.5	14.5		
20			KHELIFA	AYA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	4.16	4	6	14.16		
21			KHELIFI	ISLAM	1	1	1	1	1	1	1	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.27	3.5	1	8.77		
22			MEZIANE	MOHAMED NASSIM	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	3.75	5	13.75		
23			NADJAQUM	FERIAL	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.79	3.25	5.75	13.79		
24			OUSSAR	NOUR EL HOUDA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	5	8.5	18.5		
25			REZAM	MARIA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0.5	1	1	1	3.43	1.5	2.75	7.68		
26			SOUFI	AHMED AISSA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.79	3.75	0.5	9.04		
27			TAIBI	MALAK	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3.54	3.75	1.5	8.79		
28			TERBAK	BILAL	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.37	3.25	4	11.62		
29			ZOUBIR	ABD EL HAY	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4.37	5	8.25	17.62		
30																																		

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محضر تقييم الأعمال الموجزة/

الرقم	المجموعة	الفوج	اللقب	الاسم	2../10../2024	3../10../2024	9../10../2024	10/10../2024	13/10../2024	17../10../2024	20../10../2024	24../10../2024	27/10../2024	31/10../2024	3../11/2024	7../11/2024	10/11../2024	14/11/2024	17../11../2024	21../11/2024	24../11../2024	28../11/2024	1/12/2024	5/12../2024	8/12/2024	12../12/2024	15../12../2024	19/12../2024	نقطة الحضور (05 ن)	نقطة المشاركة/ العمل الفردي (05 ن)	نقطة امتحان التطبيق (05 + 05 ن)	علامة التقييم المستمر	
1	01	GP	ADRAOUI	LOUDJINE TESNIM	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	1	0	0	0	3.75	1.5	1	6.25	
2			AMRI	AHMED SAMER	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0.5	1	1	1	0	0	1	0	0	0	3.85	2.75	1	7.6	
3			BENAYED	ABDERRAHMENE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	3.33	1.5	0.25	5.08	
4			BENGUERREICHE	HANINE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	3.95	3	3.5	10.45	
5			BENZAOU	CHEYMA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6			BERKANI	DJESSAR TADJ EDDINE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	4.16	3.5	2.5	10.16
7			BOUMARAF	IMANE	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.5	0.5	1	1	0	0	0	0	3.95	4.5	8	16.45	
8			DIB	GHOFRANE	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	3.12	2	2.25	7.37	
9			DIB	MALAK	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	4.58	4	7	15.58	
10			HAKEM	YOUSSRA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	4.37	3.25	4.25	11.87	
11			MAMMERI	OMAR	1	1	1	1	1	0.5	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	4.47	3.75	6.5	14.72	
12			MAZIANI	HEITHEM	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	0	0	0	0	2.91	1.5	4.25	8.66	
13			MERAIHIA	ISRA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	3.54	2	4.75	10.29	
14			MEZIANE	ARIDJ CHAHINDA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15			MOKRANI	AMANI	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	0	1	1	0.5	1	0	1	0	4.06	4.25	4.5	12.81	
16			MOUERRI	ZAKARIA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17			TORCHE	ANFEL	1	1	1	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1.66	1	0	2.66
18			TOULBA	OUSSAMA	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19			ZERAFA	MAISSA	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	4.16	3.25	2.25	9.66		
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Final Exam of Physics 1

Exam duration : 1^h30 min

Course questions (10 pts)

- 1- Why we use dimensional analysis?
- 2- What are the different coordinate systems?
- 3- Define the cartesian coordinates then write the position $\vec{OM}$, velocity $\vec{v}$ and acceleration vector $\vec{a}$.
- 4- Write the relationship between **cylindrical coordinates** and **Cartesian coordinates**.
- 5- Determine that the acceleration vector in the Frenet frame is $\vec{a} = a_T \vec{u} + a_n \vec{n}$
- 6- Calculate the work of a force $F=1.5 \cdot 10^4$ N supplied to move a body at a height AB of 3 meters (AB= 3m) vertically.
- 7- What is the difference between a conservative energy and a non-conservative energy?
- 8- Write the Newton's three laws.

Exercise 01 (5 pts)

A mobile is moving along curvilinear trajectory in orthonormal basis $(\vec{u}, \vec{n})$ by the following equation :

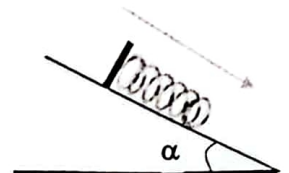
$$S(t) = t^3 + 5t$$

- 1- Find the expression of the velocity v ?
- 2- Determine the tangential acceleration vector a_T ?
- 3- If we suppose that at $t = 5s$ the acceleration is $a = 45 \text{ m/s}^2$, calculate the radius of curvature R at this time?

Exercise 02 (5 pts)

A mass $m = 22 \text{ kg}$ suspended from a spring of stiffness $K = 50 \text{ N/m}$ descends along an inclined plane which makes an angle $\alpha = 30^\circ$ with the horizontal **without friction**

- 1- Show in the diagram the forces acting on the mass.
- 2- Determine the normal reaction of the support and the acceleration of the mass when the spring is stretched by a length $x = 0.1 \text{ m}$.



Good Luck

Solution of Course questions.

1) We use dimensional analysis to:

- * Verify the homogeneity of physical laws (01)
- * Determine the unit of a physical quantity based on fundamental units (meters, second, kilogram, etc...)

2) The different Coordinate systems

- a. Cartesian coordinates
- b. Polar coordinates (01)
- c. Cylindrical Coordinates
- d. Spherical Coordinates

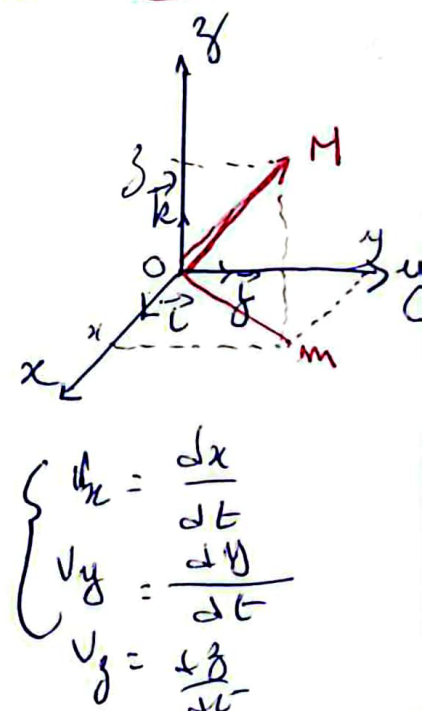
3) The Cartesian Coordinates is defined by the frame $R(O, x, y, z)$ with the unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ of a point M which gives its position in the space. (0,25)

- The position vector

$$\vec{OM} = x\vec{i} + y\vec{j} + z\vec{k} \quad (0,25)$$

- The Velocity vector

$$\vec{V} = \frac{d\vec{OM}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} \Rightarrow$$



$$\begin{cases} v_x = \frac{dx}{dt} \\ v_y = \frac{dy}{dt} \\ v_z = \frac{dz}{dt} \end{cases}$$

2-

The acceleration vector

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}}{dt} \Rightarrow$$

$$\begin{cases} a_x = \frac{dv_x}{dt} \\ a_y = \frac{dv_y}{dt} \\ a_z = \frac{dv_z}{dt} \end{cases} \quad (0, 25)$$

4. The relation between Cylindrical and Polar Coordinates

$$\begin{cases} x = \rho \cos \theta \\ y = \rho \sin \theta \\ z = z \end{cases} \quad (01)$$

5) Determination that: $\vec{a} = a_t \vec{u} + a_n \vec{n}$

$$\vec{v} = |\vec{v}| \vec{u} \quad \text{so} \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d|\vec{v}|}{dt} \vec{u} + |\vec{v}| \frac{d\vec{u}}{dt}$$

In the other hand: $\frac{d\vec{u}}{dt}$ can be written $\frac{d\vec{u}}{dt} = \underbrace{\frac{d\vec{u}}{d\theta}}_{\vec{n}} \cdot \underbrace{\frac{d\theta}{dt}}_w$

$$\text{So:} \quad \vec{a} = \frac{d|\vec{v}|}{dt} \vec{u} + |\vec{v}| \vec{n} w$$

$$v = R w \Rightarrow w = \frac{v}{R} \quad (02)$$

$$\vec{a} = \frac{d|\vec{v}|}{dt} \vec{u} + \frac{v^2}{R} \vec{n}$$

$$\Rightarrow \vec{a} = a_T \vec{u} + a_N \vec{n}$$

$$\begin{cases} a_T = \frac{d|\vec{v}|}{dt} \\ a_N = \frac{v^2}{R} \end{cases}$$

3-

6 - Calculation of the work. (Application directe)

(0.5)

$$W = \vec{F} \cdot \vec{AB} = F \cdot AB \cdot \cos 0^\circ = 1.5 \cdot 10^4 \cdot 3 \cdot \cos 0^\circ = 4.5 \cdot 10^4 \text{ J}$$

7 - Conservative energy: the variation in kinetic energy is equal to the opposite of the variation in potential energy

$$\Delta E_c = - \Delta E_p \quad / \quad \Delta E_c + \Delta E_p = 0 \quad (0.5)$$

In this case we can say that the system is isolated or the mechanical energy is conserved

$$E_m = E_c + E_p$$

non conservative

8 - Newton's three laws

(0.3)

Chaque loi sur 1 pt.

a - Newton's first law: a free particle moves in rectilinear motion with constant velocity

$$\sum \vec{F} = \vec{0}$$

b - Newton's second law: The fundamental principle of dynamics

$$\sum \vec{F} = m \vec{a}$$

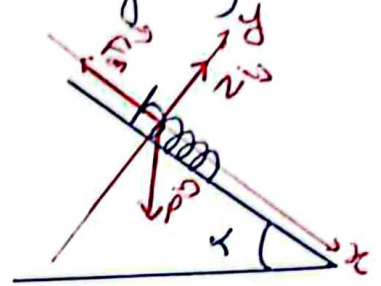
c - Newton's third law: The force applied by the first particle is opposite and equal in sign to the second particle

$$|\vec{F}_{1-2}| = |\vec{F}_{2-1}|$$

Exercise 02 (05 pt)

1 - The forces acting on the mass (see the diagram)

(01)



2 - Determination of the normal reaction by using Newton's second laws:

$$\sum \vec{F} = m\vec{a} \Rightarrow \vec{P} + \vec{N} + \vec{F}_m = m\vec{a} \quad (01)$$

Following (ox): $-F_m + P_{x1} = -F_m + mg \sin \alpha = ma \quad (01)$

Following (oy): $N - P_y = 0 \Rightarrow N = mg \cos \alpha \quad (01)$

So, $N = mg \cos \alpha$

The norm of the spring return force is given by

$$F_m = K \cdot x$$

$$a = (-kx + mg \sin \alpha) / m = 3 \text{ m/s}^2 \quad (g = 10 \text{ m/s}^2)$$

$a = 3 \text{ m/s}^2$

 (01)

Exercise 01. (05 pt)

$$S(t) = t^3 + 5t \quad (S \text{ in meters, } t \text{ in seconds})$$

1. The Velocity v :

$$v = \frac{ds}{dt} = \frac{d(t^3 + 5t)}{dt} = 3t^2 + 5 \quad (01)$$

2. The tangential acceleration a_T .

$$a_T = \frac{dv}{dt} = 6t \quad (01)$$

3. Calcul of the radius of curvature R .

$$\text{We have } a_N = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_N} \quad (01)$$

Then we must find a_N .

$$\vec{a} = \vec{a}_T + \vec{a}_N \Rightarrow a^2 = a_T^2 + a_N^2 \Rightarrow a_N^2 = a^2 - a_T^2$$

$$\text{at } t = 5s : a_T = 30 \text{ m/s}^2 : a = 45 \text{ m/s}^2$$

$$a_N^2 = (45)^2 - (30)^2$$

$$a_N^2 = 1125 \text{ (m/s}^2\text{)}^2$$

$$\boxed{a_N = 33,54 \text{ m/s}^2} \quad (01)$$

$$\text{at } t = 5s : v = 3(5)^2 + 5 = 80 \text{ m/s}$$

$$R = \frac{v^2}{a_N} = \frac{(80)^2}{33,54} = 190,81 \text{ m}$$

$$\boxed{R = 190,81 \text{ m}} \quad (01)$$