LARBI BEN M'HIDI UNIVERSITY
FACULTY OF SCIENCES AND APPLIED SCIENCES

LEVEL 2nd YEAR EPE
TEST: CHEMISTRY OF SOLUTIONS

# EXAM

**Exercise1 (4 pts)**

We mix a volume $V_1 = 200\ mL$ of potassium chloride solution ($K^+ + Cl^-$) at concentration $C_1 = 5.0.10^{-3}\ mol.\ L^{-1}$ and a volume $V_2 = 800\ mL$ of sodium chloride solution ($Na^+ + Cl^-$) at concentration $C_2 = 1.25.10^{-3}\ mol.\ L^{-1}$.

1- What is the conductivity of the solution obtained?

2- In the previous mixture, we place the cell of a conductivity meter. The surface area of the electrodes is $1.0cm^2$ and the distance between them is 1.1 $cm$. What is the value of the conductance?

The ionic molar conductivities are given:

$\lambda_{Na+} = 5.01.10{-}3\ S.\ m^2.\ mol^{-1}$

$\lambda_{Cl-} = 7.63.10{-}3\ S.\ m^2.\ mol^{-1}$

$\lambda_{K+} = 7.35.10{-}3\ S.\ m^2.\ mol^{-1}$

**Exercise2 : (8 pts)**

Consider the cell symbolized by:

$Cu / Cu^{2+}$ (0.1M) // $Fe^{2+}$ (0.1 M) /Fe

1) Calculate the potential of each electrode.

2) Indicate the anode, the cathode and give the oxidation-reduction half reactions that take place in each of these electrodes.

3) Write the equation of the overall chemical reaction of the cell operation and calculate its equilibrium constant.

4) Make a diagram of the cell on which we will specify the direction of the electric current and that of the circulation of electrons. Indicate the directions of migration of the ions in the salt bridge. Explain.

Data: E° ($Cu^{2+}$ / Cu) = 0.34V; E° ($Fe^{2+}$ /Fe) = -0.44V.

**Exercise3 : (8 pts)**

A- Consider a 0.1 mol•$L^{-1}$ hydrofluoric acid (HF) solution in which this acid is dissociated at 8%.

1) Calculate at equilibrium, the concentrations of the species HF, $F^-$ and $H_3O^+$.

2) Determine the pH of the solution considered as well as the acidity constant Ka of HF.

B- The pH of a $10^{-2}$M solution of methanoic acid (HCOOH) is equal to 2.9.

1) Write the dissociation reaction of methanoic acid.

2) Calculate the dissociation coefficient of this acid, and deduce the value

3) Determine the concentrations of the chemical species present.

4) Calculate the pH of a formic acid solution with concentrations respectively equal to $10^{-3}$ and $10^{-4}$M. Determine the dissociation coefficient in both cases and conclude.

**GOOD LUCK**

UNIVERSITE LARBI BEN M'HIDI

FACULTE DES SCIENCES ET SCIENCES APPLIQUEES

NIVEAU 2ème ANNEE ING GPC EPREUVE : CHIMIE DES SOLUTIONS

# CONTROLE

## Exercice 1 :( 4pts)

On mélange un volume $V_1 = 200\ mL$ de solution de chlorure de potassium $(K^+ + Cl^-)$ à concentration $C_1 = 5{,}0.10^{-3}\ mol.\ L^{-1}$ et un volume $V_2 = 800\ mL$ de solution de chlorure de sodium $(Na^+ + Cl^-)$ à concentration $C_2 = 1{,}25.10^{-3}\ mol.\ L{-}^{1}$.

1- Quelle est la conductivité de la solution obtenue ?

2- Dans le mélange précédent, on place la cellule d'un conductimètre. La surface des électrodes est de $1{,}0cm2$ et la distance qui les sépare est de $1{,}1\ cm$. Quelle est la valeur de la conductance ?

On donne les conductivités molaires ioniques :

$\lambda_{Na+} = 5{,}01.10{-}3\ S.\ m^2.\ mol^{-1}$

$\lambda_{Cl-} = 7{,}63.10{-}3\ S.\ m^2.\ mol^{-1}$

$\lambda_{K+} = 7{,}35.10{-}3\ S.\ m^2.\ mol^{-1}$

## Exercice 2 : (8 pts)

On considère la pile symbolisée par:

**Cu / $Cu^{2+}$ (0,1M) // $Fe^{2+}$ (0,1 M) /Fe**

1) Calculer le potentiel de chaque électrode.

2) Indiquer l'anode, la cathode et donner les demi réactions d'oxydo-réduction qui ont lieu dans chacune de ces électrodes.

3) Ecrire l'équation de la réaction chimique globale de fonctionnement de la pile et calculer sa constante d'équilibre.

4) Faire un schéma de la pile sur lequel on précisera le sens du courant électrique et celui de circulation des électrons. Indiquer les sens de migration des ions dans le pont salin. Expliquer.

Données : E° ($Cu^{2+}$ / Cu)=0,34V ; E° ($Fe^{2+}$ /Fe)= -0,44V.

## Exercice 3 :(6 pts)

**A-** On considère une solution d'acide fluorhydrique (HF) 0,1 mol·$L^{-1}$ dans laquelle cet acide est dissocié à 8 %.

1) Calculer à l'équilibre, les concentrations des espèces HF, $F^-$ et $H_3O^+$.

2) Déterminer le pH de la solution considérée ainsi que la constante d'acidité Ka de HF.

**B-** Le pH d'une solution $10^{-2}$M d'acide méthanoïque (HCOOH) est égal à 2,9.

1) Ecrire la réaction de dissociation de l'acide méthanoïque.

2) Calculer le coefficient de dissociation de cet acide, et en déduire la valeur

3) Déterminer, les concentrations des espèces chimiques présentes.

4) Calculer le pH d'une solution d'acide formique de concentrations respectivement égales à $10^{-3}$ et $10^{-4}$M. Déterminer le coefficient de dissociation dans les deux cas et conclure.

*BONNE CHANCE*

Module: Chimie des Solutions

# Corrigé type

Niveau: 2ème Année Ingenieur

## Exercice 1:

1- La conductivité de la solution obtenue:

$n(Cl^-) = C_1V_1 + C_2V_2 = 0,2 \times 5 \times 10^{-3} + 0,8 \times 1,25 \times 10^{-2} = 2 \times 10^{-3}$ mol.

(0,25) $[Cl^-] = \dfrac{n[Cl^-]}{V} = \dfrac{2 \times 10^{-3}}{0,2+0,8} = 2 \cdot 10^{-3}$ mol/l $= 2$ mol/m$^3$.

(0,25) $n(K^+) = C_2V_2 = 0,2 \times 5 \times 10^{-3} = 1 \times 10^{-3}$ mol.

(0,25) $[K^+] = \dfrac{n(K^+)}{V} = \dfrac{1 \cdot 10^{-3}}{0,2+0,8} = 1 \cdot 10^{-3}$ mol$\cdot$l$^{-1}$ $= 1 \cdot 10^{-3}$ mol $= 1$ mol/m$^3$.

$n(Na^+) = C_2V_2 = 0,8 \times 1,25 \times 10^{-3} = 1 \cdot 10^{-3}$ mol.

(0,25) $[Cl^-] = \dfrac{n(Cl^-)}{V} = \dfrac{1 \cdot 10^{-3}}{0,2+0,8} = 1 \cdot 10^{-3}$ mol/l $= 1$ mol/m$^3$.

(0,25) $\sigma = \lambda_{Cl^-} \cdot [Cl^-] + \lambda_{K^+} \cdot [K^+] + \lambda_{Na^+} \cdot [Na^+]$.

$\sigma = 7,63 \times 10^{-3} \times 2 + 7,35 \cdot 10^{-3} \times 1 + 50,1 \cdot 10^{-3} \times 1 = 2,76 \times 10^{-2}$ mol/l (0,25)

2. Valeur de la conductance G:

(0,25) $G = \sigma \cdot \dfrac{S}{L}$

$G = 2,76 \times 10^{-2} \times \dfrac{10^{-4}}{1 \cdot 1 \cdot 10^{-2}} = 2,5 \times 10^{-4}$ S.

(0,25)

## Exercice 02 :

On considère la pile symbolisée par :

$Cu \mid Cu^{+2}(0,1M) \,//\, Fe^{+2}(0,1M) \mid Fe$ .

1) Le calcul du potentiel

Demi-Réaction d'oxydo-réduction :

Electrode de cuivre.

$Cu^{+2} + 2e^- \longrightarrow Cu$.

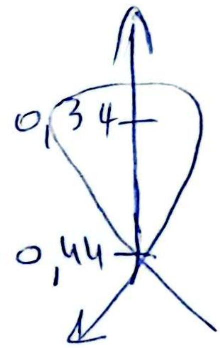
$\Delta E = E_{Cu} - E_{Fe}$ 0

Electrode fer :

$Fe^{+2} + 2e^- \longrightarrow Fe$.

2) Indiquer l'anode, la cathode et donner les demi-réactions ;

On constate que : $E_{Cu^{+2}/Cu} > E_{Fe}$ :




- l'electrode de cuivre st la : cathode 0,1

- l'elctrode de fer st l'anode 0,1

on a une oxydation de fer et réduction à la cathode (cuivre). Ceci permet d'écrire : 0,1

0,1 dans l'electrode de cuivre :

$Cu^{+2} + 2e^- \longrightarrow Cu$. 0,1

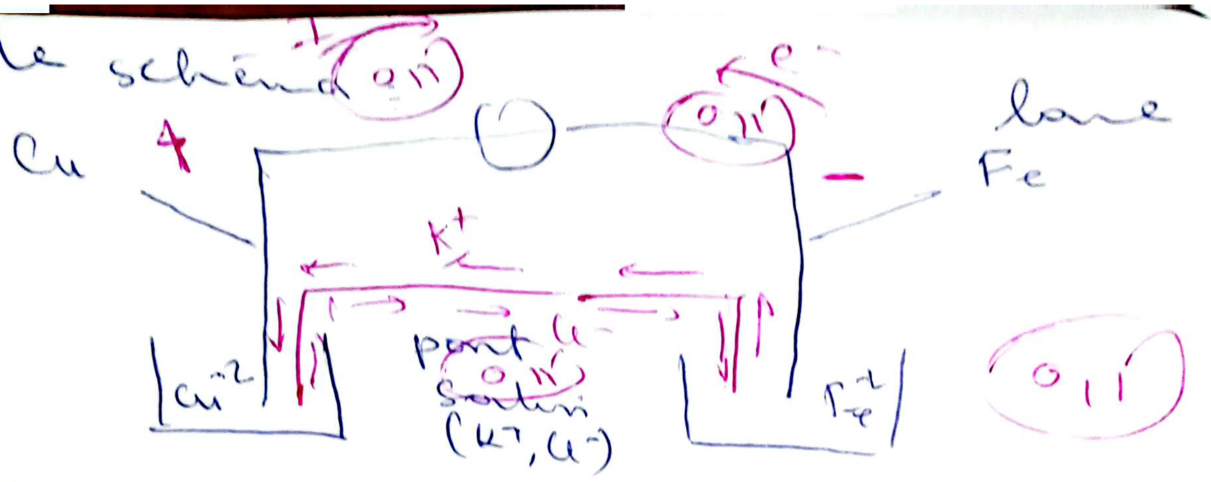
dans l'electrode de fer :

$Fe \longrightarrow Fe^{+2} + 2e^-$ 0,1

la réaction :

$Cu^{+2} + Fe \longrightarrow Cu + Fe^{+2}$.

Le schéma 0,1




cathode: reduction

$Cu^{+2} + 2e^- \rightarrow Cu$

0,1

Anode: oxydation:

$Fe \rightarrow Fe^{+2} + 2e^-$

0,1

Exercice 3 :

A - On considère une solution d'acide fluorhydrique (HF) 0,1 mol.l$^{-1}$ dans laquelle cet acide est dissocié à 8 %

1) Calculer à l'équilibre, les concentrations HF, $F^-$, $H_3O^+$

Le coefficient de dissociation noté α est défini par :

α =

$$HF \rightleftharpoons H^+ + F^- \quad 0,1$$

$\alpha = 0,08$ 0,1

| | HF | $H^+$ | $F^-$ |
|---|---|---|---|
| t = 0 | 0,1 | $10^{-7}$ | 0 |
| | $C(1-\alpha)$ | $C\alpha$ | $C\alpha$ |

1) à l'équilibre :

$[H_3O^+] = C\alpha$. 0,1

$[H_3O^+] = C\alpha = 0,1 \times 0,08 = 8 \times 10^{-3}$ M $\quad \alpha = 0,08$.

$[F^-] = [H_3O^+] = C\alpha = 0,1 \times 0,08 = 0,008$ M $= 8 \times 10^{-3}$ M

$[HF] = C(1-\alpha) = 0,1(1-0,08) = 0,092$ M. 0,1

0,1

2) Déterminer le pH de la solution considérée ainsi que $K_a$ :

$$pH = -\log[H_3O^+] = -\log(0,008) = 2,097.$$

0,1

Le calcul $pK_a$ :

0,1

$$pH = \frac{1}{2}(pK_a - \log C_A) \Rightarrow \text{Si } \alpha \leq 0,05, \ 5\%$$

0,1

$$K_a = \frac{[F^-][H_3O^+]}{[HF]} = \frac{[H_3O^+]^2}{[HF]} = \frac{(C\alpha)^2}{C(1-\alpha)} = \frac{(8,0 \times 10^{-3})^2}{0,092}$$

$$= 6,96 \times 10^{-4}.$$

$\Rightarrow pK_a = 3,16$

B - Le pH = 2,9 , [HCOOH] = $10^{-2}$ M.

$$HCOOH \longrightarrow H^+ + COO^-$$ 0,1

$$HCOOH + H_2O \rightarrow HCOO^- + H_3O^+$$

$C = 0{,}1$ 0 $10^{-7}$

$C(1-\alpha)$ $C\alpha$ $C\alpha$

2) Le calcul du coefficient de dissociation

$pH = 2{,}9 \Rightarrow [H_3O^+] = 10^{-2{,}9}\,M$ (0,1) (0,1)

$$[H_3O^+] = C\alpha \Rightarrow \alpha = \frac{[H_3O^+]}{C} = \frac{10^{-2{,}9}}{10^{-2}} \Rightarrow \alpha = 0{,}126$$

3) Déduire, les concentrations des espèces chimiques présentes :

$[HCOO^-] = [H_3O^+] = C\alpha = 0{,}1 \times 0{,}126 = 0{,}126$. (0,1)

$[HCOOH] = C(1-\alpha) = 0{,}1(1 - 0{,}0126) = 8{,}74 \times 10^{-3}\,M$.

4) Le calcul de pH d'une solution d'acide formique $10^{-3}$ et $10^{-4}\,M$. (0,1)

(0,1) $\alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{1{,}82 \times 10^{-4}}{0{,}1}} = \sqrt{0{,}182}$

$\alpha = 0{,}425 > 0{,}05$.

$$K_a = \frac{[H_3O^+]^2}{C(1-\alpha)}$$

$$K_a = \frac{C\alpha^2}{1-\alpha} =$$

$K_a = 1{,}82 \times 10^{-4}$

$pK_a = 3{,}74$.

# EXAM

**Exercise 1: (10 pts)**

Let the elements of the periodic table be : A, B, C, D, E, F and G.

| Element | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| Group | $II_A$ | $II_A$ | $II_B$ | $V_B$ | $I_A$ | $VII_A$ | $VIII_A$ |
| Période | 4 | 5 | 4 | 4 | 5 | 4 | 4 |

**1°/ Establish the electronic configuration of the elements and deduce the Z number of each element. Indicate the family to which each element belongs?**

**2°/ Give the most stable ion likely to form as well as the metallic character for each element?**

**3°/ Assign each element its atomic radius and electronegativity from the following values :**

| Radius | 2.48 | 1.74 | 1.17 | 1.25 | 1.91 | 1.22 |
|---|---|---|---|---|---|---|
| **Electronegativity** | 1.66 | 1.04 | 2.74 | 0.99 | 1.45 | 0.89 |

**5°/ Deduce the smallest element? Deduce the most oxidizing element?**

**6°/ Knowing the electronegativity of hydrogen H (2.2), predict the main character (ionic, polar and covalent) of the bonds in the following molecules: H-A, H-F. Justify your answer.**

**Exercise 2: (6 pts)**

**Copper crystallizes in the face-centered cubic system, its density ρ has the value 8920 Kg/m3.**

**1) Represent the cell?**

**2) What is the difference between a molecular crystal and a macromolecular crystal? Give two examples.**

**3) Give the projection of the cell on the (XOY) plane?**

**4) Draw the rows [011] and [201] and draw the lattice planes from the Miller indices (011) and (632)?**

**5) Calculate the atomic radius of copper? Given: Mcu=63.5 g/mol, Na=$6.02310^{23}$.**

**Exercise 3: (4 pts)**

**We oxidize 0.08 kg of methane $CH_4$ in 1.524 kg of air. (Remember that there is 21% oxygen in the air).**

**1. Demonstrate the relationship between the mass fraction and the molar fraction?**

**2. Write the equation of the reaction.**

**3. Establish the reaction progress table and determine the maximum progress and the limiting reagent?**

**4. What is the molar composition of the final state?**

**Data: MH=1 g/mol; MC=12g/mol; MO=16g/mol.**

**GOOD LUCK**

Module: Chimie Minerale

Niveau: 2ème Année GPC

# Corrigé type

## Exercice 01: 1ct / 2°

1/ Etablir la configuration electronique des éléments et déduire le numéro Z et indication de la famille

| Element | Groupe | Periode | Configuration 0,25 | Z 0,25 | La famille 0,25 | ion 0,25 | caractère métallique 0,25 |
|---|---|---|---|---|---|---|---|
| A | $II_A$ | 4 | $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$ | 20 (Ca) | Métal | $Ca^{+2}$ | Métal |
| B | $II_A$ | 5 | $5s^2$ | 38 (Sr | Métal | $Sr^{+2}$ | Métal |
| C | $II_B$ | 4 | $3d^{10} 4s^2$ | Z=30 (Zn) | Métal de Transition | $Zn^{+2}$ | Métal |
| D | $V_B$ | 4 | $3d^3 4s^2$ | 23 (V) | Métal | $V^{+3}$ | Métal |
| E | $I_A$ | 5 | $5s^1$ | 37 (Rb | Métal | $Rb^+$ | Métal |
| F | $VII_A$ | 4 | $4p^5 4s^2$ | 35 (Br) | Halogènes | $Br^-$ | non-metal |
| G | $VIII_A$ | 4 | $4p^6 4s^2$ | 36 (Kr | gaz rare | \ | gaz rare |

3/ Attribuer à chaque élément:

Classement:

| Elements | E ($_{37}Rb$) | B ($_{38}Sr$) | A ($_{20}Ca$) | D ($_{23}V$) | C ($_{30}Zn$) | F ($_{35}Br$) |
|---|---|---|---|---|---|---|
| | 0,25 | 0,25 | 0,25 | 0,25 | 0,25 | 0,25 |
| Rayons | 2,48 | 1,91 | 1,74 | 1,25 | 1,22 | 1,17 |
| Electronégativité | 0,89 | 0,99 | 1,04 | 1,45 | 1,66 | 2,74 |

Pour le rayon atomique : $r_a$ :

0,4

A, B, C, F ; même période 4 ; $r({}_{20}A) > r({}_{23}D) > r({}_{30}C) > r({}_{35}F)$

B et E ; même période ; $r({}_{37}E) > r({}_{38}B)$. 0,25

A et B ; même groupe ; $r({}_{38}B) > r({}_{20}A)$. 0,25

Donc : $r({}_{37}E) > r({}_{38}B) > r({}_{20}A) > r({}_{23}D) > r({}_{30}C) > r({}_{35}F)$

1) Pour l'électronégativité :

$E({}_{35}F) > E({}_{30}C) > E({}_{23}D) > E({}_{20}A) > E({}_{38}B) > E({}_{37}E)$. 0,25

* L'élément le plus grand est : ${}_{38}F$. 0,5
* " " " oxydant. En A ; ${}_{37}F$. 0,5

6) Connaissant d'électronégativité de H (2,2) :

H – A : H – Ca : ⇒ $\Delta\chi = 2,2 - 1,04 = 1,16$ liaison ionique 0,1

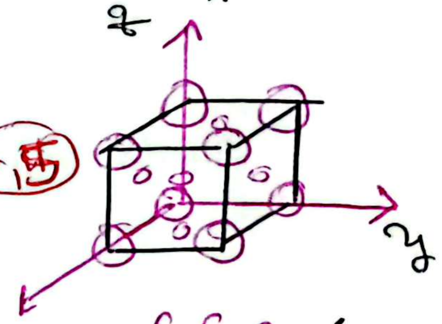
H – F : H – Br : ⇒ $\Delta\chi = 0,54$ ⇒ liaison covalente

Exercice 02 :

Le cuivre cristallise dans le système C.F.C. 0,1

$\rho = 8920\ kg/m^3$

1) La représentation de la maille. 0,5



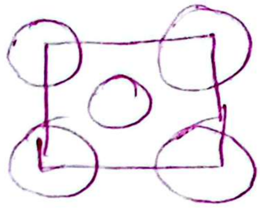
C.F.C.

2) La différence entre un cristal moléculaire et macromoléculaire est : cristal moléculaire est : sont formés des empilements réguliers de molécules ; 0,5
exple : $CO_2$, $I_2$, $H_2O$. 0,25

Cristal macromoléculaire : la notion de molécule en tant qu'entité chimique indépendante est remplacée par le cristal qui constitue la molécule. 0,5
exple : NaCl, $CaF_2$, 0,25

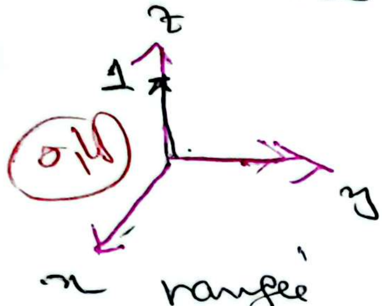
Scanné avec CamScanner

Donner la projection sur le plan XOY:

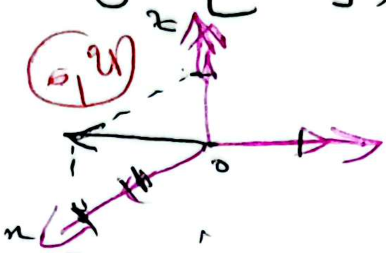


(0 1 1).

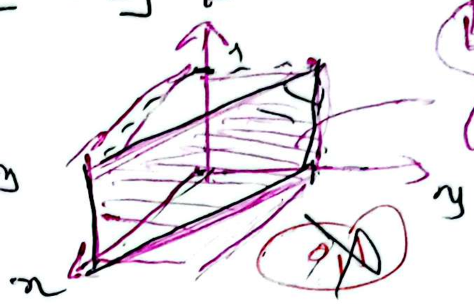
8) Tracer les rangées [001], [201]



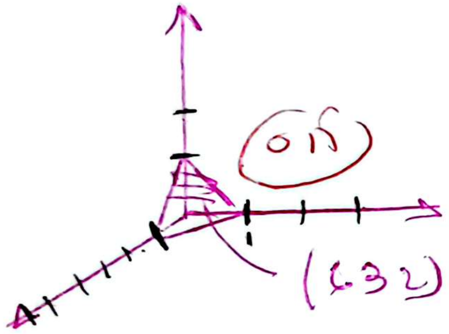

rangée [001]



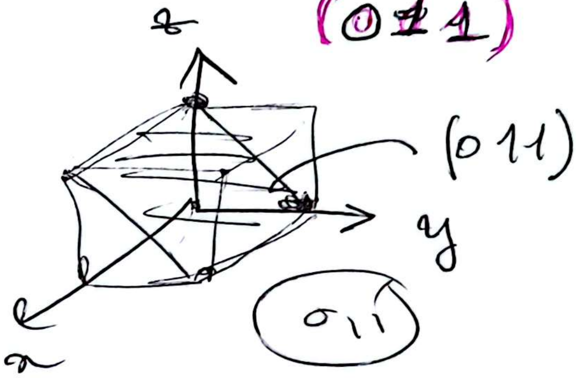

rangée [201]




plan réticule (0 1 1)




plan réticulaire (6 3 2)




Le calcul de du rayon atomique $r_{at}$

masse volumique = 8920 $Kg.m^3$ = masse des 4 atomes de cuivre propre à la maille / volume de la maille ($m^3$)

masse 4 atomes de cuivre = $4 \times 63,5 \times 10^{-3} / 6,02 \times 10^{23}$

$= 4,73 \times 10^{-29}\ m^3$.

$\boxed{a = 3,61 \times 10^{-10}\ m}$.

$r = 1,414a/4 = 1,414 \times 36,1 \times 10^{-10} / 4 = \boxed{127,8\ pm}$

Exercice 03 :

1) La relation entre la fraction massique et la fraction ~~volum~~ molaire :

$$x_i = \frac{n_i}{n_T} , \quad w_i = \frac{m_i}{m_T} .$$

$$x_i = \frac{w_i / M_i}{\sum w_i / M_i} \quad 1$$

2. Equation de la relation :

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O \quad 0,1$$

$m_{CH_4} = 0,80 \text{ Kg} = 80 \text{ g}.$

$M = 16 \text{ g/mol} \Rightarrow n_{CH_4} = \frac{80}{16} = 5 \text{ mol} \quad 0,5$

• Masse d'air : $m_{air} = 1524,21 \text{ Kg}.$

$m_{O_2} = 0,21 \times 1524,21 = 0,32 \text{ Kg} = 320 \text{ g}$ 0,1 . $M_{O_2} = 32 \text{ g/mol}$

Pour $n_{CH_4}$ 5 mol 0,1

Pour $n_{O_2} = 2 \times 5 = 10 \text{ mol}.$ 0,25

$\Rightarrow$ Alors : l'avancement maximal $x = 5 \text{ mol}.$ 0,1

| | $CH_4 + 2O_2$ | $\longrightarrow$ | $CO_2 + 2H_2O$ | |
|---|---|---|---|---|
| $t=0$ | $n_1$ | $n_2$ | 0 | 0 |
| $t$ | $n_1 - x$ | $n_2 - 2x$ | $x$ | $2x$ |
| $t_{eq}$ | $n_1 - x_{max}$ | $n_2 - 2x_{max}$ | $x_{max}$ | $2x_{max}$ |

0,1

niversité Larbi Ben M'Hidi - Oum El Bouaghi
Faculté des Sciences et Sciences Appliquées
Département des sciences et technique

ÉLECTRICITE INDUSTRIEL

Dr. Hamdi N

G MECANIQUE

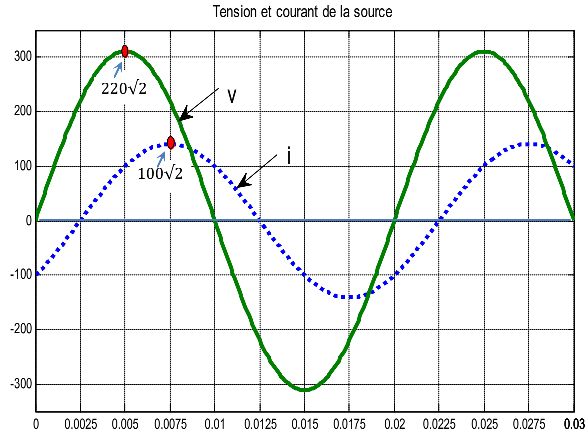
*Durée: 01H30*

# Corrigé type d'examen

**Question :** *(1pts /Q)*

1. Pourquoi transporte-on l'énergie électrique à très haute tension?
   - L'augmentation de la tension permet de diminuer le courant.
   - La réduction du courant permet d'utiliser de plus petites tailles de conducteurs.
2. Donner trois effets du courant électrique dans les systèmes physiques et un exemple d'un composant électronique qui donne lieu à chaque effet :
   - Effet thermique (calorifique) — Résistance de chauffage,
   - Effet magnétique (inductif) — Transformateur, moteur,

   .
3. Quel est le rôle du noyau ferromagnétique dans un circuit magnétique?
   - Condenser et canaliser les lignes de champ magnétique
4. Donner le principe de fonctionnement d'un transformateur électrique
   - La bobine primaire est alimentée par une source de tension variable V1, qui permet la circulation d'un courant I1, et par le principe d'Ampère, le courant primaire I1 crée un champ magnétique alternatif B dans le noyau ferromagnétique. Par le principe de Faraday, le champ magnétique variable B crée une tension induit (f.e.m) V2 dans la bobine secondaire,
5. Dans un circuit à courant continu, que deviennent le courant et la tension dans une bobine et dans un condensateur au régime permanent ?
   - Bobine considérée comme court-circuit
6. Donner brièvement le principe de fonctionnement d'une machine à courant continu (mode génératrice) ;
   - L'inducteur est alimenté par une source de tension continue réglable permettant d'adapter le courant d'excitation Iex.
   - Le rotor de la machine est entraîné par une source extérieure à la fréquence de rotation n, elle débite un courant d'intensité I dans un rhéostat de charge.
7. C'est quoi l'inducteur dans la machine à courant continu ;
   - C'est le stator ou la partie fixe de la machine
8. Comment réduire les pertes par courant de Foucault dans les matériaux ferromagnétiques
   - Par le feuilletage de circuit magnétique
9. Quel type de matériau est utilisé dans les machines électriques pour réduire les pertes magnétiques.
   - Les matériaux ferromagnétiques
10. Une charge Z est connectée à une source de tension sinusoïdale monophasée. Les formes d'onde de la tension et du courant est données sur la figure suivante.




**Déduire :**

La fréquence $f\ (Hz)$ et la période $T\ (s)$ de ces ondes

T=0.02s    f=50Hz

La valeur efficace de courant $i$et de tension$V$ ;

V=220v    i=100A

Le déphasage $\varphi\ (rad)$ entre la tension et le courant

$$\varphi = \frac{\pi}{4}$$

Type de la charge$Z$ …inductive

11. s grandeurs électriques d'une machine à café sont les suivantes : P = 2000 W  f = 50 Hz  U = 230 V , Quelle est la valeur de la résistance et de l'intensité pour ce récepteur ?

| Intensité : | | | Résistance : | | |
|---|---|---|---|---|---|
| | ➡ | 8,69 A | | | 213 |
| | | 35.6 A | | | 50 |
| | | 0,56 A | | ➡ | 26.45 |

**Ministry of Higher Education and Scientific Research**
**Larbi Ben M'hidi University, Oum El Bouaghi**

**Faculty of Sciences and Applied Sciences** **2$^{\text{nd}}$ Year Engineering (S3)**

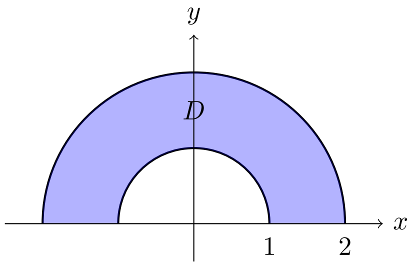
# Final Exam Correction: Applied Mathematics

**Exercice 1:** (07pts)

1. (a) Let $f$ be a continuous function on a bounded domain $D$ in $\mathbf{R}^2$. The variable change $\varphi(r,\theta) = (x,y) = (r\cos\theta, r\sin\theta)$, in polar coordinates, where $r \geq 0$ and $\theta \in [0, 2\pi]$, transforms the domain $D$ into $\Delta = \varphi^{-1}(D)$ and allows us to compute the double integral of $f$ via the formula:

$$\iint_D f(x,y)\, dxdy = \iint_\Delta f(r\cos\theta, r\sin\theta)\, |J|\, drd\theta \text{ with } |J| = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}.$$ 0.5pt

(b) Graphical representation of the domain: $D = \left\{(x,y) \in \mathbf{R}^2,\ 1 \leq x^2 + y^2 \leq 4 \text{ and } y \geq 0\right\}.$ 0.5pt




(c) $D$ is transformed into polar coordinates $(r,\theta)$ as:

$$\Delta = \left\{(r,\theta) \in \mathbf{R}^2,\ 1 \leq r \leq 2 \text{ and } 0 \leq \theta \leq \pi\right\}.$$ 1pt

$$\begin{aligned}
\iint_D (x^2 + y)\, dxdy &= \iint_\Delta \left((r\cos\theta)^2 + r\sin\theta\right) |J|\, drd\theta \\
&= \int_0^\pi \left[\int_1^2 \left(r^2\cos^2\theta + r\sin\theta\right) rdr\right] d\theta \\
&= \int_0^\pi \left[\frac{r^4}{4}\cos^2\theta + \frac{r^3}{3}\sin\theta\right]_1^2 d\theta \\
&= \int_0^\pi \left(\frac{15}{4}\cos^2\theta + \frac{7}{3}\sin\theta\right) d\theta \\
&= \int_0^\pi \left(\frac{15}{8}\left(\theta + \frac{1}{2}\sin 2\theta\right) - \frac{7}{3}\cos\theta\right) d\theta \\
&= \frac{15}{8}\pi + \frac{14}{3}.
\end{aligned}$$ 1pt

2. $D_2 = \left\{(x,y,z) \in \mathbf{R}^3, \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1\right\}$ where $a, b,$ and $c > 0$.
The volume of the domain $D_2$ is computed by the following integral:

$$Vol\,(D_2) = \iiint\limits_{D_2} dxdy.$$ 0.5pt

Using the changing of variables: $\begin{cases} X = \frac{x}{a} \\ Y = \frac{y}{b} \\ Z = \frac{z}{c} \end{cases}$

We obtain the domain: $D_2' = \left\{(X,Y,Z) \in \mathbf{R}^3, X^2 + Y^2 + Z^2 \leq 1\right\}$. Using spherical coordinates:

$$\begin{cases} X = r\cos\theta\cos\varphi \\ Y = r\sin\theta\cos\varphi \\ Z = r\sin\varphi \end{cases} \text{ where } \begin{cases} 0 \leq r \leq 1 \\ 0 \leq \theta \leq 2\pi \\ -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2} \end{cases}$$ 0.5pt

The Jacobian determinant is:

$$|J| = r^2\cos\varphi.$$

Thus, we obtain:

$$\begin{aligned} Vol\,(D_2) &= abc \iiint\limits_{D_2'} dXdYdZ \\ &= abc \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\pi} \int_0^1 r^2 \cos\varphi dr d\theta d\varphi \\ &= abc \left(\int_0^1 r^2 dr\right)\left(\int_0^{2\pi} d\theta\right)\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\varphi d\varphi\right) = \frac{4}{3}\pi abc. \end{aligned}$$ 1pt

0.25pt

3. (a) The function $t \to \frac{\sin^2 t}{1+t^2}$ is continuous, thus Riemann integrable over $[0, +\infty[$.

0.25pt

There is a convergence issue or a singularity at $+\infty$.
Moreover, the function is positive, so we will demonstrate the convergence of the integral

0.25pt

using the comparison criterion. We have $\forall t \in [0, +\infty[$:

$$0 \leq \frac{\sin^2 t}{1+t^2} \leq \frac{1}{1+t^2},$$ 0.25pt

Since $\int_0^{+\infty} \frac{1}{1+t^2} dt$ converges due to the finite limit of its primitive $\arctan t$ at $+\infty$ (which equals

0.5pt

$\frac{\pi}{2}$). This proves that $\int_0^{+\infty} \frac{\sin^2 t}{1+t^2} dt$ converges.
(b) Is the convergence absolute?
We have:

$$\int_0^{+\infty} \left|\frac{\sin^2 t}{1+t^2}\right| dt = \int_0^{+\infty} \frac{\sin^2 t}{1+t^2} dt \text{ since } \frac{\sin^2 t}{1+t^2} \geq 0,$$ 0.25pt

Thus, $\int_0^{+\infty} \left|\frac{\sin^2 t}{1+t^2}\right| dt$ converges, which means that $\int_0^{+\infty} \frac{\sin^2 t}{1+t^2} dt$ converges absolutely. 0.25pt

**Exercice 2**: (4pts)

1. If $0 \le x < 1$, $nx^n \to 0$ as $n \to +\infty$. 0.5pt

   If $x = 1$, $f_n(x) = 0$.

   Thus, the sequence of functions $(f_n)_{n\in\mathbb{N}^*}$ converges pointwise to 0 as $n \to +\infty$. 0.5pt

2. We are looking for the maximum of the function $f_n$ on the interval $[0, 1]$. To do this, we calculate:

$$f_n'(x) = nx^{n-1}(1-x)^{\alpha-1}\left(n - (n+\alpha)x\right).$$ 0.5pt

By setting $x_n = \frac{n}{n+\alpha}$, we see that $f_n$ is increasing on $[0, x]$ and decreasing on $[x, 1]$. Therefore, $f_n$ reaches its maximum at $x_n$, and this maximum is:

$$M_n = f_n(x_n) = n\left(1+\frac{\alpha}{n}\right)^{-n}\left(\frac{\alpha}{n+\alpha}\right)^{\alpha} \underset{n\to+\infty}{\rightsquigarrow} \left(\frac{\alpha}{e}\right)^{\alpha} n^{1-\alpha}.$$ 0.5pt

Since $M_n \to 0$ as $n \to +\infty$ if and only if $\alpha > 1$, the sequence of functions $(f_n)_{n\in\mathbb{N}^*}$ converges uniformly to 0 on $[0, 1]$ if and only if $\alpha > 1$. 0.5pt

3. Assume $0 < \alpha \le 1$. If $a \in [0, 1]$ is fixed, since $\lim_{n\to+\infty} x_n = 1$, for sufficiently large $n$, $x_n > a$, so the function $f_n$ is increasing on the interval $[0, a]$. Moreover, as $n \to +\infty$: 0.5pt

$$\sup_{0\le x\le a} |f_n(x)| = f_n(a) \to 0.$$ 0.5pt

Thus, the sequence of functions $(f_n)_{n\in\mathbb{N}^*}$ converges uniformly on the interval $[0, a]$ for any $a \in [0, 1[$. 0.5pt

**Exercice 3**: (09 pts) Consider the PDE:

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} = 0, \tag{E}$$

1. The equation (E) is a PDE of order 2, linear, and homogeneous. 0.75pt

2. The type of (E): the coefficients are $A = 1,\ B = 4,\ C = 0$

$$\Delta = B^2 - 4AC = 16 > 0.$$

Thus, (E) is of hyperbolic type. 1pt

**Let's find the canonical form of the PDE (E)**

The characteristic equation of (E) is:

$$\begin{aligned} A\left(\frac{dy}{dx}\right)^2 - B\frac{dy}{dx} + C &= 0 \Rightarrow \left(\frac{dy}{dx}\right)^2 - 4\frac{dy}{dx} = 0 \\ &\Rightarrow \frac{dy}{dx}\left(\frac{dy}{dx} - 4\right) = 0. \end{aligned}$$ 0.5pt

Thus, the solutions of the characteristic equation are:

$$\begin{cases} \dfrac{dy}{dx} = 0 \\ \dfrac{dy}{dx} - 4 = 0 \end{cases}$$

which are first-order differential equations with separated variables that can easily be solved

$$\begin{cases} \frac{dy}{dx} = 0 \\ \frac{dy}{dx} = 4 \end{cases} \Rightarrow \begin{cases} y = c_1 \\ y = 4x + c_2 \end{cases} \Rightarrow \begin{cases} \varphi(x, y) = y = c_1 \\ \phi(x, y) = y - 4x = c_2 \end{cases}, \text{ with } c_1,\ c_2 \in \mathbf{R}$$ 1pt

Let the change of variables be: $\begin{cases} s = c_1 = y \\ t = c_2 = y - 4x \end{cases}$

By calculating the partial derivatives, we get:

$$\begin{pmatrix} \mathbf{0.75} \\ \times\ \mathbf{3} \\ \mathbf{pts} \end{pmatrix} \begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial s}\dfrac{\partial s}{\partial x} + \dfrac{\partial u}{\partial t}\dfrac{\partial t}{\partial x} = -4\dfrac{\partial u}{\partial t}, \\ \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial u}{\partial x}\right) = -4\dfrac{\partial}{\partial x}\left(\dfrac{\partial u}{\partial t}\right) = -4\left[\dfrac{\partial}{\partial x}\left(\dfrac{\partial u}{\partial t}\right)\dfrac{\partial s}{\partial x} + \dfrac{\partial}{\partial t}\left(\dfrac{\partial u}{\partial t}\right)\dfrac{\partial t}{\partial x}\right] = 16\dfrac{\partial^2 u}{\partial t^2}, \\ \dfrac{\partial^2 u}{\partial y \partial x} = \dfrac{\partial}{\partial y}\left(\dfrac{\partial u}{\partial x}\right) = -4\left[\dfrac{\partial}{\partial s}\left(\dfrac{\partial u}{\partial t}\right)\dfrac{\partial s}{\partial y} + \dfrac{\partial}{\partial t}\left(\dfrac{\partial u}{\partial t}\right)\dfrac{\partial t}{\partial y}\right] = -4\dfrac{\partial^2 u}{\partial s \partial t} - 4\dfrac{\partial^2 u}{\partial t^2} \end{cases}$$

Finally,

$$16\frac{\partial^2 u}{\partial t^2} - 16\frac{\partial^2 u}{\partial s \partial t} - 16\frac{\partial^2 u}{\partial t^2} - 4\frac{\partial u}{\partial t} = -16\frac{\partial^2 u}{\partial s \partial t} - 4\frac{\partial u}{\partial t} = 0,$$

thus the canonical form of (E) is:

$$4\frac{\partial^2 u}{\partial s \partial t} + \frac{\partial u}{\partial t} = 0.$$ 1pt

Solution of the PDE (E) by the method of characteristics

To solve (E), we simply solve the canonical form of (E)

$$\begin{aligned} 4\frac{\partial^2 u}{\partial s \partial t} + \frac{\partial u}{\partial t} &= 0 \Leftrightarrow \frac{\partial}{\partial t}\left(4\frac{\partial u}{\partial s} + u\right) = 0 \\ &\Leftrightarrow\ 4\frac{\partial u}{\partial s} + u = f(s). \end{aligned}$$ 0.5pt

$(*)$ is a first-order differential equation with a non-homogeneous term that can easily be solved by the method of variation of constants:

The solution without the non-homogeneous term:

$$u = ke^{-\frac{1}{4}s}, \;\; s \in \mathbf{R}. \quad \text{0.5pt}$$

The solution with the non-homogeneous term:

Let $u_p = k(x)\, e^{-\frac{1}{4}s}$, then $\dfrac{\partial u}{\partial s} = k'(x)\, e^{-\frac{1}{4}s} - \frac{1}{4}k(x)\, e^{-\frac{1}{4}s}$.

Substitute into $(*)$:

$$4\left(k'(x)\, e^{-\frac{1}{4}s} - \frac{1}{4}k(x)\, e^{-\frac{1}{4}s}\right) + k(x)\, e^{-\frac{1}{4}s} = f(s)$$

$$\begin{aligned} &\Leftrightarrow \quad 4k'(x)\, e^{-\frac{1}{4}s} = f(s) \Leftrightarrow k'(x) = \frac{1}{4}f(s)\, e^{\frac{1}{4}s} \\ &\Leftrightarrow \quad k(x) = \frac{1}{4}\int f(s)\, e^{\frac{1}{4}s} ds. \end{aligned}$$

Thus,

$$u_p = \frac{1}{4}e^{-\frac{1}{4}s}\int f(s)\, e^{\frac{1}{4}s} ds. \quad \text{0.5pt}$$

Thus, the general solution of $(*)$ is:

$$u(t,s) = ke^{-\frac{1}{4}s} + \frac{1}{4}e^{-\frac{1}{4}s}\int f(s)\, e^{\frac{1}{4}s} ds. \quad \text{0.5pt}$$

The general solution is

$$u(x,y) = ke^{-\frac{1}{4}y} + \frac{1}{4}e^{-\frac{1}{4}y}\int f(y)\, e^{\frac{1}{4}y} dy. \quad \text{0.5pt}$$

# Typical answer HSE Exam

| | | | |
|---|---|---|---|
| Bony | عظمي | Complications of the disease | مضاعفات المرض |
| Articulate | مفصلي | Remnants of disease | مخلفات المرض |
| Nervous | عصبي | Comprehensive prevention | الوقاية الشاملة |
| Vascular | وعائي | Directed prevention | الوقاية الموجهة |
| Accuracy | الدقة | Targeted prevention | الوقاية المستهدفة |
| Path | مسار | Health education | التثقيف الصحي |
| Common cold | نزلات البرد | professional illness | مرض مهني |
| Synergistic effects | تأثيرات تآزرية | Psychological problems | مشاكل نفسية |
| Rescue | الإنقاذ | Ultraviolet rays | الأشعة فوق البنفسجية |
| Ruin | تلف | Infrared rays | الأشعة تحت الحمراء |
| Loads or handling | الأحمال او المناولة | Electromagnetic waves | الموجات الكهرومغناطيسية |
| Crushed | المسحوقة | Disturbances | الاضطرابات |

الخطر أو الظاهرة الخطرة هي الخاصية أو القدرة الجوهرية التي يحتمل أن يتسبب بها شيء ما (مثل المواد والمعدات وأساليب العمل 1 "الخطر هو أداة والممارسات) في حدوث ضرر (إصابة أو ضعف للصحة). وبالتالي فإن الخطر هو "سبب قادر على التسبب في ضرر". "الخطر حالة، الخطر هو احتمال حدوث الضرر المحتمل في ظل ظروف الاستخدام و / أو التعرض والمدى المحتمل للضرر. المخاطرة". تعتبر المخاطر الصناعية هي احتمال وقوع حدث عرضي في موقع صناعي ويؤدي إلى عواقب فورية خطيرة على الموظفين والسكان المحيطين والممتلكات والبيئة. والمخاطرة هو مقياسها".

المخاطر الماديه الناجمة عن بيئات العمل (البيئة الحرارية ، بيئة الضوضاء ، الاهتزازات ، بيئة الضوء) والمخاطر الناجمة عن الإشعاع (الإشعاع 2 المؤين ، الأشعة فوق البنفسجية والأشعة تحت الحمراء ، الموجات الكهرومغناطيسية). غالبا ما تندرج المخاطر المرتبطة بالتعامل مع الأحمال ضمن هذه الفئة.

**(أ) الضوضاء** الضوضاء هي ظاهرة اهتزازية ميكانيكية تنتشر في وسط مرن: الهواء. يشير مصطلح الضوضاء إلى أي ظاهرة صوتية تنتج إحساسا مزعجا. الضوضاء تتميز بشدتها وتواترها ، يمكن أن يكون لها عواقب جسدية ونفسية خطيرة على الناس. يمكن أن يكون هذا التلوث الضوضائي سببا لبعض الاضطرابات في الجسم مثل الإجهاد أو اضطرابات النوم أو انخفاض السمع. الضوضاء مزعجة إلى حد ما اعتمادا على المصدر الذي تنشأ منه وحساسية الشخص الذي يستقبلها.

**(ب) خطر الاهتزاز** إنه خطر حدوث تلف عظمي مفصلي أو عصبي أو وعائي ناتج عن استخدام الأدوات الهوائية أو قيادة المركبات أو الآلات.

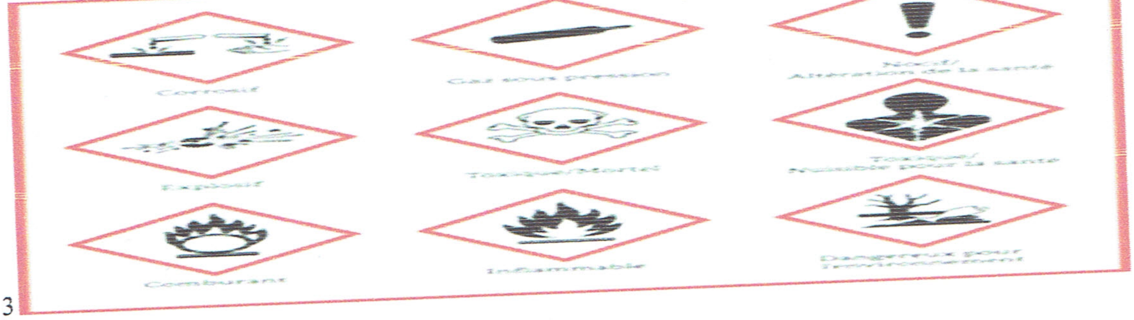
***أمثلة:*** الأدوات اليدوية الهوائية (مطرقة تعمل بالهواء المضغوط ، آلة تقطيع ، مفاتيح ربط ، إلخ.) قيادة المركبات (مركبات البضائع الثقيلة ، وسائل النقل العام) أو آلات البناء (آلات ذات المحراث الخلفي ، الضواغط ، إلخ.)

**(ج) المخاطر المرتبطة بالبيئات الحرارية** إنه مصدر لعدم الراحة ، مما قد يؤدي إلى انخفاض في اليقظة أو الدقة ، مما يزيد من خطر وقوع حادث ويمكن أن يؤدي إلى ضربة شمس أو انخفاض حرارة الجسم ، والتي يمكن أن تكون قاتلة في بعض الأحيان. في الواقع ، يمكن أن تكون الظروف الحرارية السيئة في أماكن العمل سبب الصداع وعدم الراحة في الجهاز التنفسي ونزلات البرد والألم,...

***أمثلة:*** العمل في بيئات باردة ورطبة (غرف التبريد ، المجمدات ، إلخ) تتفاقم بسبب انخفاض الحركة ؛ العمل في البيئات الحارة (أعمال الزجاج ، المسابك ، إلخ) التي تتفاقم بسبب المجهود البدني ؛العمل في عزلة في البيئات القاسية.

**(د) المناولة ومخاطر النشاط البدني** يمكن أن تسبب اتحركات سيئة التنفيذ أو المواقف غير المناسبة اضطرابات في العضلات أو المفاصل.

3



# Typical answer HSE Exam

لتنفيذ نهج الوقاية ، من الضروري الاعتماد على المبادئ العامة الرئيسية التسعة التي تحكم تنظيم الوقاية.4

أ/ لتجنب المخاطرة هو إزالة الخطر أو التعرض للخطر.

(ب) تقييم المخاطر يعني تقييم التعرض للخطر وأهميته من أجل تحديد أولويات الإجراءات الوقائية التي يتعين اتخاذها.

(ج) مكافحة المخاطر في المصدر ادراج الوقاية في أقرب وقت ممكن، ولا سيما في تصميم أماكن العمل أو المعدات أو إجراءات التشغيل.

(د) تكييف العمل مع الناس، مع مراعاة الاختلافات بين الأفراد، بهدف الحد من آثار العمل على الصحة.

(ه) مراعاة التطورات التكنولوجية يعني تكييف الوقاية مع التطورات التقنية.

و / استبدال ما هو خطير بما هو أقل خطورة يعني تجنب استخدام العمليات أو المنتجات الخطرة عندما يمكن الحصول على نفس النتيجة بطريقة ذات خطورة أقل.

ز) التخطيط للوقاية من خلال دمج التكنولوجيا والتنظيم وظروف العمل والعلاقات الصناعية والبيئة.

ح/إعطاء الأولوية لتدابير الحماية الجماعية واستخدام معدات الحماية الشخصية فقط كمكمل للحماية الجماعية إذا تبين أنها غير كافية.

ط/ إعطاء التعليمات المناسبة للموظفين يعني تدريب وإعلام الموظفين حتى يكونوا على دراية بالمخاطر والتدابير الوقائية.

: في حالة وقوع حادث في MP و AT. الخطوات التي يجب أن يتخذها الموظف في حالة إعلان حادث في العمل أو مرض مهني 5 العمل ، يستشير الضحية الطبيب أو يبلغ صاحب العمل في غضون 24 ساعة. في حالة المرض المهني ، يجب على الموظف إرسال ) في غضون 15 يوما من تشخيص المرض من قبل الطبيب CNASإعلان إلى الصندوق الوطني للتأمين الاجتماعي للعاملين بأجر ( أو التوقف عن العمل. التعويض: الحوادث في العمل والأمراض المهنية تعطي الحق في نوعين من الاستحقاقات:

- الاستحقاقات العينية: وهي تغطية بنسبة 100٪ لجميع النفقات المتكبدة نتيجة للحادث أو المرض (الأدوية ، إعادة التأهيل ، الاستشارات الطبية ، إلخ.)

- الإعانات النقدية: بدلات يومية لتعويض فقدان الراتب أو معاش العجز أو المعاش للمستحقين في حالة الوفاة

يهدف التثقيف الصحي إلى تحسين الحالة الصحية للسكان ومنع ظهور الأمراض أو تطورها أو تفاقمها من خلال تعزيز السلوكيات 6 الفردية والجماعية التي يمكن أن تساعد في الحد من مخاطر الأمراض والحوادث. وهدفها هو أن يكتسب كل مواطن طوال حياته المهارات والوسائل اللازمة لتعزيز صحته ونوعية حياته وكذلك صحة المجتمع". يمكن أن يكون التثقيف الصحي عاما أو يستهدف السكان الأكثر تعرضا للمخاطر التي يسعى إلى الحد منها.

**التعليم الجماهيري**: تم تطويره لزيادة الوعي بين جميع فئات السكان من خلال نشر الرسائل المكتوبة أو السمعية البصرية.

**التعليم الجماعي:** يستهدف مجموعات سكانية معينة.- دورات في المدارس والكليات والمدارس الثانوية (الإيدز، وما إلى ذلك)؛- الإجراءات في الشركات (الأمراض المهنية) ؛

**التعليم الفردي:** يمكن توفيره ، على سبيل المثال ، أثناء الاستشارة أو الفحص أو الحصول على منتج صحي.

**. مجالات الصحة العامة و أهدافها 7**

الصحة المهنية - الطب المهني؛ الوقاية- تعزيز الصحة (المدارس، وحركة المرور، والبيئة، وأنماط الحياة، وما إلى ذلك)؛- التطعيمات; تنظيم الرعاية-إسعاف أولي;-المستشفيات;- الطب الخاص؛ التدريب الطبي وشبه الطبي الضمان الاجتماعي

البحوث الطبية والدوائية.

اهداف الصحة العامة: 1) مراقبة الحالة الصحية للسكان.2) مكافحة الأوبئة.3) الوقاية من المرض والإصابة والعجز.4) تحسين الحالة الصحية للسكان ونوعية حياة المرضى والمعوقين والمعالين.5) المعلومات الصحية والتعليم.6) تحديد المخاطر الصحية والحد منها (البيئة ، ظروف العمل ، النقل ، إلخ.)7) الحد من التفاوتات الصحية (تعزيز الصحة، المساواة في الحصول على الرعاية).8) تنظيم النظام الصحي (الوقاية من الأمراض والإعاقات وإدارتها).

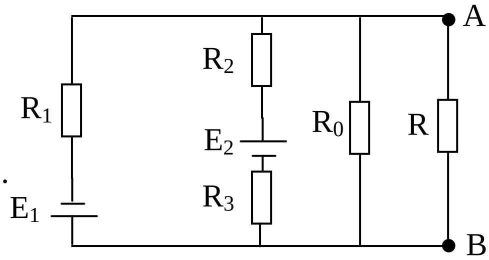
Larbi Ben M'Hidi University O.E.B — The 15/01/2025

2nd ST year Departement

Mater : Fondamental Electronic 1

# EMD1

**Exercice 01 :**




On considère le circuit de la figure ci-dessous. On donne :

$E_1$=20 V ; $E_2$=15 V ; $R_0$=30 Ω ; $R_1$=10 Ω ; $R_2$=15 Ω ; $R_3$=15 Ω ; R=30 Ω.

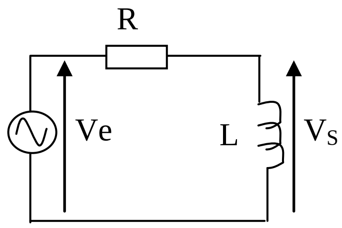
1/ On considérant la branche AB :

Déterminer l'intensité de courant de Norton $I_N$ et la résistance de Norton $R_N$.

2/ En déduire l'intensité de courant $I_R$ qui circule dans la résistance R.

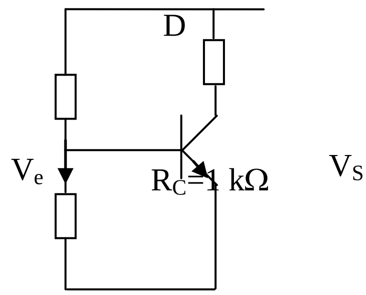
**Exercice 02 :**

Soit le filtre LR suivant :



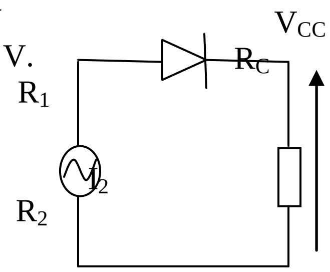

1. Quel est le type de ce filtre et quel est son ordre ?
2. Exprimer la fonction de transfert (T = Us / Ue) en fonction de R et L, ainsi que le module de T et le déphase φ.
3. La résistance R est de 1,256 kΩ et la fréquence de coupure fc est de 100 kHz. Une tension de 2 V est mesurée à la sortie du filtre lorsqu'un signal de 35,35 kHz est appliqué à l'entrée.
   Calculer la valeur de la bobine ainsi que la valeur de la tension à l'entrée du filtre

**Exercice N° 03 :**




Soit le montage suivant où la diode est au silicium avec $V_D$ = 0,7 V et de résistance dynamique $r_d$ = 20 Ω. Le signal d'entrée Ve(t) est sinusoïdale et de valeur 10 v. Déterminer la tension de sortie et le courant qui traverse $R_C$.

**Exercice 04 :**



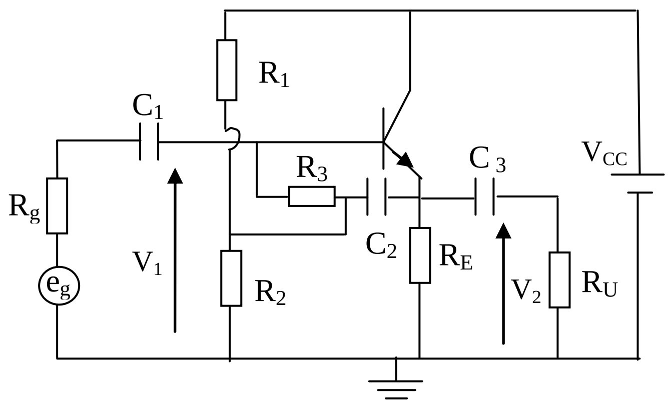

**I.** Un transistor bipolaire est utilisé dans un montage dont le point de fonctionnement a pour valeurs : $I_B$ = 2 mA ; $V_{CE}$ = 4,5 V ; $I_C$ = 200 mA ; $V_{BE}$ = 0,65 V ; $V_{CC}$ = 8 V.

- Calculer $R_C$, puis écrire l'équation de la droite de charge.
- Calculer sans négliger $I_B$, la résistance $R_1$ et $R_2$ pour que la courant $I_2$ = 13 mA

**II.** On considère l'amplificateur suivant :

a) Donner le type du montage;

b) Etablir le schéma équivalent du montage en alternatif.



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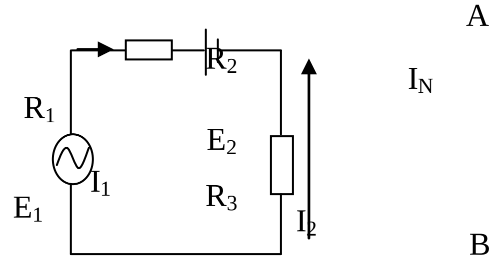
# Typical correction

**Solution Exercise 1 :**




1/ $I_N = I_1 + I_2$

$$I_N = \frac{E_2}{(R_2+R_3)} - \frac{E_1}{R_1} = -1{,}5\ A\ \ (Inverser\ le\ sens)$$

$$R_N = R_0 \parallel (R_2+R_3) \parallel R_1 = 6\ \Omega$$

2/ $I_R = \frac{R_N}{R_N+R} I_N = 0{,}25\ A$

**Solution Exercise2:**

1. See the course (Voir le cours)

   It is a first order high pass filter. (c’est un filtre passe haut du premier ordre).

2. The transfer function:

$$\underline{T}(\omega) = \frac{j\frac{\omega}{\omega_0}}{1+j\frac{\omega}{\omega_0}} \quad \text{with} \quad \omega_0 = \frac{R}{L}$$

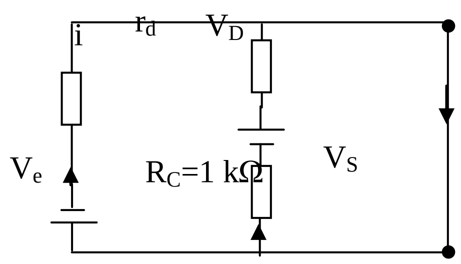
$$T(\omega) = \frac{1}{\sqrt{1+(\frac{\omega_0}{\omega})^2}}\ ;\ \arg\left(\underline{T}(\omega)\right) = \frac{\pi}{2} - arctan\frac{\omega}{\omega_0}$$

3. :

$$\omega_C = \omega_0 = \frac{R}{L} \Rightarrow L = \frac{R}{2\pi f_C} = 2\ mH$$

$$T(\omega) = \frac{1}{\sqrt{1+(\frac{f_0}{f})^2}} \Rightarrow U_e = 6{,}00\ V$$

**<u>Solution Exercice 03 :</u>**




- $V_e < 0{,}7\ V$ ; $V_S = 0$
- $V_e > 0{,}7\ V \Longrightarrow$ the diode is conducting.

$$V_S = \frac{R_C}{R_C + r_d}(V_e - V_D) = 9{,}12\ V\ \ et\ \ i = 9{,}12\ mA$$

**<u>Exercice 04 :</u>**

- <u>Calculation of $R_C$ :</u>

$$R_C = \frac{V_{CC} - V_{CE}}{I_C} = 17{,}5\ \Omega$$

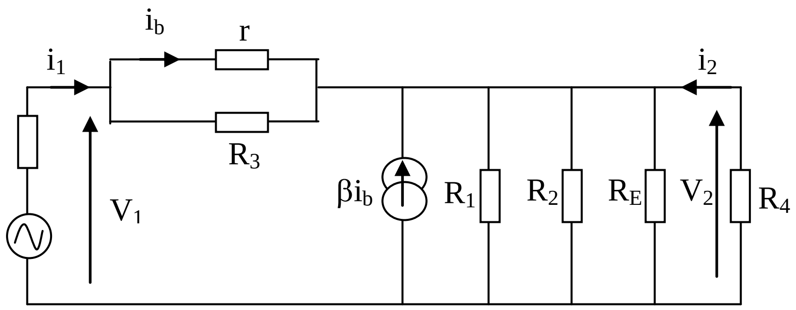
- <u>The equation of the load line</u>:

$$\boxed{I_C = -0{,}057V_{CE} + 0{,}46}$$

- <u>Calculation of $R_1$ and $R_2$</u> :

$$R_2 = \frac{V_{BE}}{I_2} = 50\ \Omega \ \ and\ R_1 = \frac{V_{CC} - R_2 I_2}{(I_2 + I_B} = 490\ \Omega$$

**Solution Exercice 04:**



ULBM FSSA Dept. ME

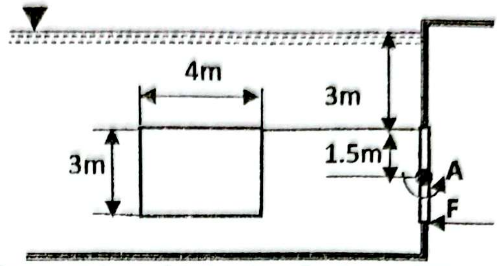
$2^{nd}$ year ST and Eng.

The 18/01/2025

# Fluid mechanics Exam

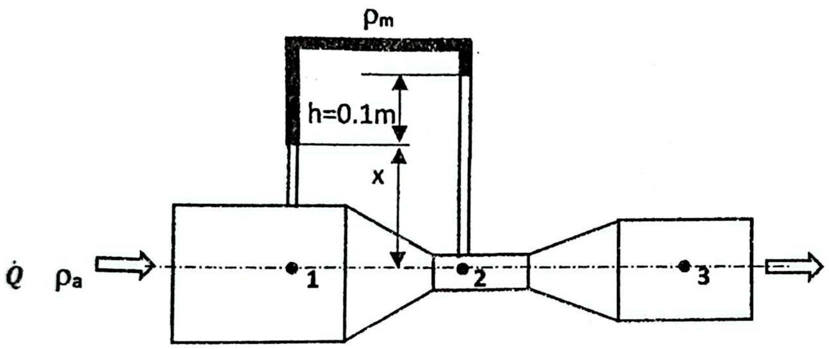
**Problem 1 (5pts)**: A rectangular gate is hinged at point A on the side of a large water tank, forming a right angle to the free surface, Figure.

1. Calculate the total force exerted on the gate.
2. Find the point of application of this force.
3. Calculate the force F that keeps the door closed.




**Problem 2 (6pts)**: The air with a density $\rho_a$=1.22 kg/m³ flows with no friction in a horizontal Venturi tube of diameters $D_1$=400 mm, $D_2$= 100 mm and $D_3$=250 mm. The Venturi tube is equipped with a manometer containing a fluid of density $\rho_m$=0.8 kg/l, attached between sections 1-2 and indicating h=0.1m. Calculate:

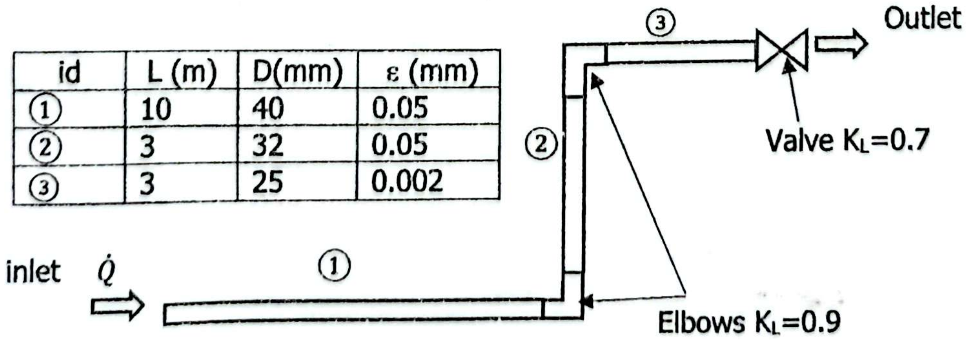
1. The pressure difference $P_1$-$P_2$ from the manometer.
2. The velocities $V_1$, $V_2$ and $V_3$ using Bernoulli's and continuity equations.
3. The volume and mass flowrates in the Venturi tube.




**Problem 3 (9pts)**: The figure shows the water (viscosity 0.113 $10^{-5}$ m²/s) distribution pipes (look at the table for dimensions). At the inlet, the pressure of water is $P_1$, and the volume flow rate is $\dot{Q}$ =4 l/s. The minor pressure loss coefficients $K_L$ are 0.9 for elbows and 0.7 for the valve, calculate the:

1. velocities $V_1$, $V_2$ and $V_3$ in the pipes.
2. Reynolds numbers $Re_1$, $Re_2$ and $Re_3$ in the pipes.
3. friction coefficients $f_1$, $f_2$ and $f_3$ from Haaland formula.
4. major pressure loss height.
5. minor pressure loss height.
6. pressure at the inlet $P_1$, if the pressure at the outlet is 2 bar.

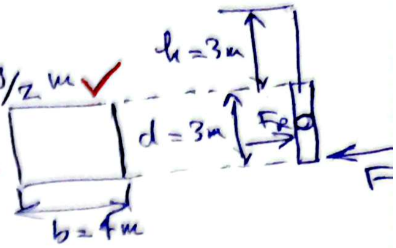
| id | L (m) | D(mm) | ε (mm) |
|---|---|---|---|
| 1 | 10 | 40 | 0.05 |
| 2 | 3 | 32 | 0.05 |
| 3 | 3 | 25 | 0.002 |



# Fluid mechanics Exam
Solution

5

## Problem 1 :




1) Calculate $F_R$ on the gate :

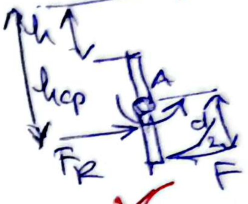
We have $F_R = \rho g h_{cg} S$ ; $h_{cg} = h + \frac{d}{2} = 9/2\ m$ ; $S = bd = 12\ m^2$

$\Rightarrow F_R = 10^3 \times 9.81 \times 4.5 \times 12 = 529740\ N$.

2) Point of application of $F_R$ :

$$h_{cp} = h_{cg} + \frac{I_{xG}}{h_{cg} S} = \left(h + \frac{d}{2}\right) + \frac{\cancel{b}d^{\cancel{3}\,2}/12}{\left(h+\frac{d}{2}\right)\cdot \cancel{b}\cancel{d}} = 4.5 + \frac{9/12}{4.5} = 4.67\ m$$

3) Calculation of $F_R$ that keeps the gate closed



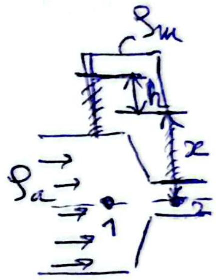

$\Sigma M/_A = 0 \Rightarrow F_R\left(h_{cp} - h - d/2\right) - F\, d/2 = 0$

$$\Rightarrow F = \frac{2\left(h_{cp} - h - d/2\right)}{d} F_R = 60037\ N.$$

V ≡ 1/2  Σ = 5 pts

6

## Problem 2




1) $P_1 - P_2$ using manometer.

$P_1 - \rho_a g(\cancel{x} + h) + \rho_m g h + \cancel{\rho_a g x} = P_2$

$\Rightarrow P_1 - P_2 = gh(\rho_a - \rho_m)$

$P_1 - P_2 = 9.81 \times 0.1(800 - 1.22) = 783.6\ Pa$

2) Re-find the $P_1 - P_2$ using Bernoulli's equation.

$P_1 + 1/2 \rho_a V_1^2 + \rho g z_1 = P_2 + 1/2 \rho_a V_2^2 + \rho g z_2$ , $z_1 = z_2$

$\Rightarrow P_1 - P_2 = 1/2 \rho_a (V_2^2 - V_1^2)$

2) Calculation of $V_1, V_2, V_3$, we have $\dot{\varphi} = VS = cte$

$V_1 S_1 = V_2 S_2 \rightarrow V_2 = V_1 \frac{S_1}{S_2}$

$$\rightarrow P_1 - P_2 = 1/2 \rho_a V_1^2 \left[\left(\frac{S_1}{S_2}\right)^2 - 1\right] \Rightarrow V_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho_a\left[\left(\frac{S_1}{S_2}\right)^2 - 1\right]}} , \frac{S_1}{S_2} = \left(\frac{D}{D}\right.$$

$$\rightarrow V_1 = \sqrt{\frac{2 \times 783.6}{1.22[4^4 - 1]}} = 2.24\ m/s.$$

$V_2 = V_1 \frac{S_1}{S_2} = 2.24\left(\frac{4}{1}\right)^2 = 36\ m/s$

$V_3 = V_1 \frac{S_1}{S_3} = 2.24\left(\frac{4}{2.5}\right)^2 = 5.73\ m/s$

3) $\dot{\varphi} = VS = 2.24 \times 3.14$
$= 0.28\ m^3/s$

$\dot{m} = \rho_a \dot{\varphi} = 0.34\ kg$

Σ = (

Problem 3 : (5)

Calculate major loss height in meters.

$$h_{L maj} = \sum_i \left(\frac{V_i^2}{2g} f \frac{l}{D}\right)_i = \frac{V_1^2}{2g} f_1 \frac{l_1}{D_1} + \frac{V_2^2}{2g} f_2 \frac{l_2}{D_2} + \frac{V_3^2}{2g} f_3 \frac{l_3}{D_3}$$

to compute the friction coefficient, we have to know the flow regime → Reynolds number & velocity

1) $V_1 = \frac{\dot{\varphi}}{S_1} = \frac{\dot{\varphi}}{\pi D_1^2/4} = \frac{4.10^{-3}}{3{,}14 \times \frac{0{,}04^2}{4}} = 3{,}18\ m/s.$

$V_2 = \frac{\dot{\varphi}}{S_2} = \frac{\dot{\varphi}}{\pi D_2^2/4} = \frac{4.10^{-3}}{3{,}14 \times \frac{0{,}032^2}{4}} = 4{,}98\ m/s.$

$V_3 = \frac{\dot{\varphi}}{S_3} = \frac{\dot{\varphi}}{\pi D_3^2/4} = \frac{4.10^{-3}}{3{,}14 \times \frac{0{,}025^2}{4}} = 8{,}15\ m/s$

2) $Re_1 = \frac{V_1 D_1}{\nu} = \frac{3.18 \times 0{,}04}{0{,}113.10^{-5}} = 112566 > 4000$

$Re_2 = \frac{V_2 D_2}{\nu} = \frac{4.98 \times 0{,}032}{0{,}113.10^{-5}} = 141026 > 4000$

$Re_3 = \frac{V_3 D_3}{\nu} = \frac{8.15 \times 0{,}025}{0{,}113.10^{-5}} = 180310 > 4000$

3) We have the Haaland formula, that gives the friction coef

$$\frac{1}{\sqrt{f}} = -1{,}8 \log\left[\left(\frac{\varepsilon/D}{3{,}7}\right)^{1.11} + \left(\frac{6.9}{Re}\right)\right].$$

$$\rightarrow \frac{1}{\sqrt{f_1}} = -1{,}8 \log\left[\left(\frac{0.05/40}{3{,}7}\right)^{1{,}11} + \left(\frac{6.9}{112566}\right)\right] = 6.65 \rightarrow f_1 = 0{,}023$$

$$\frac{1}{\sqrt{f_2}} = -1{,}8 \log\left[\left(\frac{0{,}05/32}{3{,}7}\right)^{1.11} + \left(\frac{6.9}{141026}\right)\right] = 6{,}55 \rightarrow f_2 = 0{,}023$$

$$\frac{1}{\sqrt{f_3}} = -1{,}8 \log\left[\left(\frac{0{,}002/25}{3{,}7}\right)^{1.11} + \left(\frac{6.9}{180310}\right)\right] = 7.83 \rightarrow f_3 = 0{,}016$$

$$\hookrightarrow h_{L maj} = \frac{1}{2 \times 9.81}\left[3.18^2 \times 0{,}023 \times \frac{10}{0{,}04} + 4.98^2 \times 0{,}023 \times \frac{3}{0{,}032} + 8{,}15^2 \times 0{,}016 \times \frac{3}{0{,}025}\right]$$

$= 12{,}19\ m.$

5) Minor loss height in meters :

$$h_{L_{min}} = \sum_i K_{L_i} \frac{V_i^2}{2g} = K_{L1}\frac{V_1^2}{2g} + K_{L2}\frac{V_2^2}{2g} + K_{L3}\frac{V_3^2}{2g}$$

$$= \frac{1}{2\times 9.81}\left[0.9\times 3.18^2 + 0.9\times 4.98^2 + 0.7\times 8.15^2\right] = 3.97 \text{ m.}$$

6) Calculate the pressure at the inlet $P_1$ :

$$P_1 + \tfrac{1}{2}\rho V_1^2 + \rho g z_1 = P_3 + \tfrac{1}{2}\rho V_3^2 + \rho g z_3 + \rho g h_L$$

$$P_1 = P_3 + \tfrac{1}{2}\rho\left(V_3^2 - V_1^2\right) + \rho g\left(z_3 - z_1\right) + \rho g h_L \quad , h_L = h_{L_{min}} + h_{L_{maj}}$$

$$P_1 = 2\cdot 10^5 + \tfrac{1}{2}\cdot 10^3\left(8.15^2 - 3.18^2\right) + 10^3\times 9.81(3) + 10^3\times 9.81(12.19 + 3.97)$$

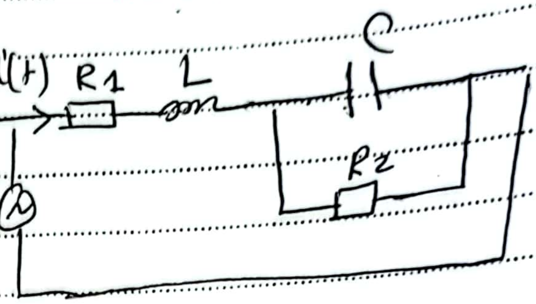
$$= 246\,227 \text{ Pa} = 2{,}46 \text{ bars}$$

$\Sigma = $ 9 pts

Fondamental electrical Engineering (1)
exam Correction
Zaamla. S.

exercice N°01: 6,75

I/ 1/ $\underline{Z}_{eq} = ?$ $\underline{Z}_{eq} = \underline{Z}_{(R_1,L)} + (\underline{Z}_C // \underline{Z}_{R_2})$




$\underline{Z}_{eq} = R_1 + jL\omega + \boxed{\dfrac{\frac{1}{jC\omega}\cdot R_2}{R_2 + \frac{1}{jC\omega}}}$ 0,25

0,25

0,5 $\underline{Z}_{eq} = R_1 + jL\omega + \dfrac{R_2}{1+jR_2C\omega} = R_1 + \dfrac{R_2}{(1+(R_2C\omega)^2)} + j\left(L\omega - \dfrac{R_2^2 C\omega}{1+(R_2C\omega)^2}\right)$

2/ $u(t)$ and $i(t)$ are in Phase $\Rightarrow \varphi(\underline{U}, \underline{I}) = 0°$ 0,25

$\Rightarrow tg\,\varphi = \dfrac{Im(\underline{Z})}{Re(\underline{Z})} = 0 \Rightarrow Im(\underline{Z}) = 0$ 0,25

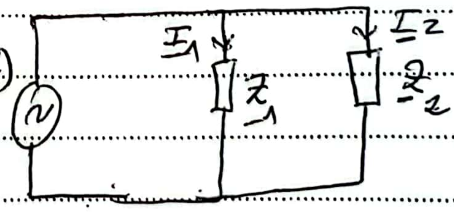
$L\omega - \dfrac{R_2^2 C\omega}{1+(R_2C\omega)^2} = 0 \Rightarrow L = \dfrac{R_2^2 C}{1+(R_2C\omega)^2}$

$\Rightarrow R_2^2 = \dfrac{L}{C(1-LC\omega^2)} \Rightarrow \boxed{R_2 = \sqrt{\dfrac{L}{C(1-LC\omega^2)}}}$ 0,5

$R_2 = f(L, C, \omega)$

II/ $\underline{Z}_1 = 4\angle 60° \ (\Omega)$, $\underline{Z}_2 = 5\angle 45° \ (\Omega)$

$\underline{V} = 20\angle 30° \ (V)$.




1/ $\underline{I}_1 = \dfrac{\underline{V}}{\underline{Z}_1} = \dfrac{20\angle 30°}{4\angle 60°} = 5\angle -30° \ (A)$ 0,5

$\underline{I}_2 = \dfrac{\underline{V}}{\underline{Z}_2} = \dfrac{20\angle 30°}{5\angle 45°} = 4\angle -15°$ 0,5

2/ $\underline{S}_1 = \underline{V}\cdot\underline{I}_1^* = 100\angle 60° = 50 + j\,50\sqrt{3}$ VA.

$\begin{cases} P_1 = 50 \text{ W} & 0{,}25 \\ Q_1 = 50\sqrt{3} \text{ VAR} & 0{,}25 \end{cases} \Rightarrow FP_1 = \dfrac{P}{S} = \dfrac{50}{100} = 0{,}5$ 0,25

$\underline{S}_2 = \underline{V}\cdot\underline{I}_2^* = 80\angle 45°$

$= 40\sqrt{2} + j\,40\sqrt{2}$

$\begin{cases} P_2 = 40\sqrt{2} \text{ W} & 0{,}25 \\ Q_2 = 40\sqrt{2} \text{ VAR} & 0{,}25 \end{cases}$

$FP_2 = \dfrac{P_2}{S_2} = \dfrac{40\sqrt{2}}{80}$ 0,25

$FP_2 = \dfrac{\sqrt{2}}{2} = 0{,}707$

3/ $\underline{P}, Q, S, \underline{S}$

$P = P_1 + P_2 = 50 + 56{,}57 = 106{,}57\,W$ (0,5)

$Q = Q_1 + Q_2 = 86{,}6 + 56{,}57 = 143{,}17\,VAR$ (0,5)

$S = \sqrt{P^2 + Q^2} = 178{,}14\,VA$. (0,50)

$\underline{S} = P + jQ = 106{,}57 + j143{,}17$ (0,5)

4/ $FP = \frac{P}{S} = 0{,}6$ (0,25)

exercise N°02 (5pts)

a balanced three-phase system in Delta Configuration ⇒

$220V/380V$, $P = 1{,}2\,KW$, $Q = 0{,}69\,KVAR$ (Capacitive)

So :

1/ Load power factor : $PF = \frac{P}{S} = \frac{P}{\sqrt{P^2+Q^2}} = \frac{1{,}2}{\sqrt{1{,}2^2+0{,}69^2}} \simeq 0{,}87$ (0,25) (0,25)

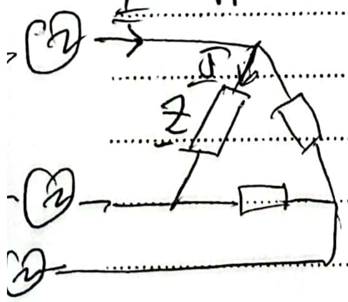
or : $PF = \cos\left(\operatorname{tg}^{-1}\left(\frac{Q}{P}\right)\right) = 0{,}87$ , $S = 1{,}38\,KVA$.

2/ $I_{eff}$, $J_{eff}$ = ? $P = \sqrt{3}\,U I \cos\varphi \Rightarrow I = \frac{P}{\sqrt{3}\,U\cos\varphi}$ (0,25)

$I_{eff} = \frac{1{,}2\times10^3}{\sqrt{3}\cdot380\cdot0{,}87} \simeq 2{,}1\,A$. (0,25) $\varphi = \arccos(0{,}87) = \begin{cases} 29{,}54 & \text{or} \\ -29{,}54 \end{cases}$

$\Rightarrow \varphi = -29{,}54$ (Capacitive Load). (0,25)

$J_{eff} = \frac{I}{\sqrt{3}} = 1{,}21\,A$ (0,25) (0,25)



$\varphi = \varphi_u - \varphi_J$ $(\varphi_V = 30° \Rightarrow \varphi_u = 60°)$

$\varphi_J = \varphi_u - \varphi = 60 + 29{,}54 = 89{,}54°$ (0,25)

$\varphi_I = \varphi_J - 30° = 59{,}54°$ (0,25)

(0,25)
$$\begin{cases} \underline{J}_1 = 1{,}21 \angle 89{,}54° \ (A) \\ \underline{J}_2 = 1{,}21 \angle -30{,}46 \ (A) \\ \underline{J}_3 = 1{,}21 \angle 209{,}54 \ (A) \end{cases} \qquad \begin{cases} \underline{I}_1 = 2{,}1 \angle 59{,}54° \ (A) \\ \underline{I}_2 = 2{,}1 \angle -60{,}46 \ (A) \\ \underline{I}_3 = 2{,}1 \angle 179{,}54° \ (A) \end{cases}$$
(0,75)

5/ $Z, R, X_C = ?$

0,5 $Z = \frac{U}{J} = \frac{380}{1{,}21} = 314{,}05\ \Omega \Rightarrow \begin{cases} R = Z\cos\varphi = 273{,}22\ \Omega & 0{,}5 \\ X_C = Z\sin\varphi = +154{,}84 & (0{,}5 \end{cases}$

0,1 $P = 3RJ^2 \Rightarrow R = \frac{P}{3J^2} = 273{,}20\ (\Omega)$

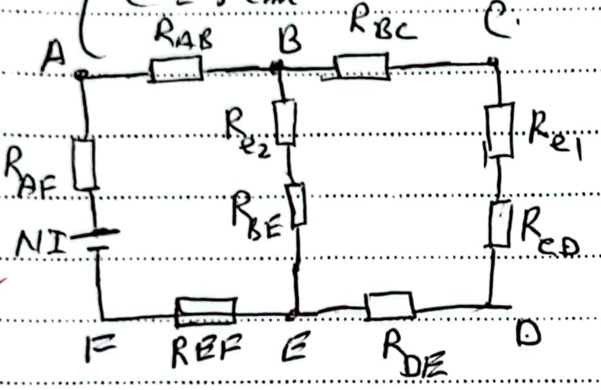
$Q = 3X J^2 \Rightarrow X = \frac{Q}{3J^2} = \ldots \simeq 157{,}1.$

Exercise N°03 (7p|5)

$N = 200$ turns, $\mu_0 = 4\pi.10^{-7}$ (H/m), $f = 50$ HZ, $\mu_r = 3000$

$\begin{cases} a = 2\ cm \\ b = 4\ cm \\ c = 5\ cm \end{cases}$ $e_1 = 1$ mm $\quad I = 2A,$

$e_2 = 1{,}5$ mm $\quad v(t) = 230\sqrt{2}\sin(2\pi f.t)$




(0,25 $\ell_{AB} = \ell_{BC} = \ell_{EF} = \ell_{ED} = 6$ cm ①

$\ell_{AF} = \ell_{BE} = \ell_{CD} = 8$ cm

(0,25 $\begin{pmatrix} e_1 \lll \ell_{CD} \\ e_2 \lll \ell_{BE} \end{pmatrix}$

(0,5

② $S_1 = S_{e_1} = S_{CD} = S_{DE} = S_{EF} = S_{AF} = S_{AB} = S_{BC} = a.c = 10\ cm^2 = 10.$

(0,25 $S_2 = S_{e_2} = S_{BE} = b \times c = 4 \times 5 = 20\ cm^2 = 2 \times 10^{-3}\ m^2.$

(0,25

$R_{e_1} = \frac{e_1}{\mu_0 . S_1} = 7{,}9577 \times 10^5\ H^{-1}$ (0,25

$R_{e_2} = \frac{e_2}{\mu_0 . S_2} = 9{,}947 \times 10^5\ H^{-1}$ (0,25

① and ② $\Rightarrow$ therefore, the equivalent circuit becomes:


$\begin{cases} R_{eq_1} = R_{BE} + R_{e_2} \\ R_{eq_2} = R_{BCDE} + R_{e_1} \end{cases}$

(0,5

$$R_{EFAB} = \frac{l_{EFAB}}{\mu . S_1} = \frac{20\times10^{-2}}{3000\times4\pi.10^{-7}\times10^{-3}} = 0,5305\times10^{5}\,(H^{-1})$$ 0,25

0,25 $R_{BCDE} = R_{EFAB} = 0,5305 \times 10^{+5}\,(H^{-1})$

0,25 $$R_{BE} = \frac{l_{BE}}{\mu . S_2} = \frac{8\times10^{-2}}{3000.4\pi.10^{-7}\times2\times10^{-3}} = 0,1061\times10^{5}\,(H^{-1})$$

0,25 $R_{eq_1} = 10,05\times10^{5}\,(H^{-1})$

0,25 $R_{eq_2} = 8,488\times10^{5}\,(H^{-1})$

So: $R_{eq} = R_{EFAB} + (R_{eq_1} // R_{eq_2}) = 5,1321\times10^{5}\,(H^{-1})$ 0,25

2/ So: $L = \frac{N^2}{R} = \frac{200^2}{5,1321\times10^5} = 77,94\ mH$ 0,5

3/ $\phi = \frac{NI}{R_{eq}} = 7,8\times10^{-4}\ Wb$ 0,5

by appling the ~~voltage~~ current divider:

$\phi_{e1} = \frac{R_{eq1}}{R_{eq1}+R_{eq2}}\,\phi = 4,23\times10^{-4}\,(Wb)$ 0,5

$\phi_{e2} = \frac{R_{eq2}}{R_{eq1}+R_{eq2}}\,\phi \simeq 3,57\times10^{-4}\,(Wb)$ 0,5

or $\{\phi_{e2} = \phi - \phi_{e1}\}$

4/ $V_{eff} = 4,44 . N . f . B_{Max} . S$

0,5 $B_{Max} = \frac{V_{eff}}{4,44 . N . f . S} = \frac{230}{4,44 . 200\times50\times10^{-3}}$

$B_{Max} = 5,2T.$

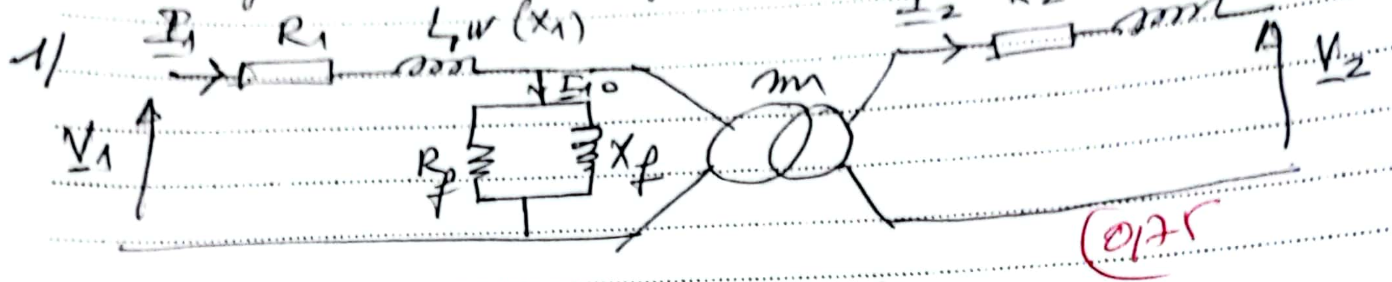
0,5 5/ $X_L = L\omega = 77\times94\times10^{-3}\times2\pi.50 = 24,49\,(\Omega)$

4

Course questions

the equivalent circuit diagramm of a real Single phase transformer

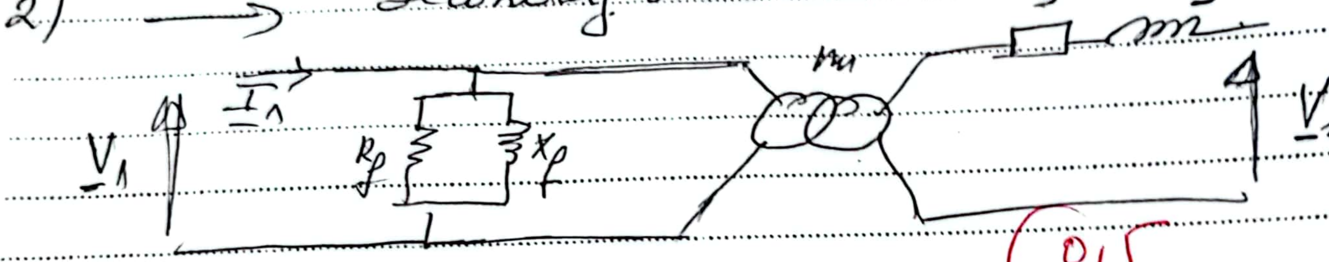
1/




0,75

equivalent circuit

2/ ⟶ Secondary side :




0,5

①

# Corrigé_type.

Ex. (1) (7pts)

(0,5) 1_ les entrées sont a, b, c, d.

les sorties sont : une seule sortie $O_C$ : ouverture du coffre.

Nb : deux considérations

1°/ B et C → 1 ; ($B=C=1$) → 1

B et D → 1 ($B=D=1$) → 1

C et D → 1 ($C=D=1$) → 1

2°/ $B=1$ et $C=1$ et $D=1$ → 1.

On doit considérer que le 1er cas.

(2,5)

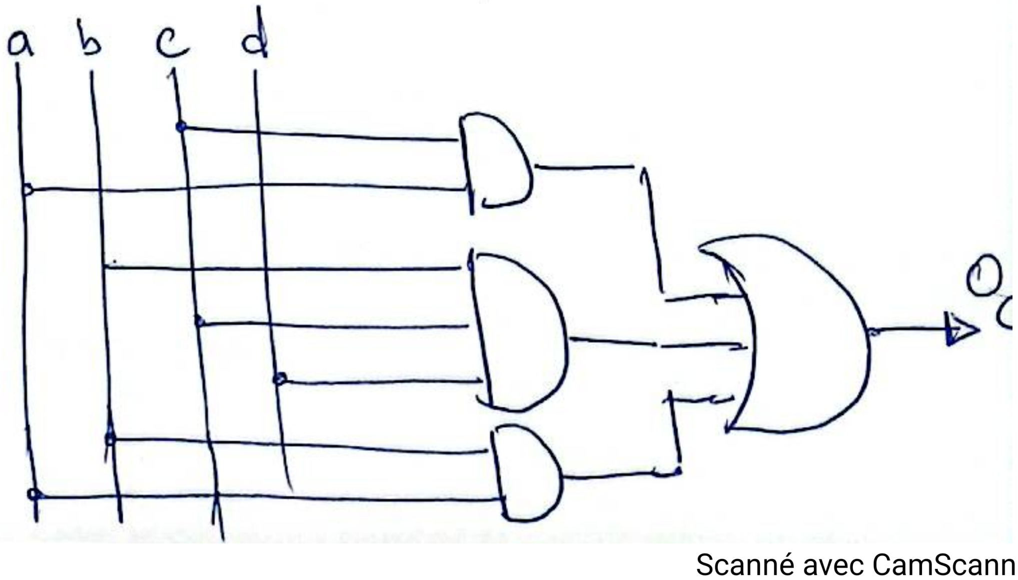
| a | b | c | d | $O_C$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

(2)

| ab \ cd | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 0 | 0 | 1 | 0 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 0 | 1 | 1 |

$O_C$

$$\boxed{O_C = ac + bcd + ab}$$ (2)



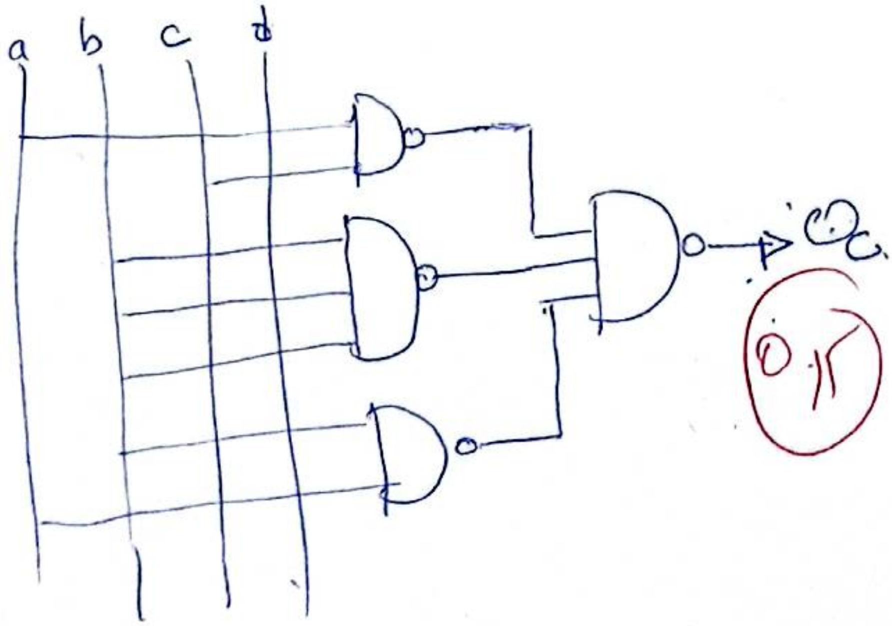
| a | b | c | d | $Q_c$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$$Q_C = ac + bcd + ab.$$

0,5

$$\overline{\overline{Q_c}} = \overline{\overline{ac + bcd + ab}}$$

$$= \overline{\overline{ac} \cdot \overline{bcd} + \overline{ab}} = \overline{X \cdot Y \cdot Z}$$

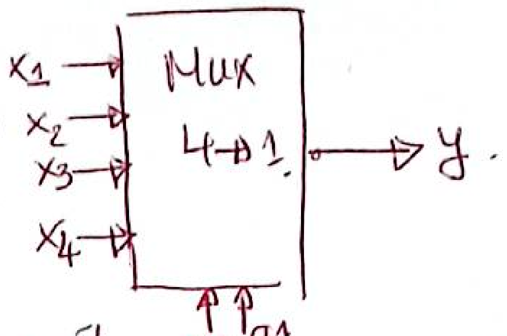



0,5

## Ex2 :

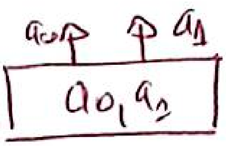
1/

(1pt)




2/ 4→1 ; le circuit on doit faire passer l'état d'une entrée à la sortie. $N = 4 = 2^n \Rightarrow n = 2$.

(1pts)



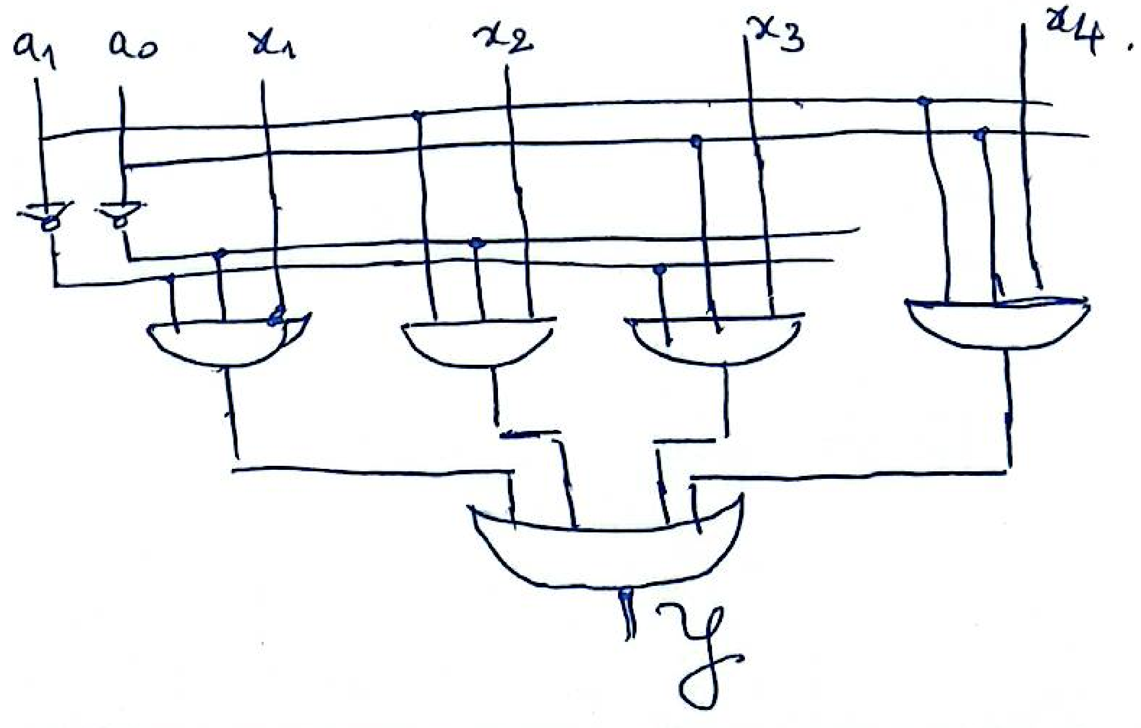

(2pts) 3/ table de vérité du Mux 4→1

$$y = x_1 \bar{a}_0 \bar{a}_1 + x_2 \bar{a}_0 a_1 + x_3 a_0 \bar{a}_1 + x_4 a_0 a_1.$$

| | $a_0$ | $a_1$ | X(t) | y |
|---|---|---|---|---|
| 0 | 0 | 0 | $x_1$ | $x_1$ |
| 1 | 0 | 1 | $x_2$ | $x_2$ |
| 2 | 1 | 0 | $x_3$ | $x_3$ |
| 3 | 1 | 1 | $x_4$ | $x_4$ |

4/ Logigramme,

(2pts)

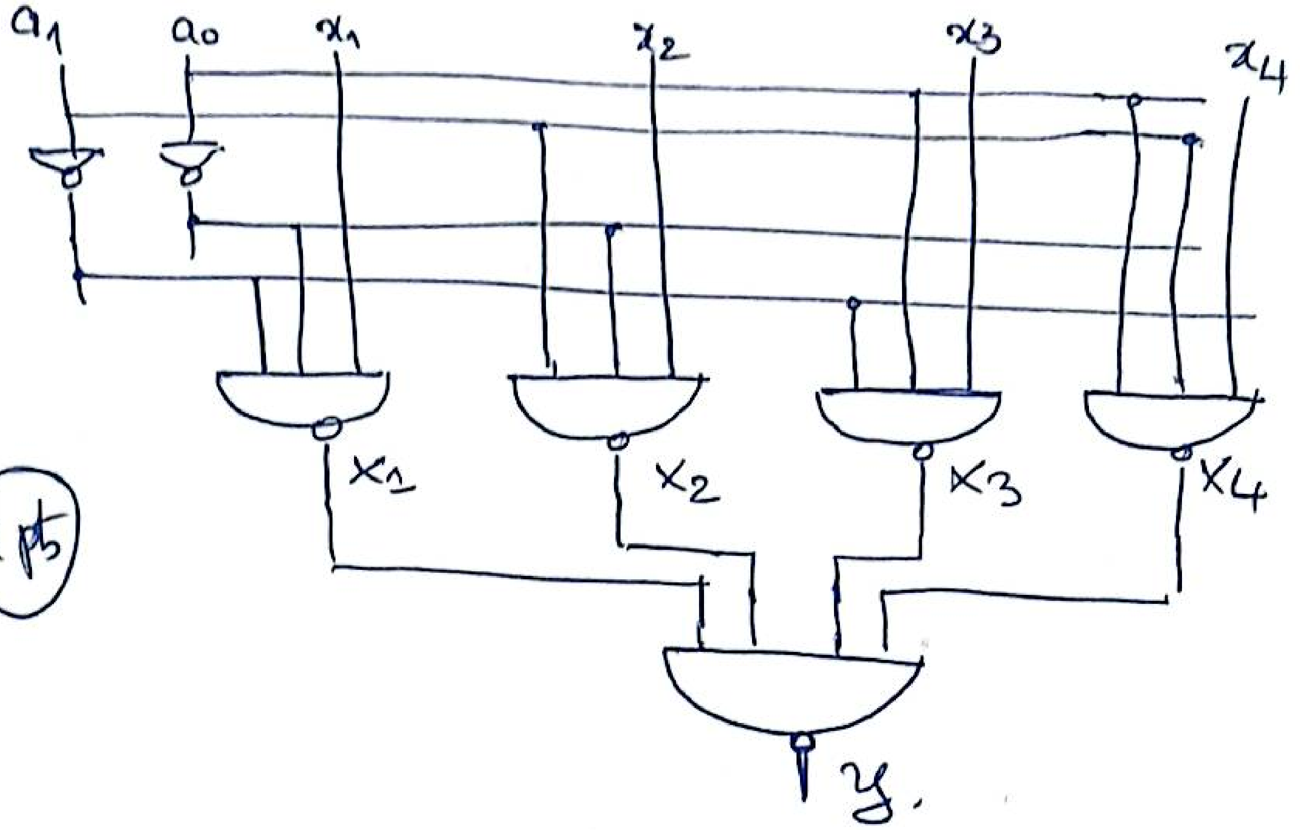


(3)

$$\bar{\bar{y}} = y = \overline{\overline{x_1 \bar{a}_0 \bar{a}_1 + x_2 \bar{a}_0 a_1 + x_3 a_0 \bar{a}_1 + x_4 a_0 a_1}}$$

(1pt)

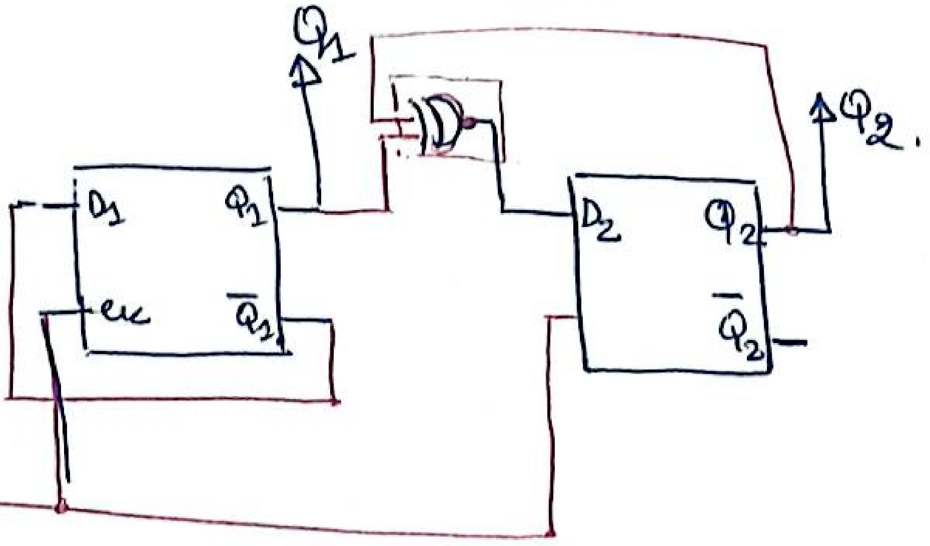
$$= \overline{\overline{x_1 \bar{a}_0 \bar{a}_1} \cdot \overline{x_2 \bar{a}_0 a_1} \cdot \overline{x_3 a_0 \bar{a}_1} \cdot \overline{x_4 a_0 a_1}}$$




(1pt)

## Ex. 3

1°/ → (1pt)




2°/ circuit C.C: (0,5 pt)  ! $D_2 = Q_1 \oplus Q_2$.



justifier :

Scanné avec CamScanner

(4)

Compteur modulo-4 : il ~~compte~~ les sequences de comptage sont : 00 01 10 11.

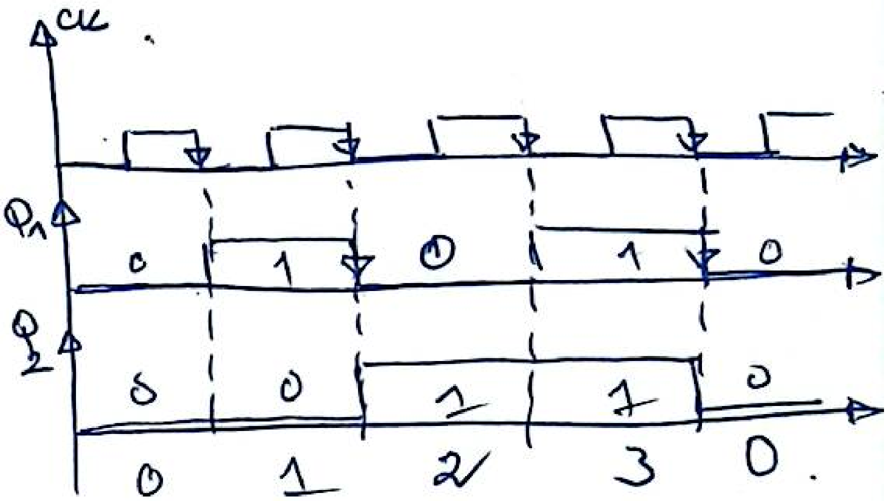
1,5

$D_2 = Q_1\overline{Q_2} + Q_2\overline{Q_1}$

$= Q_1 \oplus Q_2.$

| Etat | $Q_2$ | $Q_1$ | $D_2$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 2 | 1 | 0 | 1 |
| 3 | 1 | 1 | 0 |
| 0 | 0 | 0 | /// |

3°/. Chronogrammes :




4°/. Compteur synchrone de capacité $C=8$ ;

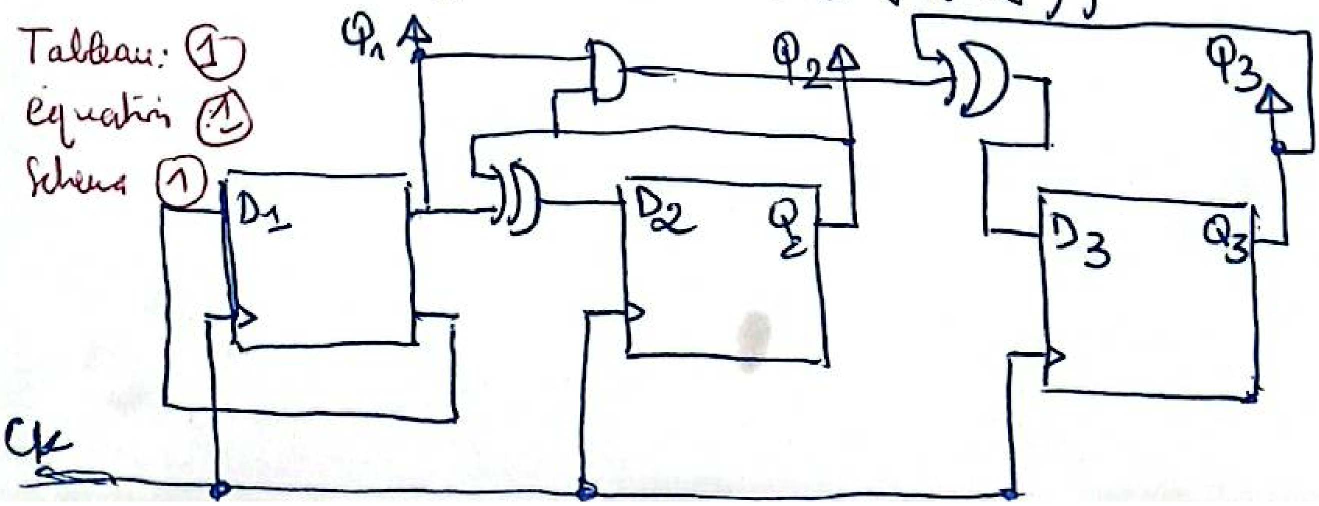
même methode ~~expliquée~~ presentée dans (2) ; la table de verite ~~contien~~ comporte 8 etats : 0, 1, .... 7.

$D_2 = Q_1 \oplus Q_2$ ; $D_3 = Q_3 \oplus (Q_1 Q_2)$ ;

Tableau : 1

equation 1

Schema 1



Université Larbi Ben M'Hidi - Oum El Bouaghi
Faculté des Sciences et Sciences appliquées
Département de 2ème Année S T

# Solution of Mathematics 03 Exam

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## Exercise 01 : (07 Pts)

1) $S_n = \sum_{k=1}^{n} \frac{3}{(2k+n)} = \frac{1}{n}\sum_{k=1}^{n} \frac{3}{(2\frac{k}{n}+1)} = \frac{b-a}{n}\sum_{k=1}^{n} f\left(a+\frac{b-a}{n}k\right)$

$$\begin{cases} b-a=1 \\ f\left(a+\frac{k}{n}\right) = \dfrac{3}{2\frac{k}{n}+1} \end{cases} \Rightarrow \begin{cases} [a \quad b] = [0 \quad 1] \\ f\left(\frac{k}{n}\right) = \dfrac{3}{2\frac{k}{n}+1} \; ; f(x) = \dfrac{3}{2x+1} \end{cases}$$

$f$ is continuous on $[0 \quad 1]$ , then $S = \int_0^1 f(x)dx$ .

$$S = 3\int_0^1 \frac{dx}{2x+1} = \frac{3}{2}\left(ln(2x+1)\right)_0^1 = \frac{3ln(3)}{2}.$$

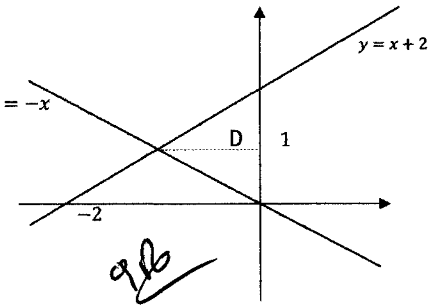
2) $\int_0^{+\infty} \frac{3x}{(2x+1)^2}dx$ ; $\frac{3x}{(2x+1)^2} \sim \frac{3}{4x}$ At the neighborhood of $(+\infty)$.

The integral $\frac{3}{4}\int_0^{+\infty} \frac{dx}{x}$ diverges (Riemann integral $\alpha = 1$) . Then $\int_0^{+\infty} \frac{3x}{(2x+1)^2}dx$ is divergent.

3) $I = \iint_D (5-y)dxdy$ ; $D: \begin{cases} x=0 \\ y=-x \\ y=x+2 \end{cases}$

$$I = \int_{-1}^{0}\left(\int_{-x}^{x+2}(5-y)dy\right)dx = \int_{-1}^{0}\left(5y-\frac{1}{2}y^2\right)_{-x}^{x+2}dx = \int_{-1}^{0}(8x+8)dx = 4.$$

$$\begin{cases} -x \le y \le x+2 \\ -1 \le x \le 0 \end{cases}$$




## Exercise 02 : (07 Pts)

$$S = \sum_{n\ge1} \frac{1}{n\,3^{n+1}}x^n$$

1) $a_n = \frac{1}{n\,3^{n+1}}$ , $\lim_{n\to+\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to+\infty}\frac{1}{3}\frac{n}{n+1} = \frac{1}{3}$ Then : $\Re = \frac{1}{l} = 3$ .

$x = 3$ , $\sum_{n\ge1}\frac{3^n}{n\,3^{n+1}} = \sum_{n\ge1}\frac{1}{3n}$ Harmonic serie diverges.

$x = -3$ , $\sum_{n\ge1}\frac{(-3)^n}{n\,3^{n+1}} = \sum_{n\ge1}\frac{(-1)^n}{3n}$ Alternating serie ,

Leibnitz : $\begin{cases} v_n = \frac{1}{3n} \text{ Decreasing} \\ \lim_{n\to+\infty} v_n = 0 \end{cases}$ , Then $\sum_{n\ge1}\frac{(-1)^n}{2n}$ converges.

Then : the interval of convergence is : $I = [-3 \quad 3[$

2) $S = \sum_{n\ge1}\frac{1}{n\,3^{n+1}}x^n = \frac{1}{3}\sum_{n\ge1}\frac{1}{n}\left(\frac{x}{3}\right)^n$

We have: $\sum_{n\ge0}\left(\frac{x}{3}\right)^n = \frac{1}{1-\frac{x}{3}} = \frac{3}{3-x} \Rightarrow \frac{1}{3}\sum_{n\ge0}\frac{1}{3^n}x^n = \frac{1}{3-x}$

By integration : $\sum_{n\ge0}\frac{1}{(n+1)3^{n+1}}x^{n+1} = \sum_{n\ge1}\frac{1}{n3^n}x^n = \int\frac{dx}{(3-x)} = -ln(3-x).$

Then : $S = -\frac{1}{3}ln(3-x)$.

3) $f(x) = \dfrac{x}{6x^2-5x+1} = \dfrac{x}{(3x-1)(2x-1)} = \dfrac{-1}{3x-1} + \dfrac{1}{2x-1}$ .

We have : $\begin{cases} \dfrac{1}{2x-1} = -\dfrac{1}{1-2x} = -\sum\limits_{n\geq 0}(2x)^n = -\sum\limits_{n\geq 0} 2^n x^n \\ \dfrac{1}{1-3x} = \sum\limits_{n\geq 0}(3x)^n = \sum\limits_{n\geq 0} 3^n x^n \end{cases}$

Then : $f(x) = \sum\limits_{n\geq 0}(3^n - 2^n)x^n$ .

**Exercise 03 : ( 06 Pts)**

$(E_1) : u_y = xuu_x$ By separation of variables , put : $u = XY$

$\begin{cases} u_x = X'Y \\ u_y = XY' \end{cases} \Rightarrow (E_1) : Y' = xY^2X' \Rightarrow \dfrac{Y'}{Y^2} = xX' = k$ (Constant)

$\begin{cases} xX' = k \\ \dfrac{Y'}{Y^2} = k \end{cases} \Rightarrow \begin{cases} X = kln(x) + C_1 \\ -\dfrac{1}{Y} = ky + C_2 \end{cases} \Rightarrow \begin{cases} X = kln(x) + C_1 \\ Y = -\dfrac{1}{ky + C_2} \end{cases}$

$u = XY = -\dfrac{kln(x) + C_1}{ky + C_2}$

$(E_2) : \begin{cases} yu_x - 2xu_y = 0 \\ u(x,0) = x^2 \end{cases}$ By the characteristics method : $\dfrac{dx}{y} = \dfrac{dy}{-2x} = \dfrac{du}{0}$ , $u = C_1$.

$\dfrac{dx}{y} = \dfrac{dy}{-2x} \Rightarrow ydy = -2xdx \Rightarrow \frac{1}{2}y^2 = -x^2 + C_2 \Rightarrow C_2 = \frac{1}{2}y^2 + x^2$

The Solution is given by : $C_1 = f(C_2) \Rightarrow u(x,y) = f\left(\frac{1}{2}y^2 + x^2\right)$ (General solution)

$u(x,0) = x^2 \Rightarrow f(x^2) = x^2 \Rightarrow f(x) = x \Rightarrow u(x,y) = \frac{1}{2}y^2 + x^2$ .

**Aim :** 0,50 ......معرفة ضغط الغاز داخل أنبوبة......

Population (Universe) Ω=....أنبوبة [illegible] T.V.4.... Statistical Variable $x$ =...ضغط الغاز

**Exercise01: Continuous Univariate**

Names : Correction Exam Normal "Regular"

Exo 1/10

| | | | | | |
|---|---|---|---|---|---|
| 0,50 | $[0, 2[$ | $[2, 4[$ | $[4, 6[$ | $[6, 8[$ | $[8, 10[$ |
| | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
| 0,50 | 1 | 3 | 5 | 7 | 9 |
| | $c_1^2$ | $c_2^2$ | $c_3^2$ | $c_4^2$ | $c_5^2$ |
| | 1 | 9 | 25 | 49 | 81 |
| | $n_1$ | $n_2$ | $n_3$ | $n_4$ | $n_5$ |
| 0,50 | 5 | 6 | 10 | 9 | 6 |
| | $\tilde{n}_1$ | $\tilde{n}_2$ | $\tilde{n}_3$ | $\tilde{n}_4$ | $\tilde{n}_5$ |
| 0,50 | 5 | 11 | 21 | 30 | 36 |
| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ |
| 0,50 | 0,14 | 0,17 | 0,28 | 0,25 | 0,17 |

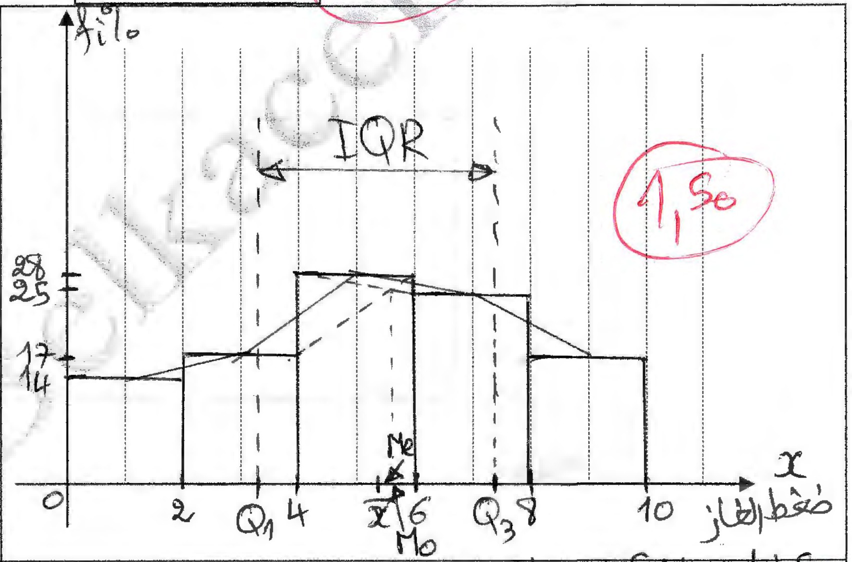
0,75 $\bar{x} = 0{,}14 \times 1 + 0{,}17 \times 3 + 0{,}28 \times 5 + 0{,}25 \times 7 + 0{,}17 \times 9 = \boxed{5{,}33}$

0,75 $\alpha = 0{,}50 \Rightarrow R = 18 \Rightarrow Me = 4 + \dfrac{(18 - 11).(6 - 4)}{21 - 11} = \boxed{5{,}40}$

0,75 $\alpha = 0{,}25 \Rightarrow R = 9 \Rightarrow Q_1 = 2 + \dfrac{(9 - 5).(4 - 2)}{11 - 5} = \boxed{3{,}33}$

0,75 $\alpha = 0{,}75 \Rightarrow R = 27 \Rightarrow Q_3 = 6 + \dfrac{(27 - 21).(8 - 6)}{30 - 21} = \boxed{7{,}33}$

0,75 $Mo = 4 + \dfrac{(10 - 6).(6 - 4)}{2 \times 10 - 6 - 9} = \boxed{5{,}60}$




1,50

- *In most of cases,* ......أغلب الأنبوبات...... *has* ......ضغط الغاز...... = 5,60
- *As* $\bar{x}$ *& Mo are* ...متقاربة جدا... ⇒ *choice of sample is* successful, ~~moderately accepted~~, ~~bad~~
- *50% of* ......الأنبوبات...... *has* ......ضغط الغاز...... *from* 3,33 *to* 7,33,
- *The* ......الأنبوبة...... *has* ......ضغط الغاز...... $= 5{,}33 \pm 2{,}51$

1

$V(x) = \boxed{6{,}28} \Rightarrow S_x = \boxed{2{,}51} \Rightarrow RSD = \boxed{47}\%$

0,75

**Exercise 02: Bivariate Statistics**

Names : Correction Exam Normal "Regular"

Exo 2/10

| X \ Y | $[b_{j-1}, b_j]$ | [2 , 34[ | | [34 , 66[ | | [66 , 98[ | | [ , [ | | [ , [ | | $n_{i*}$ & | $f_{i*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $[a_{i-1}, a_i]$ | $x_i$ \ $y_j$ | 18 | | 50 | | 82 | | 0,5 | | | | | |
| [3 , 25[ | 14 | 3 | 0,20 | 0 | | 0 | | | | | | 3 | 0,20 |
| [25 , 47[ | 36 | 1 | 0,07 | 0 | | 0 | | | | | | 1 | 0,07 |
| [47 , 69[ | 58 | 0 | | 4 | 0,27 | 0 | | | | | | 4 | 0,27 |
| [69 , 91[ | 80 | 0 | | 2 | 0,13 | 5 | 0,33 | | | | | 7 | 0,47 |
| [ , [ | 0,50 | | | | | | | | | | | | |
| $n_{*j}$ & $f_{*j}$ | 0,25 | 4 | 0,27 | 6 | 0,40 | 5 | 0,33 | | | | | 15 | 1 |

Population = غرف الاحتراق $n=$ 15 $X$ is 0,25 التركيز المولي للغاز $Y$ is 0,25 ضغط الغاز المثالي $p=$ 4 et $q=$ 3 0,25

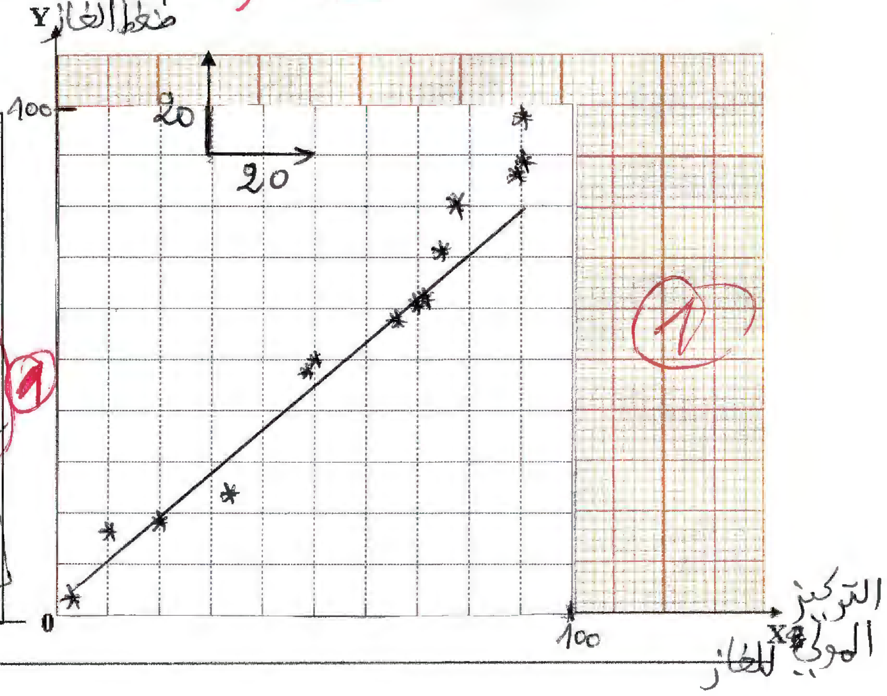
$\bar{x} =$ 58,52 $\bar{y} =$ 51,92 ; $S_x=$ 25,66 ; $S_y=$ 25,58 ; $\rho_{xy}=$ 85 %; $a=$ 0,85 ; $b=$ 2,13 ; $\Re^2=$ 0,72 6x0,25

0,50 0,50

$Y=f(x)$ can be well represented by **linear** regression ? ☒Yes ☐No Because: $R^2 > 0,70$

2x0,25

$y$ & $x$ are in ☒Direct ☐Inverse proportionality Because: $\rho_{xy} > 0$




**Exercise 03: Probabilities** Exo 3/4

1) Random variable المتغير العشوائي $x$ is:
0,25 سلسلة سير المركبات الثلاث

2) Primary Elements العناصر الأولية $E_i$ are:

0,25
$E_1$={T , T}
$E_2$={M , M , M , M}
$E_3$={C , C , C}

3) Criteria of interconnections معايير الربط:

0,50
- Ordonability الترتيب: ☒مهم ☐ غير مهم
- Repetitivity التكرار: ☒ممنوع ☐ مسموح ☐ إجباري

4) Random Event الحدث العشوائي المرغوب $X$ is
0,25 احتواء سلسلة السير على شاحنتين

5) Nb of all possibilities عدد الممكنات card(Ω)

$A_9^3 = 504$ 1

6) Possibilities of Event ممكنات الحدث card(X)

لم تحدد المسألة مكان ترتيب الشاحنتين

$3(A_2^2 \cdot A_4^1 \cdot A_3^0 + A_2^2 \cdot A_4^0 \cdot A_3^1) = 3 \cdot 14 = 42$ 1

7) Probability of event احتمال الحدث P(X):

$P(X) = \frac{card(X)}{card(\Omega)} = \frac{42}{504} = 0,08 \cong \boxed{8\%}$ 0,75

# Rational Mechanics Exam Correction

## Exercise 01 : (7 points)

**1ere Méthode : Parallélogramme des forces**

On trace le parallélogramme des forces **OABC** (Figure 2.16b), on joignant l'extrémité de chaque force une parallèle a l'autre force.

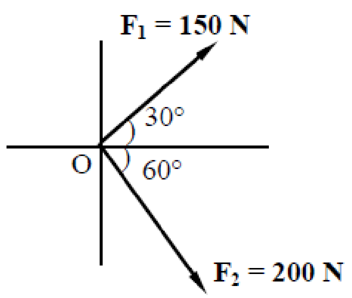



**Figure 2.16 a**

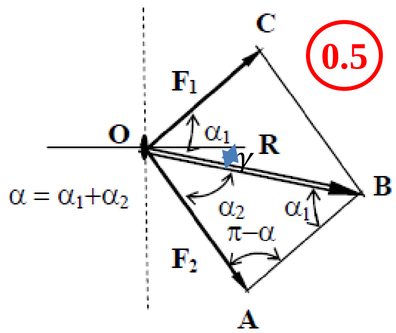



**Figure 2.16 b**

La diagonale **OB** représente la résultante des deux forces ($F_1$, $F_2$), de module :

$$\alpha = \alpha_1 + \alpha_2 = 90°$$

$$R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos(\pi - \alpha)} \quad 0.5 \quad \text{Or} \quad \mathbf{R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90°}}$$

On obtient le module de R : $R = \sqrt{F_1^2 + F_2^2} = \sqrt{(150)^2 + (200)^2} = 250\ \mathrm{N}$ 0.5

R = 250 N 0.5

La direction de R, est obtenue par l'application du théorème des sinus du triangle OAB :

$$\frac{F_1}{\sin\alpha_2} = \frac{F_2}{\sin\alpha_1} = \frac{R}{\sin(\pi - \alpha)} \quad 0.5$$

D'où :

$\alpha_1 = 53{,}13°$ **et** $\alpha_2 = 36{,}87°$ 0.5

The direction relative to the horizontal is:

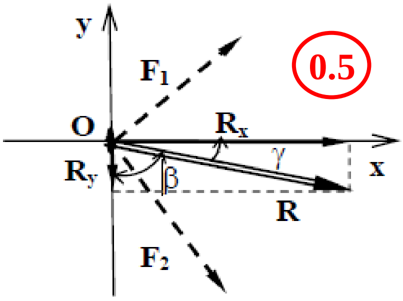
$$\gamma = 30° - \alpha_1 = 30° - 53.13° = -23.13° \quad 0.5$$

**$2^{eme}$ Méthode : projection des forces sur les axes**

La projection des composantes des forces extérieures sur les axes x et y au point O s'écrit :

$R_x = F_1 \cos 30° + F_2 \cos 60° = 229{,}9\,N$ 0.5

$R_Y = F_1 \sin 30° - F_2 \sin 60° = -98{,}21\,N$ 0.5




Les composantes de la résultante R:

$R_x = 229{,}9$ N 0.5

et $R_y$ = -98,21 N. 0.5

La résultante R:

$$\|\vec{R}\| = \sqrt{(R_X)^2 + (R_Y)^2}$$

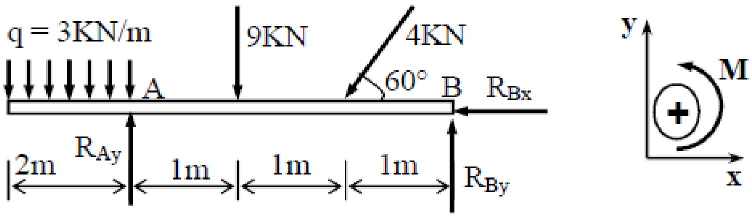
$$\|\vec{R}\| = \sqrt{(229{,}9)^2 + (-98{,}21)^2}$$ 0.5

D'où, la résultante R = 250 N

0.5

La direction de R est déterminée par : $tg\,\gamma = R_y/R_x = -0{,}43$ $\gamma = -23.13°$

**<u>Exercise 02: (7 points)</u>**




1

Pour la détermination des réactions $R_{Ay}$, $R_{Bx}$ et $R_{By}$, on écrit la projection des éléments du torseur des forces extérieurs nul en A :

$$\sum_{i=1}^{n} \vec{F}_{ix} = \vec{0}, \quad \sum_{i=1}^{n} \vec{F}_{iy} = \vec{0}, \quad \sum_{i=1}^{n} \vec{M}_A(\vec{F}_i) = \vec{0}$$

0.5 0.5 0.5

$$\sum_{i=1}^{n} \vec{F}_{ix} = \vec{0} \quad \Leftrightarrow \quad -R_{Bx} - 4\cos 60° = 0 \qquad (1)$$ 1

$$\sum_{i=1}^{n} \vec{F}_{iy} = \vec{0} \quad \Leftrightarrow \quad -3x2 + R_{Ay} - 9 - 4\sin 60° + R_{By} = 0 \qquad (2)$$ 1

$$\sum_{i=1}^{n} \overrightarrow{M}_A(\vec{F}_i)=\vec{0} \Leftrightarrow 3x2x1 - 9x1 - 4\sin 60° x2 + R_{By}x3 = 0 \qquad (3) \quad 1$$

La solution des équations d'équilibres (1), (2) et (3) donne :

| | | |
|---|---|---|
| $\mathbf{R_{Bx} = -2\ KN,}$ | $\mathbf{R_{By} = 3.31\ KN,}$ | $\mathbf{R_{Ay} = 15.15\ KN}$ |

**0.5** **0.5** **0.5**

**Exercise 03: (6.5 points)**

$$\boxed{v = 22.8\,t - 0.88\,t^2}$$

**Solution:** First convert the numbers into meters and seconds

$$22.8\frac{\text{km}}{\text{hr}}\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\text{hr}}{3600\ \text{s}}\right) = 6.33\ \text{m/s} \quad 0.5$$

$$0.88\frac{\text{km}}{\text{hr}}\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right) = 0.244\ \text{m/s} \quad 0.5$$

The governing equations are then

$$v = \frac{ds}{dt} = [6.33\,t - 0.244\,t^2](\text{m/s}) \quad 0.5$$

$$\int_0^s ds = \int_0^t v dt = \int_0^t (6.33\,t - 0.244\,t^2)dt \quad 0.5$$

$$s = \left[\frac{1}{2}(6.33)t^2 - \frac{1}{3}(0.244)t^3\right](\text{m}) \quad 0.5$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(6.33\,t - 0.244\,t^2)(\text{m/s}^2) \quad 1$$

$$a = [6.33 - 2(0.244)\,t](\text{m/s}^2) \quad 1$$

The maximum acceleration occurs at $t = 0$ (and decreases linearly from its initial value).

$$\boxed{a_{max} = 6.33\ \text{m/s}^2\ @\ t = 0} \quad 0.5$$

In the first 10 seconds the car travels a distance

$$s = \left[\frac{1}{2}\left(22.8\frac{\text{km}}{\text{hr}}\right)\frac{(10\ \text{s})^2}{s} - \frac{1}{3}\left(0.88\frac{\text{km}}{\text{hr}}\right)\frac{(10\ \text{s})^3}{\text{s}^2}\right]\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right) \quad 1$$

$$\boxed{s = 0.235\ \text{km.}} \quad 0.5$$

*Université Larbi Ben M'hidi* **Pôle Ain Beida** *Le 20/12/2025*
*Matière :* ***RDM1***
*2ème année Ingénieur*

## *Examen N°1*

***Question de cours*** ***(05 Pts) :*** donner la loi de répartition des contraintes correspondant à chaque cas de sollicitations suivantes :

- Traction-compression simple
- Flexion simple
- Flexion pure
- Torsion

***Exo 1*** ***(05 Pts) :*** Soit la barre représentée sur la figure 1, soumise à deux moments de torsion

1) Construire les diagrammes du moment de torsion.
2) Déterminer d'après la condition de résistance les dimensions des sections droites de la barre.

Données : $[\tau] = \mathbf{800\,kgf/cm^2}$ ; $I_P = \frac{\pi d^4}{32}$

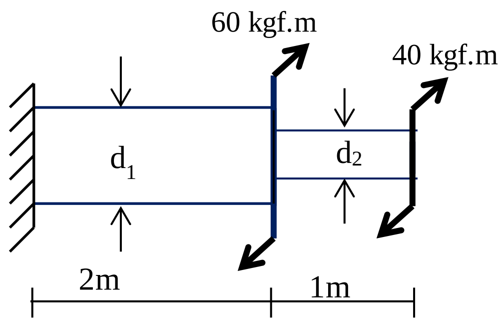


Fig.1

***Exo 2*** ***(10 Pts) :*** soit la poutre représentée sur la figure 2.

1) Déterminer les réactions des appuis.
2) Tracer les diagrammes de M et T.
3) Calculer les contraintes $\sigma_c$ $et$ $\tau_c$ dans la section $mn$ (le point $c$ se trouve à une hauteur $\frac{h}{4} = 3cm$ de l'axe Z). (voir figure).

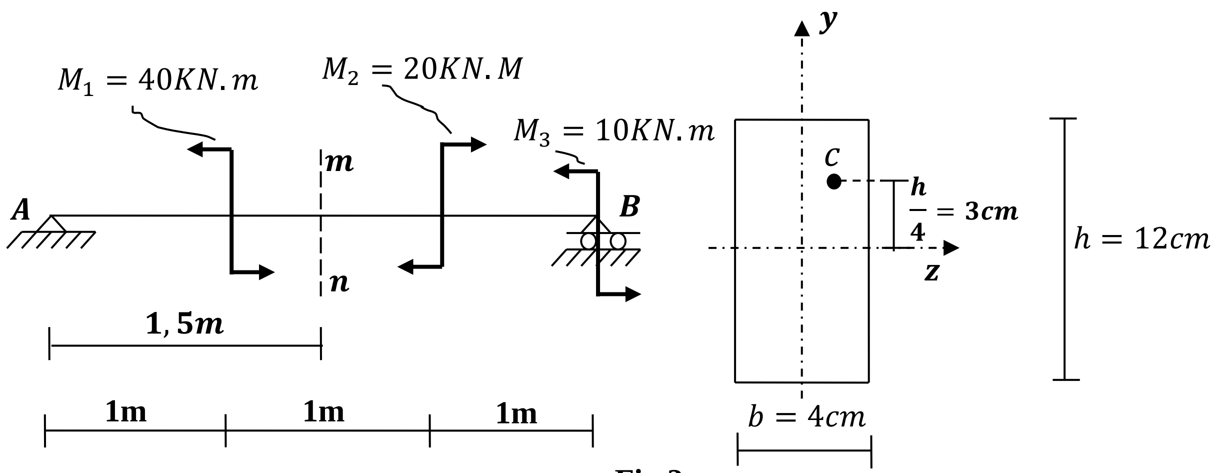


**Fig.2**

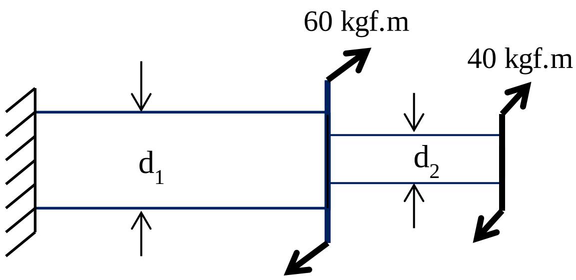
Corrigé de l'examen N°1
Matière : RDM
2ème Année ING S3

**_Question de cours_:**

- Traction-compression simple : $\sigma = \frac{N}{A}$
- Flexion simple: $\sigma = \frac{M_f . y}{I_Z}$ ; $\tau = \frac{T.S'_Z}{I_Z.b}$
- Flexion pure: $\sigma = \frac{M_f . y}{I_Z}$
- Torsion: $\tau = \frac{M_t . \rho}{I_P}$

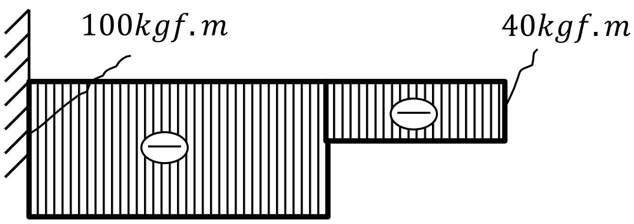
**_Exo1_ :**

1) Diagrammes de $M_t$:




$0 \leq x \leq 1m$ ; $M_t = -40\ kgf.m$

$1m \leq x \leq 3m$ ; $M_t = -40 - 60 = -100\ kgf.m$




2) Dimensionnement des sections :

$$\tau = \frac{M_t . \rho}{I_P} \leq [\tau] \; ; \quad I_P = \frac{\pi d^4}{32}$$

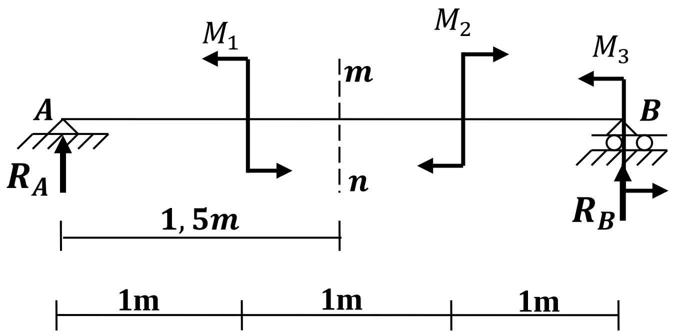
D'où : $d \geq \sqrt[3]{\frac{16.M_t}{\pi.[\tau]}}$

$$d_2 = \sqrt[3]{\frac{16.40.10^2}{3,14.800}} = \mathbf{2,94\ cm}$$

$$d_1 = \sqrt[3]{\frac{16.100.10^2}{3,14.800}} = \mathbf{3,99\ cm \approx 4cm}$$

***Exo 2 :***

1) Calcul des réactions:

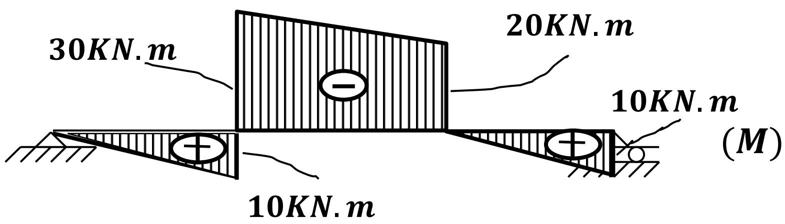



$\sum M/B = 0 \ ; R_A * 3 - 40 + 20 - 10 = 0$

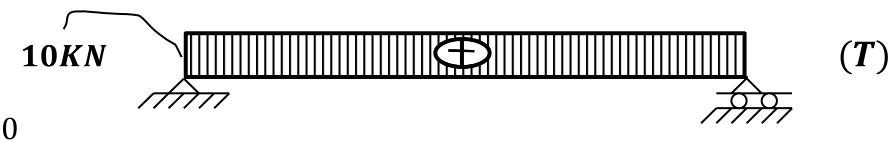
$R_A = \frac{30}{3} = \mathbf{10KN}$

$\sum M/A = 0 \ ; \ R_A * 3 + 10 - 20 + 40 = 0$

$R_B = \frac{-30}{3} = \mathbf{-10KN}$




2) Diagramme de M et T:




$0 \le x \le 1m$

$M(x) = 10x \begin{cases} x = 0\,; M(x) = 0 \\ x = 1\,;\ M(x) = 10KN.m \end{cases}$

$T(x) = \frac{\partial M(x)}{\partial x} = 10KN = const$

$1m \le x \le 2m$

$M(x) = 10.x - 40 = \begin{cases} x = 1\,; M(x) = -30KN.m \\ x = 2\,;\ M(x) = -20KN.m \end{cases}$

$T(x) = \frac{\partial M(x)}{\partial x} = 10KN$

$0 \le x \le 1m$

$M(x) = -10x \begin{cases} x = 0\,; M(x) = 0 \\ x = 1\,;\ M(x) = -10KN.m \end{cases}$

$T(x) = -\frac{\partial M(x)}{\partial x} = 10KN = const$

3) Calculer les contraintes $\sigma_c$ et $\tau_c$ **:**

La valeur du moment fléchissant de la section ***mn*** est : $M_{mn} = \frac{30+20}{2} = 25KN.m$

La valeur de l'effort tranchant de la section ***mn*** est : $T_{mn} = 10KN.m$

$\sigma_c = \frac{M_{mn} y}{I_Z}$ $\ avec \ M_{mn} = 25KN.m \ , y = \frac{h}{4} = 3cm \ et \ I_Z = \frac{bh^3}{12} = \frac{4 * 12^3}{12} = 576cm^4$

$D'ou \ \sigma_c = \frac{25 * 10^2 * 3}{576} = \mathbf{13\ KN/cm^2 = 130\ MN/m^2}$

$\tau_c = \frac{T_{mn} * S'}{I_Z * b} \ \ avec \ \ T_{mn} = 10KN \ \ ;$

$S' = b * \left(\frac{h}{2} - \frac{h}{4}\right) * \left(\frac{h}{8} + \frac{h}{4}\right) = \frac{3}{32} bh^2 = \frac{3}{32} 4 * 12^2 = 54cm^3$

$I_Z = \frac{bh^3}{12} = \frac{4 * 12^3}{12} = 576cm^4$

$T_{mn} = 10KN.m$

$b = 4cm$

$D'où\ \tau_c = \frac{10*54}{576*4} = \mathbf{0,234\ KN/cm^2 = 2,34\ MN/m^2}$

**Typical correction -Regulations and Standards**

## ***Question 1 (6 points)***

**1. Give the definition for the following statements:**

**Regulations (with example):** In simple terms, a regulation is a set of rules outlined by the government that must be followed as a minimum standard. A regulation is enforceable by law, so as workers, following regulations is mandatory**.... (0, 50)**

**Example:** Common examples of regulation include limits on environmental pollution, laws against child labor or other employment regulations**. .... (0, 50)**

Regulatory texts: **.... (1.00)**

**Standard (with example):** outline minimum industry practices that assist professionals with establishing and progressing best practices for work at height, but are not legally binding. Standards are intended to have a balanced representation of key industry stakeholders, such as government regulators, equipment manufactures, and practitioners. **.... (0, 50)**

**Example:** Management System Standards (examples: ISO 9000 and ISO 14000 Quality and Environmental Management Systems) **.... (0, 50)**

**Certification:** Procedure by which an approved body external to a company gives written assurance that a product, process or service conforms to specified requirements. **.... (0, 50)**

**ISO:** The International Organization for Standardization (ISO) is an international nongovernmental organization made up of national standards bodies that develops and publishes a wide range of proprietary, industrial, and commercial standards.

The International Organization for Standardization (ISO) was founded in 1947 and is headquartered in Geneva, Switzerland. **.... (0, 50)**

**AFNOR:** French Association of Standardization **.... (0, 50)**

The role of AFNOR: Identification of needs, Development of strategies**.... (0, 50)**

Coordination of programmes, Participation in European and global systems

**COFRAC :** French Accreditation commitee **.... (0, 25)**

**ISO 9000:** Quality Management. **.... (0, 25)**

**ISO 26000:** Social Responsibility**.... (0, 25)**

**ISO 3166:** Country Codes**.... (0, 25)**

**Typical correction -Regulations and Standards**

## Question 2 (10 points)

***What are the different types of standards* (with detail)? …. (1, 50)**

- Basic standards concern terminology, metrology, statistics, signs and symbols

- Methodology standards enable the development of guides or guidelines.

- Specification standards set the characteristics product.

***2. What are the principal of standardization (with detail)?* …. (4, 00)**

***- Inclusiveness –*** have application in all spheres, directions and levels of the economy and social life of the people;

***- Systematization –*** perform the activity by creating consistency, complexity, dynamism, systematic approach of the raw materials to the finished product;

***- General Agreement*** – compliance with the interests of consumers, manufacturers, suppliers and takes place for the general benefit, and with participation of all countries;

***- Dynamic*** – the standards are updated periodically, update and comply with the latest scientific developments in the field of research;

***3. What are the characteristics of normative documents?* …. (1.50)**

Rules, guidelines, features, Consensus between all stakeholders, Established by a recognized standardization organization, Voluntary application

***What is the importance of international standards?* …. (2.00)**

***What are the Benefits of Product Certification?* …. (1, 00)**

## Question 3 (4 points)

1. ***Give a short description of the: "Algerian Institute for Standardization (IANOR)".* …. (2.00).**

The Algerian Institute of Standardization (IANOR) was established as a public industrial and commercial establishment (EPIC) by Executive Decree No. 98-69 of February 21, 1998, amended and supplemented by Executive Decree Executive Decree No. 11-20 of January 25,

It is responsible for:

**1.** The development, publication and dissemination of Algerian standards.

**2.** The centralization and coordination of all standardization work.

**3.** The conservation and provision of all documentation or information relating to standardization.

**4**. The application of international conventions and agreements in the fields of standardization to which Algeria is a party.

***2. What are the stages of developing standards according to ISO?*** **…. (2.00)**

Identification of a need, Drafting a preliminary draft / project, Consensus between interested parties, Validation (probationary investigation: 2 months), Approval by the DG of AFNOR, Publication of the approved standard (NF).

Contrôle de V.O 13/01/2025

# Exercice N°1

6 pts / 5

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} \quad ; \quad v = \sqrt{\frac{T}{\mu}}$$

$$u = f(x) \cdot g(t) \Rightarrow v^2 g(t) \cdot f''(x) = f(x) \cdot g''(t)$$ 0,25 0,25 0,25

$$\Rightarrow \frac{f''}{f} = \frac{1}{v^2} \frac{g''}{g}$$ 0,25 0,25, The common value of the two members is a negative constant $(-w^2)$ 0,25

$$\begin{cases} f'' + w^2 f = 0 \\ g'' + v^2 w^2 g = 0 \end{cases} \Rightarrow \begin{aligned} f(x) &= A_0 \cos wx + B_0 \sin wx \\ g(t) &= A \cos vwt + B \sin vwt \end{aligned}$$ 0,25 0,25 0,25 0,25 0,25 0,25

A) Boundary conditions: $u = 0$ for $x = 0$ and $x = L$

So: $f(0) = f(L) = 0 \Rightarrow A_0 = 0$ and $\sin wL = 0$ 0,25 0,25 0,25

$w$ is a number from the series $w_n = \frac{n\pi}{L}$ 0,25

B) Initial conditions:

$\dot{u}(0) = 0 \Rightarrow g'(t=0) = -A\,vw \underbrace{\sin(v.w.0)}_{0} + B\,vw \underbrace{\cos(v.w.0)}_{1} = 0$ 0,25 0,25 0,25

$\Rightarrow B = 0$ 0,25

So the Final solution:

$$u(x,t) = \sum_{n=1}^{\infty} B_{0n} \sin w_n x \cdot A_n \cos v w_n t$$ 0,25 0,25

$$\begin{cases} u(x,t) = \sum_{n=1}^{\infty} K_n \sin w_n x \cdot \cos v.w_n t \\ w_n = n\pi/L \end{cases}$$ 0,25 0,25

Exercice n°2

8 pts / 7,5

* Kinetic energy: $T_M = \frac{1}{2}\frac{MR^2}{2}\dot{\theta}^2 = \frac{1}{4}MR^2\dot{\theta}^2$ 0,25

$$m\begin{cases} x_m = l\sin\theta & 0{,}25 \\ y_m = l\cos\theta & 0{,}25 \end{cases} \Rightarrow \begin{cases} \dot{x}_m = l\dot{\theta}\cos\theta \\ \dot{y}_m = -l\dot{\theta}\sin\theta \end{cases} \overset{0{,}25}{\Rightarrow} T_m = \frac{1}{2}ml^2\dot{\theta}^2 \quad 0{,}25$$

* Potential energy: $V_k = \frac{1}{2}ky_k^2 = \frac{1}{2}k(R\sin\theta)^2 = \frac{1}{2}kR^2\theta^2$ 0,25 0,25

$$-\frac{\partial V_m}{\partial y} = -mg \Rightarrow V_m = mgl\cos\theta \quad 0{,}25 \quad 0{,}25$$

* Lagrangian: $L = T_t - V_t = \frac{1}{4}MR^2\dot{\theta}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - \frac{1}{2}kR^2\theta^2 - mgl\cos\theta$ 0,25

* Dissipation function: $D = \frac{1}{2}\alpha\dot{y}_\alpha^2$ 0,25, $y_\alpha = R\sin\alpha \Rightarrow \dot{y}_\alpha = R\dot{\theta}\cos\alpha$ 0,25

$$\Rightarrow D = \frac{1}{2}\alpha R^2\dot{\theta}^2\cos^2\alpha \simeq \frac{1}{2}\alpha R^2\dot{\theta}^2 \quad 0{,}25$$

* Equation of motion: $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = -\frac{\partial D}{\partial\dot{\theta}}$ 0,25

$$\frac{\partial L}{\partial\dot{\theta}} = \left(\frac{1}{2}MR^2 + ml^2\right)\dot{\theta} \quad 0{,}25 \quad ; \quad \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) = \left(\frac{1}{2}MR^2 + ml^2\right)\ddot{\theta} \quad 0{,}25$$

$$\frac{\partial L}{\partial\theta} = -kR^2\theta + mgl\sin\theta = \left(-kR^2 + mgl\right)\theta \quad 0{,}25 \quad 0{,}25, \quad \frac{\partial D}{\partial\dot{\theta}} = \alpha R^2\dot{\theta} \quad 0{,}25$$

We obtain: $\left(\frac{1}{2}MR^2 + ml^2\right)\ddot{\theta} + \alpha R^2\dot{\theta} + \left(kR^2 - mgl\right)\theta = 0$ 0,25 0,25 0,25

$$\ddot{\theta} + \frac{\alpha R^2}{\frac{1}{2}MR^2 + ml^2}\dot{\theta} + \frac{kR^2 - mgl}{\frac{1}{2}MR^2 + ml^2}\theta = 0 \quad 0{,}25 \quad , \quad \ddot{\theta} + 2\delta\dot{\theta} + \omega_0^2\theta = 0$$

$$\delta = \frac{\alpha R^2}{MR^2 + 2ml^2} \quad 0{,}25 \quad , \quad \omega_0^2 = \sqrt{\frac{kR^2 - mgl}{\frac{1}{2}MR^2 + ml^2}} \quad 0{,}25$$

* Solution with initial conditions $\theta(0) = 0$, $\dot{\theta}(0) = \dot{\theta}_0$

$$\theta(t) = Ae^{-\delta t}\cos(\omega_a t + \varphi) \quad 0{,}25, \quad \omega_a = \sqrt{\omega_0^2 - \delta^2} \quad 0{,}25, \quad \theta(0) = A\cos\varphi \Rightarrow \varphi = \pm\frac{\pi}{2} \quad 0{,}25$$

$$\dot{\theta}(0) = \dot{\theta}_0 = -\delta Ae^{-\delta\cdot 0}\underbrace{\cos\left(\omega_a\cdot 0 - \frac{\pi}{2}\right)}_{\to 0} - \omega_a\cdot Ae^{-\delta\cdot 0}\underbrace{\sin\left(\omega_a\cdot 0 - \frac{\pi}{2}\right)}_{\to -1} \quad 0{,}25 \quad 0{,}25$$

$$\dot{\theta}_0 = \omega_a\cdot A \Rightarrow A = \frac{\dot{\theta}_0}{\omega_a} \quad 0{,}25 \quad \text{Then: } \theta(t) = \frac{\dot{\theta}_0}{\omega_a}e^{-\delta t}\cos\left(\omega_a t - \frac{\pi}{2}\right) \quad 0{,}25$$

if $\varphi = \frac{\pi}{2} \Rightarrow \theta(t) = \frac{-\dot{\theta}_0}{\omega_a}e^{-\delta t}\cos(\omega_a t + \pi)$

Exercice n° 3 — 1) Equations of motion. 7,5

* Lagrangian: $L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - \frac{1}{2} k_1 x_1^2 - \frac{1}{2} k_2 \left(x_2 - x_1^2\right)$

* Dissipation function: $D = \frac{1}{2} \alpha \dot{x}_2^2$

* Equations of motion:

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = F \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = \frac{-\partial D}{\partial \dot{x}_2} \end{cases} \Rightarrow \begin{cases} m_1 \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) = F \\ m_2 \ddot{x}_2 + \alpha \dot{x}_2 + k_2 (x_2 - x_1) = 0 \end{cases}$$

2) The natural frequency $\omega_1^2$ and $\omega_2^2$ : $k_1 = k_2 = k$, $m_1 = m_2 = m$, $\alpha = 0$

$$\begin{cases} \ddot{x}_1 + \frac{2k}{m} x_1 - \frac{k x_2}{m} = 0 \\ \ddot{x}_2 + \cancel{\frac{\alpha}{m} \dot{x}_2}^{\,0} + \frac{k}{m} x_2 - \frac{k}{m} x_1 = 0 \end{cases}$$

; $x_1 = a e^{j\omega t} \Rightarrow \dot{x}_1 = j\omega a e^{j\omega t}$

$\Rightarrow \ddot{x}_1 = -\omega^2 a e^{j\omega t} = -\omega^2 x_1$

$\ddot{x}_2 = -\omega^2 x_2$

we obtain :

$$\begin{cases} \left(\frac{2k}{m} - \omega^2\right) x_1 + \frac{k}{m} \omega^2 x_2 = 0 & (1) \\ \frac{k}{m} \omega^2 x_1 + \left(\frac{k}{m} - \omega^2\right) x_2 = 0 & (2) \end{cases}$$

$$\Delta(\omega) = \left(\frac{2k}{m}\,\omega^2\right)\left(\frac{k}{m} - \omega^2\right) - \frac{k^2}{m^2} = 0 = \omega^4 - \omega^2 \frac{3k}{m} + \frac{k^2}{m^2}$$

$\Delta = \frac{9k^2}{m^2} - 4\frac{k^2}{m^2} = \frac{5k^2}{m^2} > 0$ , The equation admits two solutions

$$\omega_{1,2} = \frac{1}{2}\left(\frac{3k}{m} \pm \frac{\sqrt{5}}{m} k\right) = \frac{k}{2m}\left(3 \pm \sqrt{5}\right)$$

3) The equations for speeds:

$$\begin{cases} \left(j\omega + \frac{2k}{j\omega m}\right) \dot{x}_1 - \frac{k}{j\omega m} \dot{x}_2 = \frac{F}{m} & (1) \\ \left(j\omega + \frac{\alpha}{m} + \frac{k}{j\omega m}\right) \dot{x}_2 - \frac{k}{j\omega m} \dot{x}_1 = 0 & (2) \end{cases}$$

From (2) $\dot{x}_2 = \frac{k/jm\omega}{j\omega + \frac{\alpha}{m} + \frac{k}{j\omega m}} \dot{x}_1$ in (1) we obtain

$Z^v = jm\omega + \frac{2k}{\ldots} + \frac{k^2}{\ldots}$