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Faculty of Science and Applied Sciences

1^{st} Engineer

Date 18/01/2025

Final Exam of Physics 1

Exam duration : 1^h30 min

Course questions (10 pts)

1- Why we use dimensional analysis?

2- What are the different coordinate systems?

3- Define the cartesian coordinates then write the position $\overrightarrow{OM}$,velocity $\vec{v}$ and acceleration vector $\vec{a}$.

4- Write the relationship between **cylindrical coordinates** and **Cartesian coordinates.**

5- Determine that the acceleration vector in the Frenet frame is $\vec{a} = a_T \vec{u} + \vec{n}\, a_n$

6- Calculate the work of a force **F=1.5 10^4 N** supplied to move a body at a height AB of 3 meters **(AB= 3m)** vertically.

7- What is the difference between a conservative energy and a non-conservative energy?

8- Write the Newton's three laws.

Exercise 01 (5 pts)

A mobile is moving along curvilinear trajectory in orthonormal basis $(\vec{u}, \vec{n})$ by the following equation :

S(t)= t^3+5t

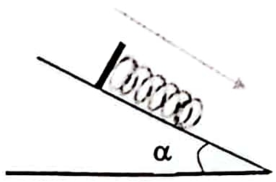
1- Find the expression of the velocity v?

2- Determine the tangential acceleration vector a_T?

3- If we suppose that at **t= 5s** the acceleration is **a= 45 m/s²**, calculate the radius of curvature R at this time?

Exercise 02 (5 pts)

A mass **m = 22 kg** suspended from a spring of stiffness **K = 50 N/m** descends along an inclined plane which makes an angle **α = 30°** with the horizontal **without friction**



1- Show in the diagram the forces acting on the mass.

2- Determine the normal reaction of the support and the acceleration of the mass when the spring is stretched by a length x = 0.1m.

Good Luck

Solution of Course questions.

1) We use dimensional analysis to:

* Verify the homogeneity of physical laws (01)
* Determine the unit of a physical quantity based on fundamental units (meters, second, kilogram, etc...)

2) The different coordinate systems

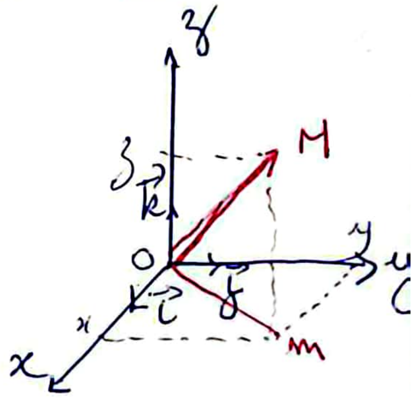
a. Cartesian coordinates

b - Polar coordinates (01)

c - Cylindrical coordinates

d - Spherical coordinates

3) The cartesian coordinate is defined by the frame $R(O, x, y, z)$ with the unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ of a point M which gives its position in the space. (0,25)



- The position vector

$\vec{OM} = x\vec{i} + y\vec{j} + z\vec{k}$ (0,25)

- The velocity vector

$$\vec{v} = \frac{d\vec{OM}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} \Rightarrow \begin{cases} v_x = \frac{dx}{dt} \\ v_y = \frac{dy}{dt} \\ v_z = \frac{dz}{dt} \end{cases}$$

(0,25)

The acceleration vector

$$\vec{a} = \frac{d^2\vec{OM}}{dt} = \frac{d\vec{v}}{dt} \Rightarrow \begin{cases} a_x = \frac{dv_x}{dt} \\ a_y = \frac{dv_y}{dt} \\ a_z = \frac{dv_z}{dt} \end{cases}$$ (0,25)

4. The relation between Cylindrical and Cartesian Coordinates

$$\begin{cases} x = \rho \cos\theta \\ y = \rho \sin\theta \\ z = z \end{cases}$$ (01)

5) Determination that: $\vec{a} = a_T \vec{u} + \vec{n}\, a_n$

$\vec{v} = |\vec{v}|\vec{u}$ so $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d|\vec{v}|\vec{u}}{dt} = \frac{d|\vec{v}|}{dt}\vec{u} + |\vec{v}|\frac{d\vec{u}}{dt}$

On the other hand: $\frac{d\vec{u}}{dt}$ can be written $\frac{d\vec{u}}{dt} = \underbrace{\frac{d\vec{u}}{d\theta}}_{\vec{n}} \cdot \underbrace{\frac{d\theta}{dt}}_{w}$

So :

$$\vec{a} = \frac{d|\vec{v}|}{dt}\vec{u} + |\vec{v}|\,\vec{n}\,w$$

$v = Rw \Rightarrow w = \frac{v}{R}$ (02)

$$\vec{a} = \frac{d|\vec{v}|}{dt}\vec{u} + \frac{v^2}{R}\vec{n}$$

$$\Rightarrow \vec{a} = a_T\,\vec{u} + a_N\,\vec{n}$$

$$\begin{cases} a_T = \frac{d|\vec{v}|}{dt} \\ a_N = \frac{v^2}{R} \end{cases}$$

6 - Calculation of the Work. (Application directe) 0,5

$W = \vec{F} \cdot \vec{AB} = F \cdot AB \cdot \cos 0° = 1,5 \cdot 10^4 \cdot 3 \cdot \cos 0° = 4,5 \cdot 10^4 \, J$

7 - Conservative energy: the variation in kinetic energy is equal to the opposite of the variation in potential energy

$\Delta E_c = -\Delta E_p \;/\; \Delta E_c + \Delta E_p = 0$ 0,5

In this case we can say that the system is isolated or the mechanical energy is conserved

$E_m = E_c + E_p$

non conservative

8 - Newton's three laws 03 Chaque lois sur 1 pt.

a - Newton's first laws: a free particles moves in rectilinear motion with constant velocity.

$\sum \vec{F} = \vec{0}$

b - Newton's second laws, The fundamental principle of dynamics

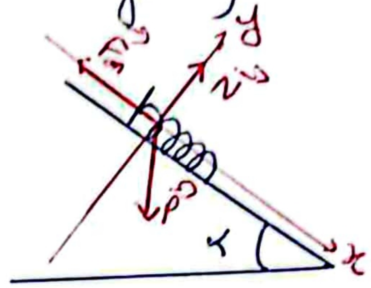
$\sum \vec{F} = m\vec{a}$

c - Newton's third law = The force applied by the first particle is opposite and equal in sign to the second particle

$|\vec{F_{1-2}}| = |\vec{F_{2-1}}|$

Exercise 02 (05 pts)

1 - The forces acting on the mass (see the diagram)



01

2 - Determination of the normal reaction by using Newton's second laws:

$\sum \vec{F} = m\vec{a} \Rightarrow \vec{P} + \vec{N} + \vec{f}_m = m\vec{a}$ 01

Following (ox): $-f_m + P_x = -f_m + mg\sin\alpha = ma$ 01

Following (oy): $N - P_y = 0 \Rightarrow N = mg\cos\alpha$ 01

So, $N = mg\cos\alpha$

The norm of the spring returnforce is given by

$f_m = K.x$

$a = (-kx + mg\sin\alpha)/m = 3\ m/s^2 \quad (g = 10\ m/s^2)$

$\boxed{a = 3\ m/s^2}$ 01

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Exercise 01. (05 pts)

$S(t) = t^3 + 5t$ (S in meters, t in seconds)

1- The velocity v:

$$v = \frac{ds}{dt} = \frac{d(t^3+5t)}{dt} = 3t^2+5 \quad 01$$

2- The tangential acceleration a_T.

$$a_T = \frac{dv}{dt} = 6t \quad 01$$

3- Calcul of the radius of curvature R.

We have $a_N = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_N}$ 01

Then we must find a_N.

$$\vec{a} = \vec{a}_T + \vec{a}_N \Rightarrow a^2 = a_T^2 + a_N^2 \Rightarrow a_N^2 = a^2 - a_T^2$$

at $t = 5s$: $a_T = 30\ m/s^2$: $a = 45\ m/s^2$

$$a_N^2 = (45)^2 - (30)^2 \quad 01$$

$$a_N^2 = 1125\ (m/s^2)^2 \qquad \boxed{a_N = 33{,}54\ m/s}$$

at $t = 5s$: $v = 3(5)^2 + 5 = 80\ m/s$

$$R = \frac{v^2}{a_N} = \frac{(80)^2}{33{,}54} = 190{,}81\ m$$

$$\boxed{R = 190{,}81\ m} \quad 01$$