Oum El Bouaghi University   Faculty of science and applied science   1st year engineering

1st Chemistry Exam   15-01-2025

Exercise 01:

Under normal conditions (CNTP), show that the following relation M= 29.d , is valid for an ideal gas.

ɟ (air) = 1.29 g/l.

Exercise 02:

Radium □□□□□ is the first radioactive element used in medicine. If the element decays into radon □□□□□ and particles Ŭaccording to the equation.

RaŸ Rn + Ŭ

- The period of this distintegration being equal to 1620 years,

Calculate.

a. The radioactive constant of radium

b. The activity of a gram of Ra, compare with the curie.

c.The time it takes for the activity to decrease to 1\8 of its value initial.

Data: 1year= 3,16.10^7 s, N_A=6.023 10^{23} , 3.59 10^{10} dps = 1 curie

Exercise 03:

The wavelength of the boundary line of the spectrum of the hydrogen atom is ゝ= 8210 Å

1- To which series does the length correspond? explain your answer.
2- Calculate the energy of the photon corresponding to the first line of this series.
3- Calculate the energy of the photon corresponding to the boundary line of this series.
4- What does the energy absorbed by the photon represent?

Data. R_H: =1.096 $10^7 m^{-1}$, E_H=-13,6eV, 1 eV= 1,61.10^{-19} J,h=6.62 10^{-34}J s , c =3.10^8m s^{-1}

Exercise 04:

1- Give the electron configuration, core electrons, valence electrons, chemical groupand the period of the following elements:$_{31}$Ga, $_{19}$K, $_{11}$Na.

Data. $_2$He ,$_{10}$Ne , $_{18}$Ar, $_{36}$Kr

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Solution
Exo 1. 4pt

d =ɟ (gas)\ɟ (air). é é é é .. **0.5 pt**
d =m (gas)\v (gas)\ɟ (air). é é é é .. **0.5 pt**

m (gas)=**d**ɟ (air) v (gas) é é é é .. **0.5 pt**

m (gas)= **n M ….. 0.5 pt**

n M=dɟ **(air) v (gas)….. 0.5 pt**
M=dɟ **(air) v (gas)\n …… 0.5 pt**

v (gas)\n=Vn= 22.4. …. 0.5pt

M=d1.29 . 22.4
M=d 29 …….0.5pt

Ex 2 6pt

Ra ð >Rn + Ŭ
a. The radioactive constant of radium

ә= Log2/ T é é 0.5 pt

= 0.69/1620*3.16 10^{7} ә= 1.347 10^{-11} S^{-1} 0.5 pt

b. The activity of a gram of Ra
A= әN. 0.5 pt
N= mN_A/M. 0.5 pt
A= әmN_A/M = 1.347 10^{-11} 1*6.023 10^{23}/226 A= 3.59 10^{10} dps0.5 pt

Note this activity corresponds to the curie value A= 3.59 10^{10} dps = 1 curie.0.5 pt

c. A_t = A_0/ 8 the corresponding time 0.5 pt

A_0/8 = A_0 $e^{-әt}$0,5 pt
Log 1/8 = - әt. 0.5 pt
So t = 1/әlog 8 = 3log2 / әt = 3log2 /log2 . T=3T1pt
t= 4860 years 0.5 pt

Ex3 .5.5pt

Boundary line or limit line, n1=?, n2 =Ð. 0.5 pt

$1\backslash\lambda_{lim} = R_H\,(1\backslash n^2 - 1\backslash p^2)$. 0.5 pt
$1\backslash\lambda_{lim} = R_H\backslash n^2$ $n = \sqrt{\square_{\square}\lambda_{\square\square\square}}$ $n = 3$ 0.5 pt

Pashen series 0.5 pt

2. The energy of photon (the first line of this series) Pashen. $n = 3$ $p=4$. 0.5 pt
$\varphi E=E_{ph}=Z^2\backslash n^2 * E_H - Z^2\backslash p^2 * E_H$. 0.5 pt
$Z=1$

$E_{ph} = E_H\,(1\backslash n^2 - 1\backslash p^2) = 0.66$ eV 0.5 pt

$E_{ph} = 1.062\ 10^{-19}$ J $= 0.66$ eV. 0.5 pt
3. E_{ph} of the limite boundary line

$E_{ph} = E1\ \backslash 9$. 0.5 pt

$= 1.51$ eV 0.5 pt

4. the energy absorbed by the photon representthe ionization energy é .. 0.5 pt

Ex 4. 4.5 pt

the electronconfiguration. 0.5 * 3 elements pt é é é 1.5 pt
core electrons 0.25 pt* 3 é é é 0.75 pt
valence electrons0.25pt *3é é é 0.75 pt
chemical group 0.25 pt*3é é é 0.75 pt
the period0.25 pt*3é é é 0.75 pt

Analysis 1: solutions of the Final Exam:

Exercise 01: (08 pts)

Let: $A = \left\{ \frac{n+2}{n-1},\ \forall n \in \mathbb{N},\ n \geq 2 \right\}$.

a) • We have: $\frac{n+2}{n-1} = \frac{n-1+3}{n-1} = 1 + \frac{3}{n-1}$.

and: $n \geq 2 \Rightarrow n-1 \geq 1 \Rightarrow \frac{1}{n-1} \leq 1 \Rightarrow \frac{3}{n-1} \leq 3$.

$\Rightarrow 1 + \frac{3}{n-1} \leq 4 \longrightarrow (1)$ (1pt)

Also we have: $n-1 \geq 0 \Rightarrow \frac{3}{n-1} > 0 \Rightarrow \frac{3}{n-1} + 1 > 1 \longrightarrow (2)$

from (1) and (2) we obtain:

$\forall n \geq 2: \quad 1 < \frac{n+2}{n-1} \leq 4 \Rightarrow A$ is bonded.

• 4 is an upper bond, and $4 \in A$ (0,5) (0,5)

because: if $\frac{n+2}{n-1} = 4 \Leftrightarrow n+2 = 4n-4 \Leftrightarrow 3n = 6 \Leftrightarrow n = 3$

so: $\sup A = \max A = 4$. (0,5)

• 1 is a lower bound, but: $1 \notin A$. (0,5)

because: if $\frac{n+2}{n-1} = 1 \Leftrightarrow n+2 = n-1 \Leftrightarrow 2 = -1$ contradiction. (0,5)

so we must prove that: $\inf A = 1$.

$$\inf A = 1 \Leftrightarrow \begin{cases} \forall n \geq 2: \frac{n+2}{n-1} > 1 \text{ (proved)}. \\ \wedge \\ \forall \varepsilon > 0: \exists n \geq 2: \frac{n+2}{n-1} < 1 + \varepsilon. \end{cases}$$ (0,5)

so; Let: $\varepsilon > 0$; $\quad 1 + \frac{3}{n-1} < 1 + \varepsilon \Leftrightarrow \frac{3}{n-1} < \varepsilon$.

$\Leftrightarrow n - 1 > \frac{3}{\varepsilon}$

$\Leftrightarrow n > \frac{3}{\varepsilon} + 1$

by taking: $n = \left[\frac{3}{\varepsilon} + 1\right] + 1 \geq 2$. (0,5)

so: $\forall \varepsilon > 0: \exists n = \left[\frac{3}{\varepsilon} + 1\right] + 1 \geq 2$: $\frac{n+2}{n-1} < 1 + \varepsilon$. (its the definition)

$\Rightarrow \inf A = 1$

but: min A not exist because $1 \notin A$. (0,5)

• solution of: $\left|\frac{1}{2}x + 2\right| + 6 \leq 15 \rightarrow (2)$

1st method $\Rightarrow \left|\frac{1}{2}x + 2\right| < 9 \Rightarrow -9 < \frac{1}{2}x + 2 < 9$ (1,5)

$\Rightarrow -11 < \frac{1}{2}x < 7 \Rightarrow -22 < x < 14$

$\Rightarrow S_1 =]-22, 14[$

2nd method:

$$\left|\tfrac{1}{2}x+2\right| = \begin{cases} \tfrac{1}{2}x+2 & \text{if } \tfrac{1}{2}x+2 \geq 0 \\ -\tfrac{1}{2}x-2 & \text{if } \tfrac{1}{2}x+2 \leq 0 \end{cases} = \begin{cases} \tfrac{1}{2}x+2 & \text{if: } x \geq -4 \\ -\tfrac{1}{2}x-2 & \text{if } x \leq -4 \end{cases}$$

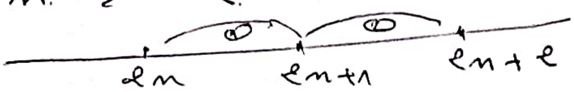
so: $(2) \iff \begin{cases} \tfrac{1}{2}x+2+6 \leq 15 & \text{if } x \geq -4 \\ \vee \\ -\tfrac{1}{2}x-2+6 \leq 15 & \text{if } x \leq -4 \end{cases}$

$\iff \begin{cases} x < 14 \wedge x \geq -4 \Rightarrow x \in [-4,14[\\ \vee \\ x > -22 \wedge x \leq -4 \Rightarrow x \in]-22,-4] \end{cases}$

so, $S =]-22,-4] \cup [-4,14[=]-22,14[$.

• solution of: $[2x]+2[x]=13 \longrightarrow (1)$

we put: $[x]=n \Rightarrow n \leq x < n+1 \Rightarrow 2n \leq 2x < 2n+2$ 1.5



if: $2n \leq 2x < 2n+1$, then: $[2x]=2n$.

so: $(1) \iff 2n+2n = 13 \iff 4n=13$

$\iff n = \frac{13}{4} \notin \mathbb{N} \iff S_1 = \emptyset$.

if: $2n+1 \leq 2x < 2n+2$

then: $[2x]=2n+1$.

so: $(1) \iff 2n+1+2n = 13 \Rightarrow 4n=12 \Rightarrow n=3 \in \mathbb{N}$.

$[x]=3 \Rightarrow 3 \leq x < 4 \Rightarrow S_2 = [3,4[$

$\Rightarrow S = S_1 \cup S_2 = [3,4[$.

Exercise 02: (4 pts): $u_n = \sum_{k=1}^{n} \frac{1}{k^2}$ and $v_n = u_n + \frac{2}{n}$.

• $u_{n+1}-u_n = \sum_{k=1}^{n+1} \frac{1}{k^2} - \sum_{k=1}^{n} \frac{1}{k^2} = \sum_{k=1}^{n} \frac{1}{k^2} + \frac{1}{(n+1)^2} - \sum_{k=1}^{n} \frac{1}{k^2}$ 1.5

$= \frac{1}{(n+1)^2} > 0 \Rightarrow u_n$ is increasing $\rightarrow (1)$

• $v_{n+1}-v_n = u_{n+1} + \frac{2}{n+1} - u_n - \frac{2}{n} = u_{n+1}-u_n + \frac{2n-2n-2}{n(n+1)}$

$= \frac{1}{(n+1)^2} + \frac{-2}{n(n+1)} = \frac{n-2(n+1)}{n(n+1)^2}$ 1.5

$= \frac{-n-2}{n(n+1)^2} < 0 \Rightarrow v_n$ is decreasing $\rightarrow (2)$

• $\lim_{n\to+\infty}(u_n-v_n) = \lim_{n\to+\infty} \frac{2}{n} = 0 \longrightarrow (3)$ 1

from (1), (2) and (3): u_n and v_n are adjacent sequences.

Exercise 03: (8 Pts):

- $g(x) = x^3 + 3x - e$ is a continuous function on $[0,1]$ (0,5)
  $g(0) = -e$ and $g(1) = \tau$. (0,5)
  and $g(0) \cdot g(1) < 0$ (0,5)
  by intermediate value theorem we obtain:
  $\exists\, c \in]0,1[: g(c) = 0$. (0,5)
  but: $g'(x) = 3x^2 + 3 > 0 \Rightarrow g$ is monotonic.
  So: $\exists!\, c \in]0,1[: g(c) = 0$. (there is only one solution c in $]0,1[$) (0,5)
- $\lim_{x\to 0} \left(\frac{(x^3+x^2)\sqrt{x^2+3}}{\ln(x^2+1)}\right) = \frac{0}{0}$ (IF) (0,5)

$\ln(x^2+1) \underset{x\to 0}{\sim} x^2$ because: $\lim_{x\to 0} \frac{\ln(x^2+1)}{x^2} \overset{x^2=t}{=} \lim_{t\to 0} \frac{\ln(t+1)}{t} = \boxed{1}$

So: $\lim_{x\to 0} \frac{(x^3+x^2)\sqrt{x^2+3}}{\ln(x^2+1)} = \lim_{x\to 0} \frac{x^2(x+1)\sqrt{x^2+3}}{x^2} = \sqrt{3}$. (0,5)

- we have: $D_f = \mathbb{R}^*$ so f is continuous on $\mathbb{R}^*$. (0,5)
  yes f admit a continuous extension on $\mathbb{R}$, because: $\lim_{x\to 0} f(x) = \sqrt{3}$ (exists and unique)

So: $\tilde{f}(x) = \begin{cases} f(x) & \text{if } x \in \mathbb{R}^* \\ \sqrt{3} & \text{if } x = 0 \end{cases}$. (0,5)

- $h(x) = \begin{cases} ax^2+1 & \text{if } x \geq 1 \\ bx-3 & \text{if } x < 1 \end{cases}$ ax^2+1 is continuous on $(x \geq 1)$, $bx-3$ is continuous on $(x < 1)$ (0,5)

$h(x)$ is continuous on $\mathbb{R} \Rightarrow h(x)$ is continuous at $x_0 = 1$.

$h(x)$ is continuous $\Rightarrow \lim_{x \overset{>}{\to} 1} h(x) = \lim_{x \overset{<}{\to} 1} h(x) = h(1) = a+1$. (0,5)

and $\lim_{x \overset{<}{\to} 1} h(x) = b-3$.

we have: $\lim_{x \overset{>}{\to} 1} h(x) = a+1$ (0,5)

So, $h(x)$ is continuous on $\mathbb{R} \Rightarrow a+1 = b-3 \rightarrow (1)$

- Also we have: ax^2+1 is differentiable on $(x \geq 1)$
  and: $bx-3$ is " " on $(x < 1)$ (0,5)

$h(x)$ is differentiable on $\mathbb{R} \Rightarrow h(x)$ is differentiable at $x_0 = 1$

So: $h(x)$ is differentiable $\Rightarrow \lim_{x \overset{>}{\to} 1} \frac{h(x)-h(1)}{x-1} = \lim_{x \overset{<}{\to} 1} \frac{h(x)-h(1)}{x-1} = L$ (exists and unique) (0,5)

Remark: if: $a+1 \neq b-3 \Rightarrow h(x)$ is not continuous at $x_0 = 1$
$\Rightarrow h(x)$ is not differentiable at $x_0 = 1$ (0,5)

So: $h(x)$ is differentiable at $x_0 = 1$
$\Rightarrow a+1 = b-3$

$$\lim_{x \overset{>}{\to} 1} \frac{h(x) - h(1)}{x - 1} = \lim_{x \overset{>}{\to} 1} \frac{ax^2 + 1 - a - 1}{x - 1} = \lim_{x \overset{>}{\to} 1} \frac{a(x^2 - 1)}{x - 1} =$$

$$= \lim_{x \overset{>}{\to} 1} a(x+1) = 2a.$$

and: $\lim_{x \overset{<}{\to} 1} \frac{h(x) - h(1)}{x - 1} = \lim_{x \overset{<}{\to} 1} \frac{bx - 3 - (a+1)}{x - 1}$

$= \lim_{x \overset{<}{\to} 1} \frac{bx - 3 - (b-3)}{x - 1}$ (because $a+1 = b-3$)

$= \lim_{x \overset{<}{\to} 1} \frac{bx - b}{x - 1} = b$

0,n

$h(x)$ is differentiable on $\mathbb{R}$ => diff at $x_0 = 1$.

=> $2a = b \longrightarrow$ (2)

so: $h(x)$ is continuous and differentiable on $\mathbb{R}$

$$\Rightarrow \begin{cases} a + 1 = b - 3 \rightarrow (1) \\ \wedge \\ 2a = b \rightarrow (2) \end{cases}$$

0,n

(2) in (1): $a + 1 = 2a - 3 \Rightarrow \boxed{a = 4}$

from (2): $b = 2a \Rightarrow \boxed{b = 8}$.