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Faculté des sciences exactes et des sciences de la nature et de la vie
Département des Mathématiques et de l'informatique

الجمهورية الجزائرية الديمقراطية الشعبية
وزارة التعليم العالي و البحث العلمي
جامعة أم البواقي
كلية العلوم الدقيقة وعلوم الطبيعة والحياة
قسم الرياضيات و الاعلام الالي

Année Académique: 2025/2026 **Domaine:** Mathématiques et Informatique
Filière: Mathématiques
Spécialité: mathématiques
Niveau: Licence 2ème Année **Période:** Semestre 4
Matière Analyse complexe
Section/Groupe Section **Enseignant:** MERAD Ahcene

PV des notes des examens par matière (Enseignant)

#	Matricule	Nom	Prénom	Note Examen	Note corrigée	Signature
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2	242434010405			2.5		
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Signature de l'enseignant: (MERAD Ahcene)
Le: 17/05/2026

UNIVERSITY OF OUM EL BOUAGHI-FINAL EXAM -COMPLEX ANALYSIS

L2 MATHEMATICS-MAY10, 2026

Exercice 1. (05 pts)

1) State the Cauchy Riemann Equations.

Let : $\Omega = \{z \in \mathbb{C}, \quad \operatorname{Re} z > 0\}$,and consider the real valued function

$$v(x,y) = 1 + \arctan \frac{y}{x}.$$

2) Find, if it exist a holomorphic function $f : \Omega \to \mathbb{C}$ where

$$f(z) = u(x,y) + iv(x,y),$$

satisfying each of the following conditions

$$\begin{aligned} a)\ f(3) &= \ln 3 + 6 + i \\ b)\ f(e) &= 1 - i \end{aligned}$$

Exercice 2. Compute the integral
(05 pts)

$$\int_\gamma |z|\,\overline{z}dz,$$

where γ is a simple, closed, positively oriented, piecewise smooth path defined by

$$\gamma(t) = \begin{cases} 2e^t, & \text{if } t \in [0,\pi] \\ t, & \text{if } t \in [-2,2] \end{cases}$$

Exercice 3. Find the Laurent series expansion of the function
(05 pts)

$$f(z) = \frac{1}{z^2 - 2z - 3},$$

on the annulus (ring)

$$\{z \in \mathbb{C} : \quad 1 < |z| < 3\}.$$

Exercice 4. (05 pts)

1) State the Cauchy Residue theorem

2) Using the Cauchy Residue theorem, evaluate the integral

$$\int_\gamma \frac{\sin z}{(z-i)(z+1)^2} dz,$$

where the path γ is defined by

$$\gamma(t) = 3e^{it} + 1, \qquad \text{if } t \in [0, 2\pi].$$

Good Luck

Correction of the Final Exam
Complex Analysis - L2 Mathematics

Exo1 (05pts):

1) Cauchy Riemann equations: (see the lesson). 1

2) $\Omega = \{z \in \mathbb{C};\ \mathrm{Re}\, z > 0\}$, $v(x,y) = 1 + \arctan\frac{y}{x}$

$$\frac{\partial v}{\partial y} = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} = \frac{x}{x^2+y^2} = \frac{\partial u}{\partial x},$$ therefore 0,5 0,5

$u(x,y) = \frac{1}{2}\ln(x^2+y^2) + \varphi(y)$, and by the second Cauchy-Riemann equation 0,5

$$-\frac{\partial v}{\partial x} = \frac{-1}{1+\frac{y^2}{x^2}} \cdot \frac{-y}{x^2} = \frac{y}{x^2+y^2} = \frac{\partial u}{\partial y} = \frac{y}{x^2+y^2} + \varphi'(y),$$ 0,5 0,5

we get: $\varphi(y) = c$, $c \in \mathbb{R}$. The sought function on the set Ω is: $f(x+iy) = \ln\left(\sqrt{x^2+y^2}\right) + c + i\arg(x+iy)$; 0,5

$c \in \mathbb{R}$, which is: $f(z) = \ln z + c$, $c \in \mathbb{R}$.

a) we have: $f(3+0i) = \ln 3 + c + i = \ln 3 + 6 + i \Rightarrow$ 0,5

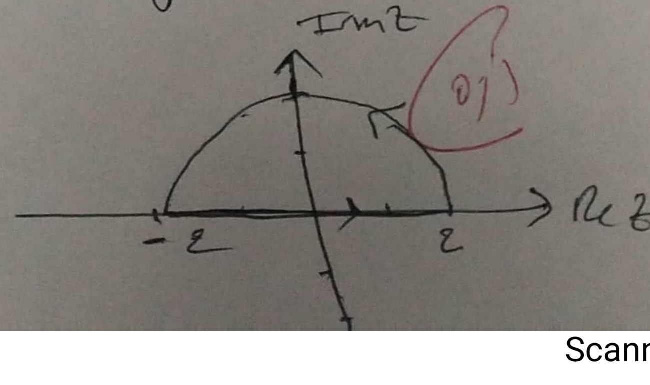
$c = 6$, and therefore,

$f(x+iy) = \ln\sqrt{x^2+y^2} + 6 + i\left(\arctan\frac{y}{x} + 1\right)$ in Ω.

b) $\forall c \in \mathbb{R}$: $f(e+0i) = 1 + c + i \neq 1 - i$; 0,5

the sought function f does not exist.

Exo2 (05pts):



Scanné avec CamScanner

Let $\gamma_1(t)=2e^{it}$; $t\in[0,\pi]$ and $\gamma_2(t)=t$; $t\in[-2,2]$

then : $\gamma_1'(t)=2ie^{it}$; $\gamma_2'(t)=1$ and therefore

$$\int_\gamma |z|\bar{z}\,dz=\int_{\gamma_1}|z|\bar{z}\,dz+\int_{\gamma_2}|z|\bar{z}\,dz=\int_0^\pi 2\cdot 2e^{-it}\cdot 2ie^{it}\,dt+\underbrace{\int_{-2}^{2}|t|t\,dt}_{=0}=8i\int_0^\pi dt=8\pi i.$$

<u>Exo3</u> (<u>05pts</u>) : Let $f(z)=\frac{1}{z^2-2z-3}=\frac{1}{(z-3)(z+1)}=\frac{1}{4}\frac{1}{z-3}-\frac{1}{4}\frac{1}{z+1}$.

* <u>If $|z|>1$</u> : $-\frac{1}{4}\frac{1}{z+1}=\frac{-1}{4z}\frac{1}{1-(-\frac{1}{z})}=-\frac{1}{4z}\sum_{n\geq 0}\left(-\frac{1}{z}\right)^n$

$=\frac{1}{4}\sum_{n\geq 0}\frac{(-1)^{n+1}}{z^{n+1}}=\frac{1}{4}\sum_{n\geq 1}\frac{(-1)^n}{z^n}$.

* <u>If $|z|<3$</u> : $\frac{1}{4}\frac{1}{z-3}=-\frac{1}{12}\frac{1}{1-\left(\frac{z}{3}\right)}=-\frac{1}{12}\sum_{n\geq 0}\left(\frac{z}{3}\right)^n=$

$-\frac{1}{4}\sum\frac{z^n}{3^{n+1}}$. Consequently, Laurent series of f given by : $f(z)=-\frac{1}{4}\sum_{n\geq 0}\frac{z^n}{3^{n+1}}+\frac{1}{4}\sum_{n\geq 1}\frac{(-1)^n}{z^n}$ on

$1<|z|<3$.

<u>Exo4</u> <u>(05pts)</u> :

1) Cauchy Residue theorem (See the lesson).

$\boxed{P2}$

② $f(z) = \dfrac{\sin z}{(z-i)(z+1)^2}$ has two singularities $z_1 = i$ and $z_2 = -1$. The points z_1 is simple pole and z_2 is the pole of order 2. Now, calculate residues at these points: $\text{Res}(f, z_1) = \lim_{z \to i} \frac{1}{0!} \left(f(z)(z-i)\right) = \lim_{z \to i} \frac{\sin z}{(z+1)^2} = \frac{\sin i}{(i+1)^2}$, $\text{Res}(f, z_2) = \lim_{z \to -1} \frac{1}{1!} \left(f(z)(z+1)^2\right)' =$

$$\lim_{z \to -1} \left(\frac{\sin z}{z-i}\right)' = \frac{\cos(-1)(-1-i) - \sin(-1)}{(i+1)^2}.$$

z_1, z_2 inside γ, by Residue theorem:

$$\int_\gamma f(z)\, dz = 2\pi i \left(\text{Res}(f, z_1) + \text{Res}(f, z_2)\right) =$$

$$\frac{2\pi i}{(1+i)^2} \left[\sin(i) - \cos(1)(1+i) + \sin(1)\right].$$

End