SOULA Yaminamathématiques/Semestre 2/Algèbre 2/Groupe 1						
Matricule	Note	Absent	Absence Justifiée	Observation	Section	Groupe
242534110810	0.0				Section	Groupe 1
242434008001	11.0				Section	Groupe 1
242434071807	13.5				Section	Groupe 1
232334031113	10.5				Section	Groupe 1
242534111906	17.0				Section	Groupe 1
252534079903	19.0				Section	Groupe 1
222234091402	3.5				Section	Groupe 1
222434105207	14.0				Section	Groupe 1
242434107816	3.5				Section	Groupe 1
232334015904	0.0				Section	Groupe 1
242434042118	1.5				Section	Groupe 1
242434044510	14.5				Section	Groupe 1
222534098701	0.0				Section	Groupe 1
212434106116	0.0				Section	Groupe 1
242434049306	0.0				Section	Groupe 1
222234031107	0.0				Section	Groupe 1
242434031619	10.0				Section	Groupe 1
242434049207	13.5				Section	Groupe 1
232334139218	0.0				Section	Groupe 1
242434070102	10.5				Section	Groupe 1
242534111901	0.0				Section	Groupe 1
242434043817	10.0				Section	Groupe 1
242434010203	9.0				Section	Groupe 1
242434081609	17.5				Section	Groupe 1
242434100919	13.0				Section	Groupe 1
242434065515	2.0				Section	Groupe 1
242434009716	14.5				Section	Groupe 1

SOULA Yaminamathématiques/Semestre 2/Algèbre 2/Groupe 3						
Matricule	Note	Absent	Absence Justifiée	Observation	Section	Groupe
252534031502	12.5				Section	Groupe 3
252534052205	2.0				Section	Groupe 3
222234018714	0.0				Section	Groupe 3
252534011219	0.0				Section	Groupe 3
252534034502	10.0				Section	Groupe 3
252534062102	12.0				Section	Groupe 3
252534074407	11.5				Section	Groupe 3
252534040810	15.5				Section	Groupe 3
252534062606	8.75				Section	Groupe 3
252534047405	8.25				Section	Groupe 3
252534073812	10.75				Section	Groupe 3
252534043111	17.0				Section	Groupe 3
252534079908	14.5				Section	Groupe 3
252534017514	0.0				Section	Groupe 3
252534062604	13.5				Section	Groupe 3
252534062103	6.5				Section	Groupe 3
252534010502	2.0				Section	Groupe 3
252534060406	0.0				Section	Groupe 3
252534065007	9.0				Section	Groupe 3
242434042419	0.0				Section	Groupe 3
252534043004	14.0				Section	Groupe 3
252534018416	0.0				Section	Groupe 3
252534064616	10.0				Section	Groupe 3
252534080606	0.0				Section	Groupe 3
252534034907	11.5				Section	Groupe 3
252534056220	10.0				Section	Groupe 3
252534072409	10.5				Section	Groupe 3
252534052801	14.0				Section	Groupe 3
252534023118	10.0				Section	Groupe 3
252534073819	9.5				Section	Groupe 3
252534043818	10.0				Section	Groupe 3

252534065316	0.0				Section	Groupe 3

SOULA Yaminamathématiques/Semestre 2/Algèbre 2/Groupe 2						
Matricule	Note	Absent	Absence Justifiée	Observation	Section	Groupe
252534007201	12.5				Section	Groupe 2
242534102310	16.5				Section	Groupe 2
242434065517	16.0				Section	Groupe 2
252534010003	0.0				Section	Groupe 2
252534017519	10.0				Section	Groupe 2
252534051717	0.0				Section	Groupe 2
242434033204	10.5				Section	Groupe 2
252534010004	15.5				Section	Groupe 2
252534010015	13.0				Section	Groupe 2
252534006818	10.0				Section	Groupe 2
252534042414	11.5				Section	Groupe 2
252534042915	15.5				Section	Groupe 2
252534007510	10.0				Section	Groupe 2
252534008314	5.0				Section	Groupe 2
242534108110	13.5				Section	Groupe 2
252534010705	16.5				Section	Groupe 2
252534008310	8.5				Section	Groupe 2
252534042907	10.0				Section	Groupe 2
252534010101	11.5				Section	Groupe 2
252534110709	1.0				Section	Groupe 2
252534035304	14.5				Section	Groupe 2
252534108808	8.0				Section	Groupe 2
252534010105	10.0				Section	Groupe 2
252534007704	15.0				Section	Groupe 2
252534071502	13.5				Section	Groupe 2
252534081916	0.0				Section	Groupe 2
252534010103	13.0				Section	Groupe 2
252534009912	7.5				Section	Groupe 2
252534052409	14.0				Section	Groupe 2

252534069406	12.5				Section	Groupe 2
252534005906	3.0				Section	Groupe 2
252534043804	11.0				Section	Groupe 2
252534051801	13.0				Section	Groupe 2

SOULA Yamina/mathématiques/Semestre 2/Algèbre 2/Section							
Matricule	Note	Absent	Absence Justifiée	Observation	Section	Groupe	
242534110810	1.0					Section/Groupe 1	
242434008001	2.0					Section/Groupe 1	
242434071807	7.5					Section/Groupe 1	
232334031113	5.0					Section/Groupe 1	
242534111906	7.5					Section/Groupe 1	
252534079903	14.5					Section/Groupe 1	
222234091402						Section/Groupe 1	
222434105207	8.5					Section/Groupe 1	
242434107816	1.5					Section/Groupe 1	
232334015904						Section/Groupe 1	
242434042118	1.0					Section/Groupe 1	
242434044510	12.0					Section/Groupe 1	
222534098701						Section/Groupe 1	
212434106116						Section/Groupe 1	
242434049306						Section/Groupe 1	
222234031107						Section/Groupe 1	
242434031619	7.5					Section/Groupe 1	
242434049207	10.0					Section/Groupe 1	
232334139218						Section/Groupe 1	
242434070102	7.0					Section/Groupe 1	
242534111901						Section/Groupe 1	
242434043817	12.0					Section/Groupe 1	
242434010203	1.0					Section/Groupe 1	
242434081609	15.0					Section/Groupe 1	
242434100919	7.5					Section/Groupe 1	
242434065515						Section/Groupe 1	
242434009716	5.5					Section/Groupe 1	
252534007201	12.5					Section/Groupe 2	
242534102310	18.5					Section/Groupe 2	
242434065517	18.5					Section/Groupe 2	
252534010003						Section/Groupe 2	

252534017519	7.0					Section/Groupe 2
252534051717						Section/Groupe 2
242434033204	5.5					Section/Groupe 2
252534010004	8.5					Section/Groupe 2
252534010015	5.5					Section/Groupe 2
252534006818	2.0					Section/Groupe 2
252534042414	13.0					Section/Groupe 2
252534042915	7.5					Section/Groupe 2
252534007510	1.0					Section/Groupe 2
252534008314	1.0					Section/Groupe 2
242534108110	12.0					Section/Groupe 2
252534010705	13.5					Section/Groupe 2
252534008310	7.0					Section/Groupe 2
252534042907	12.0					Section/Groupe 2
252534010101	5.0					Section/Groupe 2
252534110709	1.0					Section/Groupe 2
252534035304	11.5					Section/Groupe 2
252534108808	6.0					Section/Groupe 2
252534010105	11.5					Section/Groupe 2
252534007704	12.0					Section/Groupe 2
252534071502	10.0					Section/Groupe 2
252534081916						Section/Groupe 2
252534010103	10.5					Section/Groupe 2
252534009912	2.0					Section/Groupe 2
252534052409	12.0					Section/Groupe 2
252534069406	8.5					Section/Groupe 2
252534005906	1.0					Section/Groupe 2
252534043804	4.0					Section/Groupe 2
252534051801	8.0					Section/Groupe 2
252534031502	5.0					Section/Groupe 3
252534052205	1.0					Section/Groupe 3
222234018714						Section/Groupe 3
252534011219						Section/Groupe 3

252534034502	4.0					Section/Groupe 3	
252534062102	10.5					Section/Groupe 3	
252534074407	1.0					Section/Groupe 3	
252534040810	6.0					Section/Groupe 3	
252534062606	1.0					Section/Groupe 3	
252534047405	4.0					Section/Groupe 3	
252534073812	7.5					Section/Groupe 3	
252534043111	19.0					Section/Groupe 3	
252534079908	13.0					Section/Groupe 3	
252534017514						Section/Groupe 3	
252534062604	10.0					Section/Groupe 3	
252534062103	1.0					Section/Groupe 3	
252534010502	1.0					Section/Groupe 3	
252534060406						Section/Groupe 3	
252534065007	1.0					Section/Groupe 3	
242434042419						Section/Groupe 3	
252534043004	19.0					Section/Groupe 3	
252534018416						Section/Groupe 3	
252534064616	8.0					Section/Groupe 3	
252534080606						Section/Groupe 3	
252534034907	3.5					Section/Groupe 3	
252534056220	8.5					Section/Groupe 3	
252534072409	7.5					Section/Groupe 3	
252534052801	11.0					Section/Groupe 3	
252534023118	1.0					Section/Groupe 3	
252534073819	4.0					Section/Groupe 3	
252534043818	3.0					Section/Groupe 3	
252534065316						Section/Groupe 3	

L'ARBI BEN M'HIDI UNIVERSITY, OUM EL BOUAGHI
Department of Mathematics and Computer Science
First Year Mathematics **Module Algebra 2**
Professor: Soula **2025/2026**

Exam in: 13/05/2026

Exercise 1 (5 points)
1) Give the definition of a vector subspace of a vector space over the field $\mathbb{R}$.
2) Define what it means for a set of vectors to be:
a) linearly independent.
b) a basis of a vector space.
3) State the relationship between:
a) dimension
b) basis of a finite-dimensional vector space
4) State a condition under which a square matrix is invertible.
5) Define the rank of a matrix.

Exercise 2 (7 points)
Let

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

defined by:

$$f\ (x, y, z) = (x - y + z,\ \ 2x + y - z)$$

1) Verify that f is a linear application.
2) Determine the kernel of f , $(\ker(f))$
3) Determine the image of f , $(\text{Im}(f))$
4) Compute:
a) the dimension of the kernel
b) the dimension of the image.
5) Determine the rank of the linear application f.

Exercise 3 (8 points)
Let the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

1) Compute: A^2
2) Compute: $\det(A)$
3) If A is invertible, find $\mathbf{A}^{-1}$.
4) Determine the rank of the matrix A.

Good luck!

L'ARBI BEN M'HIDI UNIVERSITY, OUM EL BOUAGHI
Department of Mathematics and Computer Science
First Year Mathematics **Module Algebra 2**
Professor: Soula **2025/2026**
Date: 13/05/2026

Model Answer for Exam

Exercise 1 (5 points)

1) Definition of a vector subspace...**(1.5)**

A nonempty subset F of a vector space E over the field $\mathbb{R}$ is called a vector subspace of E if it satisfies the following conditions:

- $0_E \in F$
- For all U, $V \in F$, we have $U + V \in F$ (closed under addition).
- For all $\lambda \in \mathbb{R}$ and For all $U \in F$, we have $\lambda U \in F$ (closed under scalar multiplication)

2)...**(1.5pt)**

a) Linearly independent

A set of vectors $\{v_1, v_2, \ldots, v_n\}$ in a vector space E is said to be linearly independent if:

For all $\lambda_1, \lambda_2, \cdots, \lambda_n \in \mathbb{R}$,

$\lambda_1 v_1 + \lambda_2 v_2 + \cdots + \lambda_n v_n = 0$ implies $\lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$.

b) Basis of a vector space

A family of vectors $\{v_1, v_2, \ldots, v_n\}$ is called a basis of a vector space E if:

- the vectors are linearly independent, and
- they generate the space E (that is, every vector of E can be written as a linear combination of them).

3) Relationship between dimension and basis..**(1pt)**

In a finite-dimensional vector space:

- All bases have the same number of vectors.
- This number is called the dimension of the vector space.

In other words:

The dimension of a finite-dimensional vector space is equal to the number of vectors in any basis of that space.

4) Condition for a square matrix to be invertible A **square matrix**

A is invertible if and only if: $\det(A) \neq 0$...**(0.5pt)**

(Equivalently, the rank of A is equal to its size).

i.e. if a square matrix A of order $(n \times n)$ then :

$$A \text{ is invertible.} \Leftrightarrow \det(A) \neq 0. \Leftrightarrow rank(A) = n$$

5) Definition of the rank of a matrix ....**(0.5pt)**

The rank of a matrix A is the dimension of the vector space generated by its rows (or columns).

Equivalently:

The rank of a matrix is the maximum number of linearly independent rows (or columns) of the matrix.

Exercise 2 (7 points)

$$f : \mathbb{R}^3 \to \mathbb{R}^2$$

$$f\ (x, y, z) = (x - y + z,\ \ 2x + y - z)$$

1) Show that f is a linear application..**(2pts)**

We verify linearity:

For all $U = (x_1, y_1, z_1)$, $V = (x_2, y_2, z_2)$, and for all $\lambda \in \mathbb{R}$

• $f(U + V) = f((x_1, y_1, z_1) + (x_2, y_2, z_2))$

$= f((x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2))$

$= (x_1 + x_2 - y_1 - y_2 + z_1 + z_2,\ \ 2x_1 + 2x_2 + \ y_1 + y_2 - \ z_1 - z_2)$

$= ((x_1 - y_1 + z_1) + (x_2 - y_2 + z_2),,\ \ (2x_1 + y_1 - z_1) + (2x_2 + y_2 - z_2)))$

$= ((x_1 - y_1 + z_1),\ (2x_1 + y_1 - z_1)) + (\ (x_2 - y_2 + z_2),\ (2x_2 + y_2 - z_2))$

$= f(x_1, y_1, z_1) + f(x_2, y_2, z_2)$

$= f(U) + f(V)$.

• $f(\lambda U) = f(\lambda(x_1, y_1, z_1)) = (\lambda x_1 - \lambda y_1 + \lambda z_1,\ 2\lambda x_1 + \lambda y_1 - \lambda z_1)$

$= \lambda f(U)$

Therefore, f is a linear application.

2) Kernel of f...**(1pt)**

$ker(f) = \left\{(x, y, z) \in \mathbb{R}^3 / \quad f(x, y, z) = 0_{\mathbb{R}^2}\right\}$

$= \left\{(x, y, z) \in \mathbb{R}^3 / \quad (x - y + z,\ \ 2x + y - z) = (0, 0)\right\}$

So,

$$\begin{cases} x - y + z = 0 \\ 2x + y - z = 0 \end{cases}$$

Add equations:

$x - y + z + 2x + y - z = 0 \Rightarrow x = 0$

Substitute

$-y + z = 0 \Rightarrow y = z$

So,

$ker(f) = \{(0, y, y) \ \ /y \in \mathbb{R}\} = \{y(0, 1, 1) \ \ /y \in \mathbb{R}\}$

3) Image of f...**(1pt)**

$\text{Im}(f) = \left\{f(x, y, z) \ \ / \quad (x, y, z) \in \mathbb{R}^3\right\}$

$= \left\{(x - y + z,\ \ 2x + y - z) \ \ / \quad (x, y, z) \in \mathbb{R}^3\right\}$

$= \left\{x(1, 2) + y(-1, 1) + z(1, -1) \ \ / \quad (x, y, z) \in \mathbb{R}^3\right\}$

Tow vectors are clearly linearly independent.

4)

$dim(ker(f)) = 1$...**(1pt)**

$dim(Im(f)) = 2$...**(1pt)**

5)

$rank(f) = dim(Imf) = 2$..**(1pt)**

<u>Exercise 3</u> (8 points)

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

1) Compute A^2..**(2pt)**

$$A^2 = A \times A$$

Compute row by column:

$$A^2 = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

<u>Row 1:</u>
- 1(1)+1(2)+0(1)=3
- 1(1)+1(1)+0(0)=2
- 1(0)+1(1)+0(1)=1

<u>Row 2:</u>
- 2(1)+1(2)+1(1)=5
- 2(1)+1(1)+1(0)=3
- 2(0)+1(1)+1(1)=2

<u>Row 3:</u>
- 1(1)+0(2)+1(1)=2
- 1(1)+0(1)+1(0)=1
- 1(0)+0(1)+1(1)=1

So,

$$A^2 = \begin{pmatrix} 3 & 2 & 1 \\ 5 & 3 & 2 \\ 2 & 1 & 1 \end{pmatrix}$$

2) Compute $\det(A)$..**(2pt)**

$$\det(A) = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

Expand along first row:

$$\det(A) = 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

So,

$$\det(A) = 1(1(1) - 0(1)) - 1(2(1) - 1(1)) + 0(2(0) - 1(1))$$

Then,

$$\det(A) = 1(1) - 1(1) + 0(-1)$$

Therfore,

$$\det(A) = 0$$

3)..**(2pt)**
$\det(A) = 0 \Rightarrow\ A$ is NOT invertible
So the matrix has no inverse
Then
A^{-1} does not exist.
4)..**(2pt)**
Since determinant of A is zero then $rank\ (A) < 3$.
Rows are not multiples, and two rows are independent.
So,

$$rank\ (A) = 2$$