



Exam

2 janvier 2026

Durée : 1h30

Tous les documents, autres que ceux fournis dans le sujet, sont interdits. ¹

Exercice 1 .

1. Consider the first-order ordinary differential equation

$$y' = (y - x)^2.$$

– Without explicitly solving this equation, sketch the slope field associated with this equation and draw several representative solution curves to illustrate the qualitative behavior of solutions.

2. Solve the following first-order ODE :

$$\frac{dy}{dx} = -\frac{2xy + y^2 + 1}{x^2 + 2xy}.$$

Exercice 2 .

Consider the first-order quasilinear partial differential equation

$$(y^2 - u^2)u_x + (u^2 - x^2)u_y = x^2 - y^2.$$

1. Solve the PDE using the method of characteristics.
2. Determine the integral surface of the PDE that contains the line

$$(P) : \begin{cases} z = 0, \\ y = 2x. \end{cases}$$

Exercice 3 .

1. Plot the 2π -periodic extension of the function

$$f(x) = x^2, \quad x \in (0, 2\pi).$$

2. Determine the Fourier series of the 2π -periodic extension of $f(x) = x^2$.
3. Using the convergence of the Fourier series at $x = 0$, deduce the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

1. Barème : 7-6-7 pts.