



Exams in physics I

Name:	Group:	Date: 18/01/2026
Surname:		Duration: 01h30

Exercise 1 (08)

The motion of a point M is described in polar coordinates by:

$$\begin{cases} r = b \\ \varphi = k\omega t^2 \end{cases} \quad \text{Where } b, k \text{ and } \omega \text{ are positive constants.}$$

1. Write the position vector of point M in polar coordinates.

$$\vec{OM} = r \vec{U}_r$$

☐ $\vec{OM} = x\vec{i} + y\vec{j}$

☐ $\vec{OM} = b(\sin\varphi\vec{i} + \cos\varphi\vec{j})$

☒ $\vec{OM} = b(\cos\varphi\vec{i} + \sin\varphi\vec{j})$

☒ $\vec{OM} = b\vec{U}_r$

2. Calculate the velocity vector of point M in polar coordinates.

$$\vec{v} = \dot{r}\vec{U}_r + r\dot{\varphi}\vec{U}_\varphi$$

☐ $\vec{v} = x\vec{i} + y\vec{j}$

☒ $\vec{v} = b\dot{\varphi}\vec{U}_\varphi$

☐ $\vec{v} = b\dot{\varphi}\vec{U}_r$

☒ $\vec{v} = 2bk\omega t\vec{U}_\varphi$

3. Calculate the magnitude of the velocity vector of point M in polar coordinates.

☒ $v = \sqrt{v_\varphi^2}$

☒ $v = 2bk\omega t$

☐ $v = b\dot{\varphi}$

☐ $v = \sqrt{\dot{x}^2 + \dot{y}^2}$

4. Calculate the acceleration vector of point M in polar coordinates.

$$\vec{a} = (\ddot{r} - r\dot{\varphi}^2)\vec{U}_r + (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\vec{U}_\varphi$$

☐ $\vec{a} = b t^4 \vec{U}_r + 2bt \vec{U}_\varphi$

☐ $\vec{a} = 2bk\omega \vec{U}_\varphi$

☒ $\vec{a} = -4bk^2\omega^2 t^2 \vec{U}_r + 2bk\omega \vec{U}_\varphi$

☐ $\vec{a} = b\ddot{U}_r$

5. Calculate the magnitude of the acceleration vector of point M in polar coordinates.

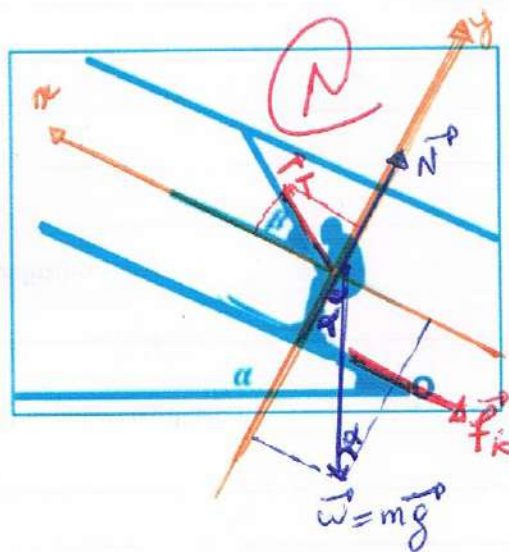
☒ $a = \sqrt{a_r^2 + a_\varphi^2}$
☒ $a = \sqrt{(-4bk^2\omega^2 t^2)^2 + (2bk\omega)^2}$
☐ $a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$
☐ $a = \sqrt{(2bk\omega)^2}$

6. Calculate the Cartesian coordinates (x, y) of point M.

☒ $x = b \cos k\omega t^2$
☐ $y = r \cos k\omega t^2$
☒ $y = b \sin k\omega t^2$
☐ $x = b \sin k\omega t^2$

Exercise 2 (08)

Pulled by a ski lift, a skier of mass $M = 60$ kg has a rectilinear movement on a slope inclined at an angle $\alpha = 10^\circ$ with the horizontal. The pole makes an angle $\beta = 25^\circ$ with the slope and exerts on the skier a tension $T = 300$ N. The skier is subjected to friction equivalent to a force of value $f_k = 50$ N. Initially, the skier is at point O of the frame ($x_0 = 0$ m) (Ox, Oy) without initial velocity ($v_0 = 0$ m/s).



1. Represent the forces acting on the skier.

2. Find the normal force N applied to the skier.

$\sum \vec{F}_{ext} = \vec{0} \Rightarrow \vec{W} + \vec{f}_k + \vec{N} + \vec{T} = \vec{0}$
 the projection on an axis (Oy): $-W \cos \alpha + 0 + N + T \sin \beta = 0$

$N = 60 \cdot (9.81) \cos 10 + 300 \sin 25 =$

☐ $N = 320$ N
 ☒ $N = 452.87$ N
 ☐ $N = 165.03$ N
 ☐ $N = 505.92$ N

3. Find the coefficient of kinetic friction μ_k for the skier?

$f_k = \mu_k \cdot N \Rightarrow \mu_k = \frac{f_k}{N} = \frac{50}{452.87}$

☐ $\mu_k = 0.09$
☐ $\mu_k = 0.15$
☒ $\mu_k = 0.11$
☐ $\mu_k = 0.3$

4. What is the acceleration of the skier?

$$\sum F_{ext} = m \vec{a} \Rightarrow \vec{W} + \vec{f}_k + \vec{N} + \vec{T} = m \vec{a}$$

the projection on an axis (ox): $-W \sin \alpha - f_k + 0 + T \cos \beta = ma$

$$a = (-60 \cdot (9.81) \sin 10 - 50 + 300 \cos 25) / 60$$

☒ $a = 1.99 \text{ m/s}^2$

☐ $a = 2 \text{ m/s}^2$

☐ $a = 2.38 \text{ m/s}^2$

☐ $a = 1.2 \text{ m/s}^2$

5. Show that the skier has a uniformly accelerated movement.

$$a = 1.99 (\text{m/s}^2) = \text{Constant} \Rightarrow \text{uniformly accelerated M} \checkmark$$

6. In what is the time "t" required for the skier in order to ke much time will the skier have travel 300 m ?

$$dv = a dt \Rightarrow \int_{v_0=0}^v dv = \int_0^t 1.99 dt \Rightarrow v = 1.99t \quad (1)$$

$$dx = v \cdot dt \Rightarrow \int_{x_0=0}^x dx = \int_0^t 1.99t \cdot dt \Rightarrow x = \frac{1.99}{2} t^2$$

$$\Rightarrow t = \sqrt{\frac{2 \times 300}{1.99}}$$

☐ $t = 20.04s$

☐ $t = 9.93s$

☒ $t = 17.36s$

☐ $t = 11.22s$

Exercise 2 (04)

A material point M is moving along a trajectory in space R (O; $\vec{i}, \vec{j}, \vec{k}$). The Cartesian coordinates of M are:

$$\bullet \quad x(t) = \sin\left(\frac{\pi}{2}t\right) + 2; \quad y(t) = 2 \cos\left(\frac{\pi}{2}t\right) + 1; \quad z(t) = 0$$

1. Determine the equation of the trajectory of point M. What is its nature?

$$\begin{cases} x-2 = \sin\left(\frac{\pi}{2}t\right) \\ y-1 = 2 \cos\left(\frac{\pi}{2}t\right) \end{cases} \Rightarrow \begin{cases} (x-2)^2 = \sin^2\left(\frac{\pi}{2}t\right) \quad (1) \\ \frac{(y-1)^2}{2^2} = \cos^2\left(\frac{\pi}{2}t\right) \quad (2) \end{cases}$$

$$(1) + (2) \Rightarrow$$

$$\frac{(x-2)^2}{1^2} + \frac{(y-1)^2}{2^2} = 1 \Rightarrow \text{ellipse trajectory}$$

2. Draw the trajectory of the mobile.

centre of ellipse is
(2, 1)

