



Exams in physics I

Name:	Group:	Date: 18/01/2026
Surname:		Duration: 01h30

Exercise 1 (08)

The motion of a point M is described in polar coordinates by:

$$\begin{cases} r = b \\ \varphi = k\omega t^2 \end{cases} \quad \text{Where } b, k \text{ and } \omega \text{ are positive constants.}$$

1. Write the position vector of point M in polar coordinates.

$$\overrightarrow{OM} = r \vec{U}_r$$

$\overrightarrow{OM} = x\vec{i} + y\vec{j}$ $\overrightarrow{OM} = b(\sin\varphi \vec{i} + \cos\varphi \vec{j})$ $\overrightarrow{OM} = b(\cos\varphi \vec{i} + \sin\varphi \vec{j})$ $\overrightarrow{OM} = b \vec{U}_r$

2. Calculate the velocity vector of point M in polar coordinates.

$$\vec{v} = \dot{r} \vec{U}_r + r \dot{\varphi} \vec{U}_\varphi$$

$\vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j}$ $\vec{v} = b\dot{\varphi} \vec{U}_\varphi$ $\vec{v} = b\dot{\varphi} \vec{U}_r$ $\vec{v} = 2bk\omega t \vec{U}_\varphi$

3. Calculate the magnitude of the velocity vector of point M in polar coordinates.

$v = \sqrt{v_\varphi^2}$

$v = 2bk\omega t$

$v = b\dot{\varphi}$

$v = \sqrt{\dot{x}^2 + \dot{y}^2}$

4. Calculate the acceleration vector of point M in polar coordinates.

$$\vec{a} = (\ddot{r} - r\dot{\varphi}^2) \vec{U}_r + (2\dot{r}\dot{\varphi} + r\ddot{\varphi}) \vec{U}_\varphi$$

$\vec{a} = b t^4 \vec{U}_r + 2bt \vec{U}_\varphi$

$\vec{a} = 2bk\omega \vec{U}_\varphi$

$\vec{a} = -4bk^2 \omega^2 t^2 \vec{U}_r + 2bk\omega \vec{U}_\varphi$

$\vec{a} = b \ddot{\vec{U}}_r$

5. Calculate the magnitude of the acceleration vector of point M in polar coordinates.

$a = \sqrt{a_r^2 + a_\phi^2}$ $a = \sqrt{(-4bk^2 \omega^2 t^2)^2 + (2bk\omega)^2}$ $a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$ $a = \sqrt{(2bk\omega)^2}$

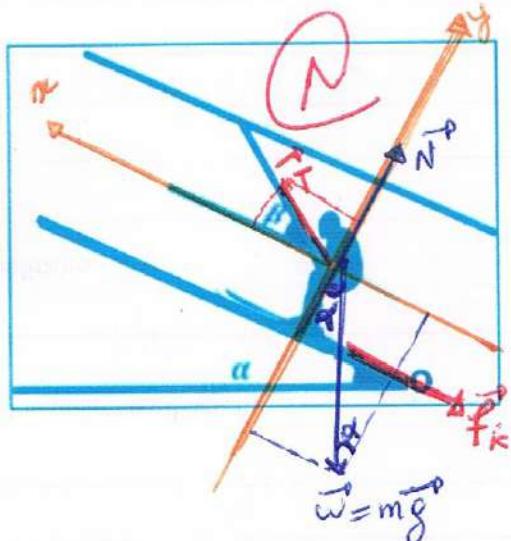
6. Calculate the Cartesian coordinates (x, y) of point M.

$x = b \cos k\omega t^2$ $y = r \cos k\omega t^2$ $y = b \sin k\omega t^2$ $x = b \sin k\omega t^2$

Exercise 2 (08)

Pulled by a ski lift, a skier of mass $M = 60 \text{ kg}$ has a rectilinear movement on a slope inclined at an angle $\alpha = 10^\circ$ with the horizontal. The pole makes an angle $\beta = 25^\circ$ with the slope and exerts on the skier a tension $T = 300 \text{ N}$. The skier is subjected to friction equivalent to a force of value $f_k = 50 \text{ N}$. Initially, the skier is at point O of the frame $(x_0 = 0 \text{ m})$

(Ox, Oy) without initial velocity ($v_0 = 0 \text{ m/s}$).



1. Represent the forces acting on the skier.

2. Find the normal force N applied to the skier.

$$\sum \vec{F}_{ext} = \vec{0} \Rightarrow \vec{w} + \vec{f}_k + \vec{N} + \vec{T} = \vec{0}$$

the projection on an axis (Oy): $-w \cos \alpha + 0 + N + T \sin \beta = 0$

$$N = 60 \cdot (9,81) \cos 10 + 300 \sin 25 =$$

$N = 320 \text{ N}$ $N = 452.87 \text{ N}$ $N = 165.03 \text{ N}$ $N = 505.92 \text{ N}$

3. Find the coefficient of kinetic friction μ_k for the skier?

$$f_k = \mu_k \cdot N \Rightarrow \mu_k = \frac{f_k}{N} = \frac{50}{452,87}$$

$\mu_k = 0.09$ $\mu_k = 0.15$ $\mu_k = 0.11$ $\mu_k = 0.3$

4. What is the acceleration of the skier?

$$\sum F_{\text{ext}} = m \ddot{a} \Rightarrow \vec{w} + \vec{f}_k + \vec{N} + \vec{T} = m \vec{a}$$

the projection on an axis (ox) is $a_x = -w \sin \alpha - f_k + 0 + T \cos \beta = m a$

$$a = (-60 \cdot (9,81) \sin 10 - 50 + 300 \cos 25) / 60$$

$a = 1.99 \text{ m/s}^2$

$a = 2 \text{ m/s}^2$

$a = 2.38 \text{ m/s}^2$

$a = 1.2 \text{ m/s}^2$

OR

5. Show that the skier has a uniformly accelerated movement.

$$a = 1,99 \text{ (m/s}^2) = \text{Constant} \Rightarrow \text{Uniformly accelerated M}$$

6. In what time "t" required for the skier in order to travel 300 m?

$$dv = a dt \Rightarrow \int_{v_0=0}^v dt = \int_{t=0}^t 1,99 t \Rightarrow v = 1,99 t \quad \text{OR} \quad \text{①}$$

$$dx = v \cdot dt \Rightarrow \int_{x_0=0}^x dt = \int_{t=0}^t 1,99 t \cdot dt \Rightarrow x = \frac{1,99}{2} t^2$$

$$\Rightarrow t = \sqrt{\frac{2 \times 300}{1,99}}$$

$t = 20.04 \text{ s}$

$t = 9.93 \text{ s}$

$t = 17.36 \text{ s}$

$t = 11.22 \text{ s}$

OR

Exercise 2 (04)

A material point M is moving along a trajectory in space R (O; $\vec{i}, \vec{j}, \vec{k}$). The Cartesian coordinates of M are:

- $x(t) = \sin\left(\frac{\pi}{2}t\right) + 2;$ $y(t) = 2 \cos\left(\frac{\pi}{2}t\right) + 1;$ $z(t) = 0$

1. Determine the equation of the trajectory of point M. What is its nature?

$$\left\{ \begin{array}{l} x - 2 = \sin\left(\frac{\pi}{2}t\right) \\ y - 1 = 2 \cos\left(\frac{\pi}{2}t\right) \end{array} \right. \Rightarrow \left\{ \begin{array}{l} (x-2)^2 = \sin^2\left(\frac{\pi}{2}t\right) \\ \frac{(y-1)^2}{2^2} = \cos^2\left(\frac{\pi}{2}t\right) \end{array} \right. \quad \text{①} \quad \text{②}$$

$$\text{①} + \text{②} \Rightarrow$$

$$\frac{(x-2)^2}{1^2} + \frac{(y-1)^2}{2^2} = 1 \Rightarrow \text{Ellipse trajectory}$$

2. Draw the trajectory of the mobile.

centre of ellipse is

(2, 1)

