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Linear Programming – Final Examination

Exercise 1 (08pts): Consider the following linear programming problem:

$$\begin{aligned} &\boldsymbol{Max}\, Z = 400\,x + 800\,y \\ &\boldsymbol{s.t}:\; x + y \le 10\,000 \quad \textit{Resource 1} \\ &\quad 2x + 6y \le 48\,000 \quad \textit{Resource 2} \\ &\quad 3x + y \le 24\,000 \quad \textit{Resource 3} \\ &\quad x \ge 0, y \ge 0 \end{aligned}$$

1. Solve the problem graphically.
2. Identify the binding and nonbinding constraints.
3. Find the range of values that the coefficient of x in the objective function can assume without changing the optimal solution.
4. Find the shadow price for resource 1.

Exercise 2 (12pts): A company manufactures two computer models, desktop and portable computers. Each computer requires one identical processor; however, the two models differ in their memory and assembly requirements. A desktop computer requires 2 memory modules and 3 minutes of assembly time, whereas a portable computer requires 6 memory modules and 1 minute of assembly time. For the next quarter, the company has the following limited resources: 10 000 processors, 48 000 memory modules and 400 hours of assembly time.

The profit obtained from selling one desktop computer is 400 $, while one portable computer generates 800 $.

1. Formulate the linear programming model that enables the company to maximize its total profit, clearly defining the decision variables, the objective function, and all relevant constraints.
2. Solve the formulated linear programming problem using the simplex method, and provide a detailed interpretation of the optimal solution obtained, including its economic meaning.
3. Formulate the dual linear programming problem and deduce its optimal solution.
4. Suppose that the company obtains 10 additional processors. Analyze the impact of this additional resource on the total profit, and provide a clear economic interpretation.
5. A new supplier offers to sell memory modules to the company at a price of 99 $ per unit. Is this offer economically attractive? Justify your answer.

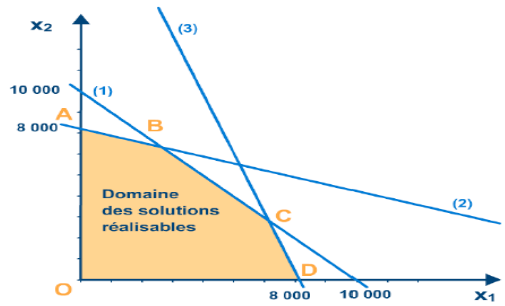
Exercise 1: (8 pts)

1. Graphical Solution (4 pts)

Step 1: Intercepts of constraints (1 pt)

- (1) $x + y = 10\,000 \rightarrow (10\,000, 0), (0, 10\,000)$
- (2) $2x + 6y = 48\,000 \rightarrow (24\,000, 0), (0, 8\,000)$
- (3) $3x + y = 24\,000 \rightarrow (8\,000, 0), (0, 24\,000)$

Step 2: draw of Feasible region (2 pts)



Vertex	Z= 400 x + 800 y
$O(0, 0)$	0
$A(0, 8000)$	6 400 000
$B(3000, 7000)$	6 800 000
C(7000, 3000)	5 200 000
$D(8000, 0)$	3 200 000

Step 3: Method of Corners (1 pt)

From the table, we see that the **maximum of *Z*** occurs at the vertex ***B*(3000, 7000)** and has a value of **6 800 000**.

2. Binding and Non-Binding Constraints (1.5 pts)

Evaluate constraints at $(3000, 7000)$:

Constraint (1) : $3000 + 7000 = 10\,000 \rightarrow$ binding
Constraint (2) : $2(3000) + 6(7000) = 48\,000 \rightarrow$ binding
Constraint (3) : $3(3000) + 7000 = 16\,000 < 24\,000 \rightarrow$ non-binding

3. Range of optimality for the Coefficient of x (1.5 *pts*)

Let the objective function be: $Z = c_x x + 800y$

Slopes of the binding constraints at the optimal point:

- Constraint (1): slope = –1
- Constraint (2): slope = –1/3

For optimality to remain at the same vertex: $-1 \leq -\frac{c_x}{800} \leq -\frac{1}{3} \rightarrow$ $\boxed{267 \leq c_x \leq 800}$

4. Shadow price for resource 1 (1 pt)

Increasing Resource 1 by one unit changes the constraint to:

$$x + y \leq 10\,000 + 1$$

Since the slopes of the constraints and the objective function remain unchanged, the optimal solution stays at the intersection of constraints (1) and (2). Solving:

$$\begin{cases} x + y = 10\,001 \\ 2x + 6y = 48\,000 \end{cases} \Rightarrow x = 3000 + \frac{3}{2}, y = 7000 - \frac{1}{2}$$

New profit: $Z = 400x + 800y = 6\,800\,000 + 200$

$$\boxed{\textbf{Shadow price of Resource 1} = 200}$$

Exercise 2: (12 pts)

1. Linear Programming Formulation (3 pts)

Decision variables

- x_1: number of desktop computers
- x_2: number of portable computers

Objective function $\max Z = 400x_1 + 800x_2$

Constraints

$$
\begin{aligned}
x_1 + x_2 &\le 10\,000 \quad \text{(processors)} \\
2x_1 + 6x_2 &\le 48\,000 \quad \text{(memory)} \\
3x_1 + x_2 &\le 24\,000 \quad \text{(assembly)} \\
x_1, x_2 &\ge 0
\end{aligned}
$$

2. Optimal Solution (Simplex) (5 pts)

Max Z= $400x_1 + 800x_2$

Subject to :

$x_1 + x_2 + e_1 = 10\,000$ (0,5 pt)

$2x_1 + 6x_2 + e_2 = 48\,000$

$3x_1 + x_2 + e_3 = 24\,000$

$x_1 \ge 0$ $x_2 \ge 0$ $e_1 \ge 0$ $e_2 \ge 0$ $e_3 \ge 0$

Basic	x_1	x_2	e_1	e_2	e_3	rhs	ratio	
e_1	1	1	1	0	0	10000	10000	
e_2	2	6	0	1	0	48000	8000	1pt
e_3	3	1	0	0	1	24000	24000	
-Z	400	800	0	0	0	0		
e_1	2/3	0	1	-1/6	0	2000	3000	
x_2	1/3	1	0	1/6	0	8000	24000	1pt
e_3	8/3	0	0	-1/6	1	16000	6000	
-Z	400/3	0	0	-400/3	0	-6400000		
x_1	1	0	3/2	-1/4	0	3000		
x_2	0	1	-1/2	1/4	0	7000		1pt
e_3	0	0	-4	1/2	1	8000		
-Z	0	0	-200	-100	0	-6800000		

$$x_1 = 3000, y_2 = 7000, Z = 6\,800\,000$$

Economic interpretation (1,5 pt)

- Produce **3000 desktops** and **7000 portables**
- Maximum profit: **6.8 million dollars**
- Processors and memory are fully used
- Assembly time has slack (8000 min)

3. Dual Problem (2 pts)

$$
\begin{aligned}
\min\ W &= 10\,000y_1 + 48\,000y_2 + 24\,000y_3 \\
y_1 + 2y_2 + 3y_3 &\ge 400 \\
y_1 + 6y_2 + y_3 &\ge 800 \\
y_1, y_2, y_3 &\ge 0
\end{aligned}
$$

Optimal dual solution $y_1 = 200,\ y_2 = 100,\ y_3 = 0$

4. Effect of 10 Additional Processors (1 pt)

Shadow price of processor = 100

$$\Delta Z = 10 \times 200 = 2\,000$$

Interpretation:

Total profit increases by **2000** \$.

5. Memory Modules at 99 \$ (1 pt)

- Shadow price of memory = 100\$
- Market price = 99\$

$$99 < 100 \Rightarrow \text{Accept the offer}$$