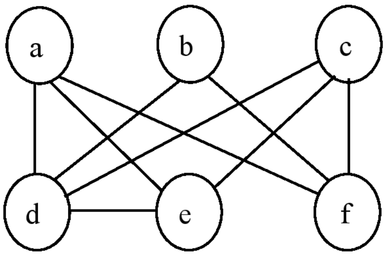
Larbi Ben Mhidi University - Oum El Bouaghi
Faculty of Exact Sciences, Natural and Life Sciences
Department of Mathematics and Computer Science

2nd Year Bachelor of Computer Science – 3rd Semester
Saturday 17/01/2026
Duration : 1 H 30 mn

Graph Theory Exam

Exercise n°=1 : (4 pts)

Let be the following graph G :



Determine **(justifying your answer)** whether the graph G is a planar graph. **(1 pt)**

If yes,

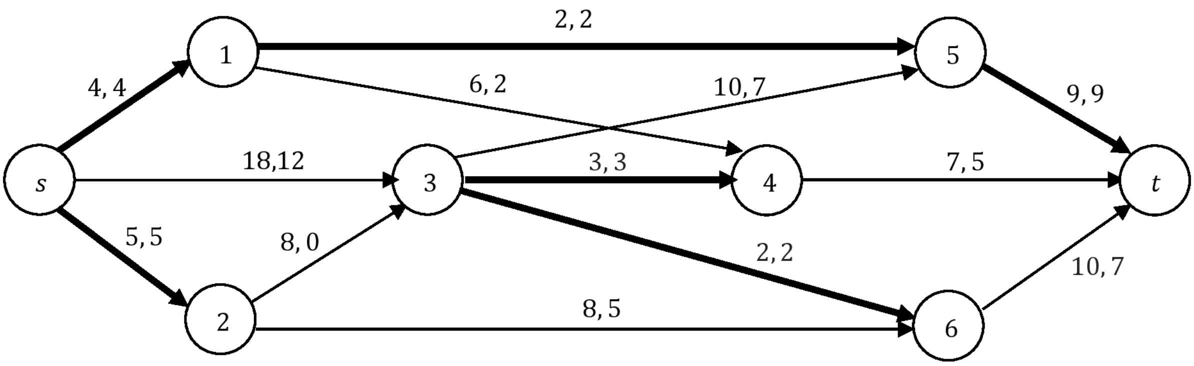
a. give its planar representation, **(0.5 pt)**

b. determine the number of faces and the degree of each face, **(1.5 pts)**

c. and give the corresponding dual graph G^*. **(1 pt)**

Exercise n°=2 : (7.25 pts)

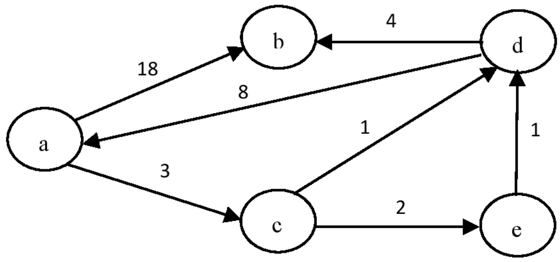
Let f be the following flow :



1. Is the proposed flow f feasible? Justify your answer. **(0.25 pt + 0.5 pt)**
2. Is the proposed flow f complete? Justify your answer. **(0.25 pt + 0.5 pt)**
3. Calculate the value of the flow $\boldsymbol{\varphi}^?$ of this iteration. **(0.5 pt)**
4. Is this flow f maximal? Justify your answer. If not, increase it and calculate the maximal flow. **(0.25 pt + 0.5 pt + 2 pts)**
5. calculate a minimum cut. **(2.5 pts)**

Exercise n°=3 : (4 pts)

Let be the following graph G :



1. Determine a minimal weight path from vertex $\boldsymbol{a}$ to each of the other vertices of the graph $\boldsymbol{G}$, indicating the different steps. **(2.5 pts)**
2. Determine the shortest path $\boldsymbol{\mu_{ab}}$ joining vertex $\boldsymbol{a}$ to $\boldsymbol{b}$, as well as its length $l(\mu_{ab})$. **(1 pt + 0.5 pt)**

Turn the sheet

Last name :………………………	First name :……………………………	Group :………	Mark (MCQ) :	4.75

comprehension questions (MCQ) : (4.75 pts) Check the correct answer in the following :

1) **In the context of directed graphs, what does it mean if a vertex has an in-degree of zero?**

	It has at least one outgoing edge to another vertex
	It has no incoming edges from other vertices
	It is part of a directed cycle
	It is guaranteed to be connected to all other vertices

2) **A subgraph $H = (V_1, E_1)$ of a graph $G = (V, E)$ is called a spanning subgraph if:**

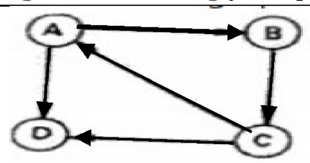
	$E_1 = \emptyset$
	$V_1 = \emptyset$
	$V_1 = V$
	$E_1 = E$

3) **If G is a cycle-free graph with n verticies and k components then G has:**

	$n + 1$ edges
	$n - 1$ edges
	$n + k$ edges
	$n - k$ edges

4) **Consider the graph shown in the figure below :**
Which of the following is a valid strongly component ?

	$\{A, C, D\}$
	$\{A, C, B\}$
	$\{A, D, B\}$
	$\{B, C, D\}$



5) **A complete graph of five vertices is :**

	Planar graph
	Non-planar graph
	Null graph
	Bipartite graph

6) **Minimum number of linearly independent vectors that generates the vectors in a vector space is called :**

	Basis of vector space
	Dimension of vector space
	Space
	None of these

7) **What is the definition of a spanning tree in the context of graph theory?**

	A subgraph that includes some vertices and is acyclic
	A graph that includes all edges of the original graph
	A graph that contains at least one cycle
	A subgraph that includes all vertices and is acyclic

8) **A graph can have more than one Minimum Spanning Tree:**

	The graph is disconnected
	All edge weights are distinct
	There are equal-weight edges
	The graph is directed

9) **In the induction step of proving the number of connected components, what happens when an edge e is removed from the graph ?**

	The number of components always increases by one
	The number of components always decreases by one
	The number of components remains unchanged
	The number of components may increase or stay the same

10) **What characterizes a bipartite graph as described in the text?**

	It contains cycles of odd length
	Every vertex connects to every other vertex
	All vertices have the same degree
	Edges connect vertices from two distinct sets

11) **If G is a tree and an edge is added, what is the result regarding cycles in the graph?**

	The graph becomes disconnected
	Exactly one cycle is created
	No cycles are created
	Multiple cycles are created

12) **How many edges are there in a complete graph K_n?**

	n
	$n - 1$
	$\frac{n(n-1)}{2}$
	n^2

13) **Which vertex violates flow conservation?**

	Source only
	Sink only
	Source and sink
	All vertices

14) **The value of a flow is equal to:**

	Flow entering the source
	Flow leaving the sink
	Flow leaving the source
	Maximum edge capacity

15) **A cut (S, T) in a flow network is:**

	A path from source to sink
	A partition of vertices with $s \in S$, $t \in T$
	A minimum flow
	A cycle

16) **The adjacency matrix of a simple undirected graph is always:**

	Diagonal
	Upper triangular
	Symmetric
	Anti-symmetric

17) **Which of the following is TRUE?**

	Every cocycle is a cocircuit
	Every cocircuit is a cocycle
	Cocycle and cocircuit are unrelated
	Cocircuit contains all edges of the graph

18) **What does the cyclomatic number represent?**

	Number of connected components
	Number of independent cycles
	Number of vertices in a cycle
	Number of edges in a tree

19) **A tree with v vertices has:**

	Cyclomatic number $v - 1$
	Cyclomatic number 0 and cocyclomatic number $v - 1$
	Cyclomatic number 1 and cocyclomatic number 0
	Cyclomatic number v

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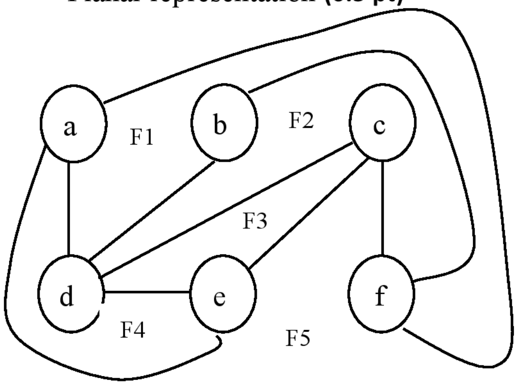
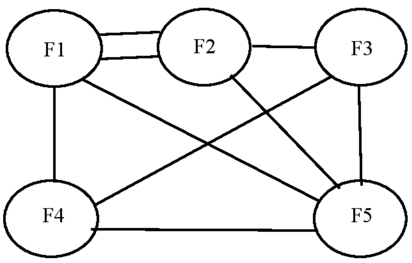
Graph Theory Exam

Exercise n°=1 : (4 pts)

Planar representation **(0.5 pt)**



The corresponding dual graph G^*. **(1 pt)**



The graph has a triangle, so we apply Euler's property 1. We have :

$$\boldsymbol{m \leq 3 \times n - 6}$$

$$\frac{3 \times 4 + 2 + 4}{2} \leq 3 \times 6 - 6 \Rightarrow 9 \leq 12 \Rightarrow TRUE$$

so the graph is planar. **(1 pt)**

Number of faces :

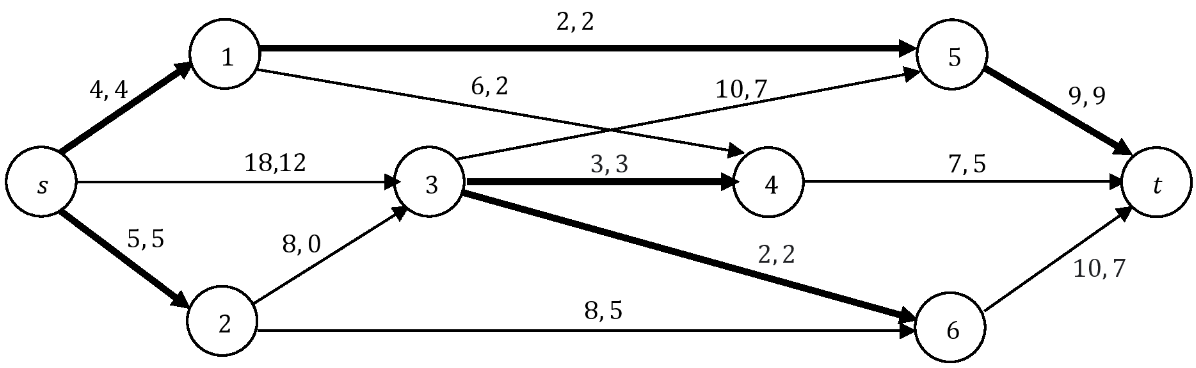
$$f = 2 - n + m \rightarrow \ f = 2 - 6 + 9 = 5\, faces$$ **(0.5 pt)**

and the degree of each face, **(1 pt)**

$deg(F1) = length(a - f - b - d - a) = 4$
$deg(F2) = length(b - f - c - d - b) = 4$
$deg(F3) = length(d - c - e - d) = 3$
$deg(F4) = length(e - d - a - e) = 3$
$deg(F5) = length(a - f - c - e - a) = 4$

Exercise n°=2 : (7.25 pts)

Let f be the following flow :



1. The proposed flow f is feasible **(0.25 pt)** because the following conditions :
   - $\varphi(u_j) \geq 0 \ \ \forall j \in \{1, \dots, m\} \ \Leftrightarrow \ \varphi \geq 0$
   - $\varphi(u_j) \leq \ c(u_j), \forall\, u_j \in U$

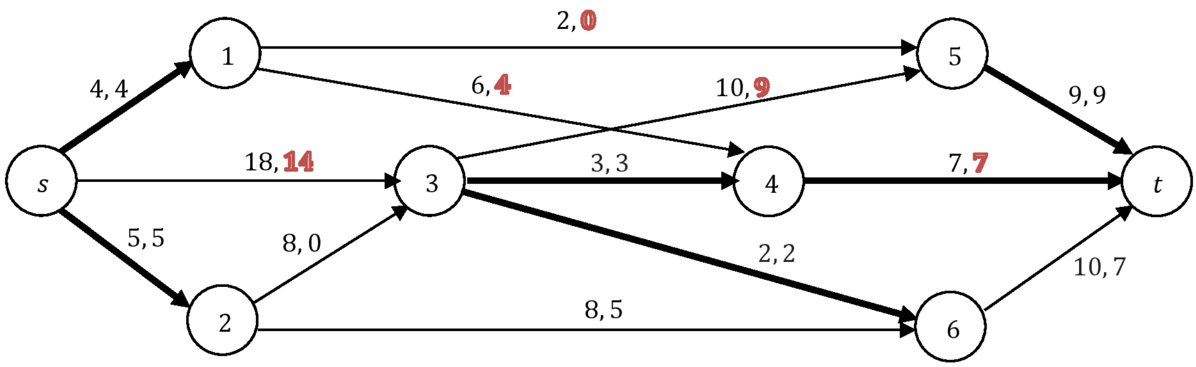
   are verified **(0.5 pt)**

2. The proposed flow f is complete **(0.25 pt)** because :
   for every path from the source s to the sink t, there is at least one saturated arc **(1 pt)**

3. $\varphi^? = \sum\left(\varphi(s,x)/x \in \Gamma_R^+(s)\right) = \sum(\varphi(x,t)/x \in \Gamma_R^-(t))$
$= \varphi(s,1) + \varphi(s,3) + \varphi(s,2) = \varphi(5,t) + \varphi(4,t) + \varphi(6,t)$
$= \mathbf{4 + 12 + 5 = 9 + 5 + 7 = 21}$ **(0.5 pt)**

4. This flow f is not maximal **(0.25 pt)** because :
There exists an augmenting chain from s to the sink t : $A = \{s, 3, 5, 1, 4, t\}$ **(0.5 pt)**

Calculating the maximal flow : **(2 pts)**
$A = \{s, 3, 5, 1, 4, t\}$
t is marked, so the flow is not maximal
$\varepsilon = min\{18 - 12, 10 - 7, ,2, 6 - 2, 7 - 5\} = min\{6, 3, 4, 2\} = 2$
$\boldsymbol{\varphi^{new}} = \boldsymbol{\varphi}^? + \varepsilon = 21 + 2 = 23$



$A = \{s, 3, 5, STOP\}$
t is not marked, THEN **finished, the flow is maximum** :
$\varphi_{max} = \boldsymbol{\varphi^{new}} = \mathbf{23}$

$$\varphi_{max} = \sum\left(\varphi(s,x)/x \in \Gamma_G^+(s)\right) = \varphi^{new}(s,1) + \varphi^{new}(s,3) + \varphi^{new}(s,2) = 4 + 14 + 5 = 23$$

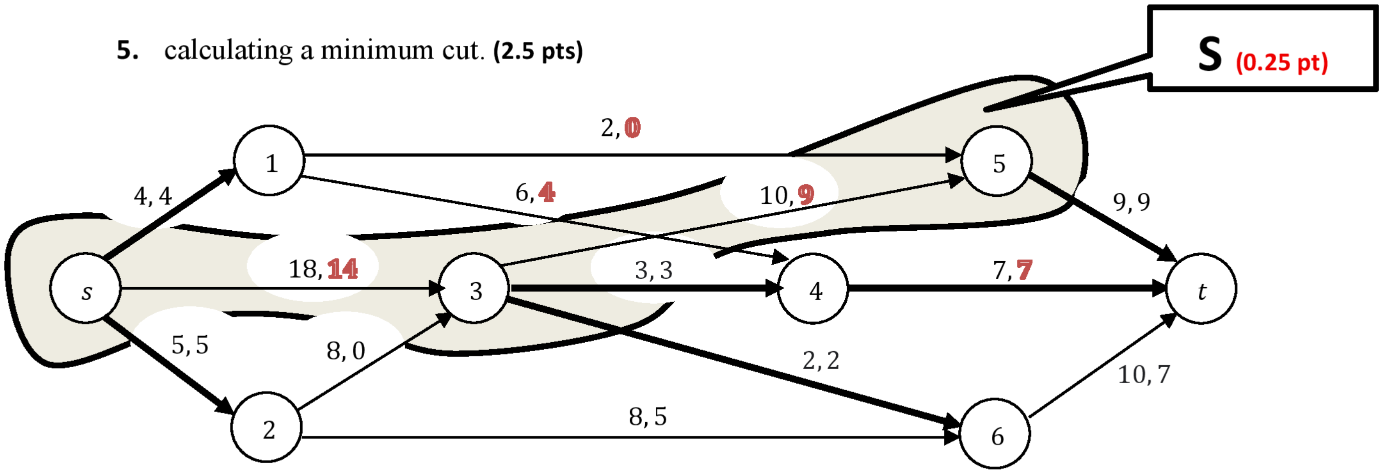
or

$$\varphi_{max} = \sum(\varphi(x,t)/x \in \Gamma_G^-(t)) = \varphi^{new}(5,t) + \varphi^{new}(4,t) + \varphi^{new}(6,t) = 9 + 7 + 7 = 23$$

Then :

$$\boxed{\boldsymbol{\varphi_{max} = 23}}$$

5. calculating a minimum cut. **(2.5 pts)**



a) $(s - t) - cut = \{(s,1), (s,2), (3,4), (3,6), (5,t)\}$ **(0.25 pt)**

b) $C_p = S \cup P = \{s, 3, 5\} \cup \{2, 4, 6, t\}$ **(0.25 pt)**

c) $C(C_p) = \sum_{\substack{x\in S \\ y\in P}} c(x,y) = c(s,1) + c(s,2) + c(3,4) + c(3,6) + c(5,t) = 4+5+3+2+9 = 23$

(0.5 pt)

d) This cut C_P is *minimal* ($C_{p_{min}}$) because : **(0.5 pt)**

- The arcs $(s,1), (s,2), (3,4), (3,6), (5,t)$ outcoming the cut are saturated.
- $(2,3)$ *and* $(1,5)$ are incoming arcs the cut with a flow $= 0$, so this condition is *explicitly* verified.

e) C_P is *minimal* ($C_{p_{min}}$), so : **(0.25 pt)**

$$|\varphi_{max}| = |C(C_{p_{min}})|$$

$$23 = 23$$

f) $\left|\varphi_{C_{p_{min}}}\right| = \sum_{\substack{x\in S \\ y\in P}} \varphi(x,y) - \sum_{\substack{x\in P \\ y\in S}} \varphi(x,y)$ **(0.5 pt)**

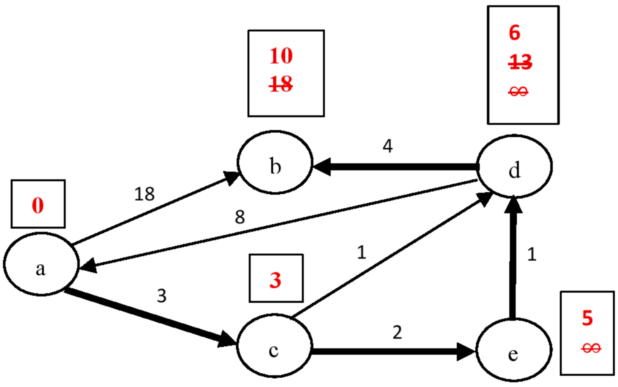
$$\left|\varphi_{C_{p_{min}}}\right| = \big(\varphi(s,1) + \varphi(s,2) + \varphi(3,4) + \varphi(3,6) + \varphi(5,t)\big) - \big(\varphi(2,3) + \varphi(1,5)\big)$$

$$\left|\varphi_{C_{p_{min}}}\right| = (4+5+3+2+9) - 0 = 23 - 0 = \mathbf{23}\ .$$

Exercise n°=3 : (4 pts)

(2.5 pts)

		Verticies				
Steps (k)	***D***	**a**	**b**	**c**	**d**	**e**
1	$\{a\}$	0	18	**<u>3</u>**	$+\infty$	$+\infty$
2	$\{a, \mathbf{c}\}$	0	18	3	13	**<u>5</u>**
3	$\{a, c, \mathbf{e}\}$	0	18	3	**<u>6</u>**	5
4	$\{a, c, e, \mathbf{d}\}$	0	**<u>10</u>**	3	6	5
5	$\{a, c, e, d, \mathbf{b}\}$	0	10	3	6	5



The shortest path μ_{ab} joining vertex$\boldsymbol{a}$ to $\boldsymbol{b}$ is given by the sequence of vertices :

$$\mu_{ab} = (a - c - e - d - b)$$ **(1 pt)**

with a minimum length equal to :

$$l(\mu_{ab}) = 3 + 2 + 1 + 4 = \lambda_b = \mathbf{10}.$$ **(0.5 pt)**

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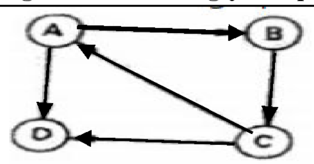

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