University of oum el bouaghi **08/01/2026**
Department : Mathématics and Computer Sciences
1^{st} years Computer Sciences (Debt's section) **Module: Algebra1**

Correction of exam of Algebra 1

Exercise 1 (7 *pts*) *Let be the statements*

$$a)\ \forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x - y < 0$$
$$b)\ \forall x \in \mathbb{N},\ \exists y \in \mathbb{N},\ x = y^2$$

1)
a) is true because, for all $x \in \mathbb{R}$, *we choose* $|y| > x$....... $\boxed{1\ pts}$
b) is false, because for example, there exists $x = 5 \in \mathbb{N}$, *such that* $5 \neq y^2$, *for all* $y \in \mathbb{N}$...... $\boxed{1\ pts}$
2) the negation

$$\overline{a} : \quad \exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x - y \geqslant 0 .... \boxed{1\ pts}$$
$$\overline{b} : \quad \exists x \in \mathbb{N},\ \forall y \in \mathbb{N},\ x \neq y^2 .... \boxed{1\ pts}$$

3) Fill in the blanks with the appropriate logical connector: $\Longleftarrow, \Longrightarrow, \Longleftrightarrow$

$$1)\ \forall x \in \mathbb{R},\ x^2 = 4 \Longleftarrow x = 2 .... \boxed{1\ pts}$$
$$2)\ \forall z \in \mathbb{C},\ z = \overline{z} \Longleftrightarrow z \in \mathbb{R} .... \boxed{1\ pts}$$
$$3)\ \forall x \in \mathbb{R},\ x = \pi \Longrightarrow e^{2ix} = 1 .... \boxed{1\ pts}$$

Exercise 2 (4 *pts*) *1) By reasoning case-by-case, Show that:*

$$\forall n \in \mathbb{N},\ \frac{n(n+1)}{2} \in \mathbb{N}.$$

- if n *is even*
so $\exists k \in \mathbb{N},\ n = 2k$, *then* $\dfrac{n(n+1)}{2} = \dfrac{2k\,(2k+1)}{2} = k\,(2k+1) \in \mathbb{N}$...... $\boxed{1\ pts}$
- if n *is odd*
so $\exists k \in \mathbb{N},\ n = 2k+1$, *then* $\dfrac{n(n+1)}{2} = \dfrac{(2k+1)\,(2k+2)}{2} = (2k+1)\,(k+1) \in \mathbb{N}$...... $\boxed{1\ pts}$
2)

$$A = x^3 + 5x^2 + 2x - 8, \qquad B = x^2 - 1$$

$$\begin{array}{c|l} x^3 + 5x^2 + 2x - 8 & \underline{x^2 - 1} \\ \underline{x^3 + x} & \\ 5x^2 + 3x - 8 & x + 5 \quad .... \boxed{0.75\ pts} \\ \underline{-5x^2 + 5} & \\ 3x - 3 & \end{array}$$

$$\begin{array}{c|l} x^2 - 1 & \underline{3x - 3} \\ \underline{-x^2 + x} & \\ x - 1 & \frac{1}{3}x + \frac{1}{3} \quad .... \boxed{0.75\ pts} \\ \underline{-x + 1} & \\ 0 & \end{array}$$

Hence the

$$GCD\,(A,B) = x-1 \text{..........} \boxed{\textit{0.5 pts}}$$

Exercise 3 (9 *pts*) *1) On définit la relation binaire* $\Re$ *sur* $\mathbb{R}$ *par :*

$$\forall\,(x,y)\in\mathbb{R}^2 : x\Re y \Leftrightarrow f\,(x) = f\,(y)\,.$$

$\Re$ *is an equivalence relation if and only if it is : reflexive, symetric and transitive*.................. $\boxed{\textit{1.5 pts}}$

1) $\forall x\in\mathbb{R} : f\,(x) = f\,(x) \Longrightarrow x\Re x$, *donc* $\Re$ *is* <u>*reflexive*</u> .... $\boxed{\textit{1 pts}}$.

2) $\forall\,(x,y)\in\mathbb{R}^2$:

$$\begin{aligned} x\Re y &\Longrightarrow f\,(x) = f\,(y)\,. \\ &\Longrightarrow f\,(y) = f\,(x) \\ &\Longrightarrow y\Re x \\ &\Longrightarrow \Re \text{ is } \underline{\text{symmetric}} \text{ ....} \boxed{\textit{1 pts}} \end{aligned}$$

3) $\forall x,y,z\in\mathbb{R}$:

$$\left\{\begin{array}{l} x\Re y \\ y\Re z \end{array}\right. \Longrightarrow \left\{\begin{array}{l} f\,(x) = f\,(y) \\ f\,(y) = f\,(z) \end{array}\right. \Longrightarrow f\,(x) = f\,(z) \Longrightarrow x\Re z \Longrightarrow \Re \text{ is } \underline{\text{transitive}}\text{...........}\boxed{\textit{1 pts}}$$

Then, $\Re$ *is an equivalence relation*

2)

$$\forall x\in\mathbb{R} : f\,(x) = x^3-3x+2$$

a)

$$A = \{-2,-1,0,1,2\}\,, \qquad B = \{2\}.$$

$$\begin{aligned} f\,(A) &= \{f\,(-2)\,,\ f\,(-1)\,,\ f\,(0)\,,\ f\,(1)\,,\ f\,(2)\} \\ &= \{0,\ 4,\ 2,\ 0,\ 4\} \text{ ....} \boxed{\textit{1.25 pts}} \end{aligned}$$

$$\begin{aligned} f^{-1}\,(B) &= \{x\in\mathbb{R},\ f\,(x) = 2\} \\ &= \left\{x\in\mathbb{R},\ x^3-3x+2 = 2\right\} \\ &= \left\{0,\ \ +\sqrt{3},\ \ -\sqrt{3}\right\} \text{ ....} \boxed{\textit{0.75 pts}} \end{aligned}$$

f *is not injective because* $f\,(-2) = f\,(1) = 0$ *but* $-2\neq 1$.................... $\boxed{\textit{1 pts}}$

Since f *is not injective, it is not bijective*.................... $\boxed{\textit{1 pts}}$

$$\begin{aligned} \dot{0} &= \{x\in\mathbb{R},\ x\Re 0\} \\ &= \{x\in\mathbb{R},\ f\,(x) = f\,(0)\} \\ &= \{x\in\mathbb{R},\ f\,(x) = 2\} \\ &= \left\{0,+\sqrt{3},-\sqrt{3}\right\} \text{........................} \boxed{\textit{0.25 pts}} \end{aligned}$$

$$\begin{aligned} \dot{2} &= \{x\in\mathbb{R},\ x\Re 2\} \\ &= \{x\in\mathbb{R},\ f\,(x) = f\,(2)\} \\ &= \{x\in\mathbb{R},\ f\,(x) = 4\} \\ &= \{-1,\ 2\} \text{ ....} \boxed{\textit{0.25 pts}} \end{aligned}$$