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Faculty of exact sciences, natural and life science

Department of Mathematics and Computer Science **Academic year: 2024/2025**

Level: L1 and Computer Science **Duration:1h30**

Exam: Analysis 2

Exercise 1(... pts)

1) Find a limited development of order 3 in a neighborhood of 0 for the function

$$f(x) = \sqrt{1+x}\ln(1+x) - \sin x.$$

2) Using the previous limited developments, calculate the following limit:

$$\lim_{x\to 0}\left(\frac{\sqrt{1+x}\,\ln(1+x) - \sin x}{x^3}\right).$$

3) Let g be a function defined on $\mathbb{R} - \{0,1\}$ by $g(x) = \frac{x^2}{x-1} e^{\frac{1}{x}}$, we denote the graph that represents it by (Cg).

a) Find a limited development of order 2 in a neighborhood of ∞ for the function

$$h(x) = \frac{x}{x-1} e^{\frac{1}{x}}.$$

b) Calculate the limit $\lim_{x\to\infty}(g(x) - x - 2)$ and conclude that the graph (Cg) has an asymptote (denoted by (Δ)), write an equation for (Δ).

c) Determine the relative position of (Cg) and (Δ).

Given:

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + o(x^3)\ ;\ \ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + o(x^3)\ ;$$

$$\sin x = x - \frac{1}{6}x^3 + o(x^3)\ ;\ e^x = 1 + x + \frac{1}{2}x^2 + o(x^2)\ ;\ \frac{1}{1-x} = 1 + x + x^2 + o(x^2).$$

Exercise 2(... pts)

1) Calculate the integral:

$$I = \int \frac{4}{(x+1)^2(x-1)} dx.$$

2) Using integration by Change of variable calculate the integral:

$$J = \int_0^3 \frac{1}{1+\sqrt{x+1}} dx.$$

Exercise 3 (... pts)

1) Find the general solution of the differential equation:

$$y' - y = x \quad \dots\dots\dots\dots\dots\dots (E).$$

2) Deduce the particular solution to equation (E) that achieves $y(0) = 1$.

Good luck.