L'arbi Ben M'hidi University

Faculty: Exact sciences and sciences of nature and life Departement: MI Academic year : 2024/2025 Module: Algebra 2

Exam n 2

Exercise 1: Let the following sets

$$F_1 = \{(x, y, z) \in \mathbb{R}^3 / 3x - y + z = 0\}$$

$$F_2 = \{(x, y) \in \mathbb{R}^2 / 2x - y = 0\}$$

 Prove that F₁, F₂ are vectors subspaces.
Determine a basis for each vector subspace.
Deduce the dimension of F₁ and F₂.
Exercise 2 : Let the following vectors

$$v_1 = (0, 1, -2), v_2 = (1, 1, 0), v_3 = (-2, 0, -2)$$

1. Prove that v_1 , v_2 , v_3 form a basis of \mathbb{R}^3 .

2. Determine the coordinates of u_1 , u_2 in the basis $\{v_1, v_2, v_3\}$ where

$$u_1 = 7v_2 - 5v_3 u_2 = (-2, 6, 0)$$

Exercise 3 : Let the matrix :

$$P = \left(\begin{array}{cc} 2 & 0\\ 1 & 3 \end{array}\right)$$

(1) Find a and b where

$$P^2 + aP + bI_2 = 0_2$$

(2) Deduce that P is an invertible matrix and calculate P^{-1}

(3) Let the linear application $f : \mathbb{R}^2 \to \mathbb{R}^2$ where

$$f(x,y) = (2x, x+3y)$$

Prove that f^{-1} exists and find its formula.

Bonne chance.

Pr. Rezzag.S