L'arbi Ben M'hidi University

Faculty: Exact sciences and sciences of nature and life
Departement: MI
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Module: Algebra 2

Exam n 2

Exercise 1:
Are the following sets vector subspaces ?

$$\begin{aligned} F_1 &= \left\{(x,y) \in \mathbb{R}^2 / 3x + y = 0\right\} \\ F_2 &= \left\{(x,y) \in \mathbb{R}^2 / y \leq 0\right\}. \end{aligned}$$

Exercise 2 :
Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ an application with

$$f(x,y,z) = (-3x - y + z, 8x + 3y - 2z, -4x - y + 2z)$$

1. Prove that f is a linear application.
2. Determine a basis of $\ker f$ and its dimension.
3.Is f an injective application?
4. Give the rank of f.. Is f a surjective application?
Exercise 3 : Let the matrix :

$$A = \begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix}, B = \begin{pmatrix} 7 & -5 \\ -3 & 2 \end{pmatrix}, C = \begin{pmatrix} 3 & 1 & 1 \\ 1 & -3 & 3 \\ 2 & 3 & 1 \end{pmatrix},$$

(1) Calculate the product AB, AC (if it is possible)
(2) Is the matrix C invertible? Justify. If yes, Determine C^{-1}.

Good luck.

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