

Subject :	Level :	Exam :
Modeling, Simulation and Performance Evaluation	1 st Year Master (Distributed Architectures)	Final
Documents are not authorized	Time : 01h 30mn	Scientific calculator allowed

Saturday May 17, 2025

Answer clearly and succinctly

Exercise 01 (COMPARISON OF QUEUES : 05 Marks)

Consider an $M/M/1$ queue with a service rate $\mu > 0$ and an $M/M/2$ queue with a service rate $\frac{\mu}{2}$. Both queues have an arrival rate $\lambda > 0$ and hence the same server utilization $\rho = \frac{\lambda}{\mu}$.

1. Give the values of $P_{0M/M/1}$ and $P_{0M/M/2}$ in terms of ρ .
2. Compare $P_{0M/M/1}$ and $P_{0M/M/2}$.
3. Give the values of $\bar{Q}_{M/M/1}$ and $\bar{Q}_{M/M/2}$ in terms of ρ .
4. Compare $\bar{Q}_{M/M/1}$ and $\bar{Q}_{M/M/2}$.
5. Give the values of $\bar{N}_{M/M/1}$ and $\bar{N}_{M/M/2}$ in terms of ρ .
6. Compare $\bar{N}_{M/M/1}$ and $\bar{N}_{M/M/2}$.

Exercise 02 (QUEUEING SYSTEM : 05 Marks)

We want to design a web server to ensure the reception of at least 90% of the incoming requests (according to the Poisson process). The arrival rate of requests is 5 requests/second, and the service rate (exponential) is 6 requests/second. The average size of a request is 1500 bytes.

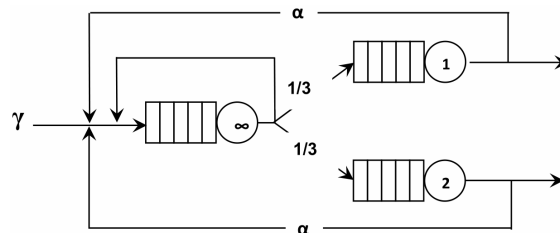
1. What is the queue that correctly models our system (with justification) ?
2. What is the minimum memory size (in bytes) required to achieve our objective ?
3. What is the average number of daily processed requests ?
4. What is the average number of waiting requests ?

Exercise 03 (QUEUEING NETWORK : 10 Marks)

Consider the following Jackson network, where $\gamma = 4, m_1 = +\infty, m_2 = 1, m_3 = 2, \mu_1 = 5, \mu_2 = 8, \mu_3 = 6$

1. Give the internal and external routing probability matrices.
2. Find the effective arrival rates λ_i as a function of α .
3. Find the values of α that ensure network stability.

Let's take $\alpha = \frac{1}{2}$. Find in this case :



4. The average number of waiting clients at each station and in the network.
5. The average number of clients at each station and in the network.
6. The average residence time in each station and in the network.
7. The average waiting time at each station and in the network.
8. The probability that the network is not empty.

Note 1 : use at least 6 digits after the decimal point in all your calculations.