Larbi Ben Mhidi University - Oum El Bouaghi Faculty of Exact Sciences, Natural and Life Sciences Department of Mathematics and Computer Science	2nd Year Bachelor of Computer Science – 3rd Semester Sunday 11/05/2025 Duration : 1 H 30 mn
FULL NAME : ……………………………………………………………..	GROUP : …………

Language Theory Exam (LT)

Exercise n°=1 (5 pts)

Choose the correct answer(s) :

1) Which of the following words is a sub-word of $abcabcabc$?

	$aabbcc$
	$abacbab$
	$acbba$
	$acbac$

2) $\{a, b\}^*$ is an infinite language :

	True
	False

3) two words u, w defined on the alphabet V are equal if and only if : $|u| = |w| = n$

	True
	False

4) Let the word $w = aabcbbca$. The factor $u = bcbb$ is not proper.

	True
	False

5) Languages equivalent to $\{a, b, aab, bab\}$ are :

	$\{a, b\}.\{ab, \varepsilon\}$
	$\{a\}.\{ab, \varepsilon\}.\{b\}$
	$\{\varepsilon\}.\{a, aab, b, bab\}$
	$\{a, b, aab, bab\}.\{\}$
	$\{a, aa\}\{\varepsilon, b, bb\}$

6) The grammar :
$G = (\{a, b, c\}, \{S, A, R, T\}, S, P)$ with :
$P = \{S \rightarrow aRbc \mid abc \mid \varepsilon$;
$R \rightarrow aRTb \mid aTb$;
$Tb \rightarrow bT$; $Tc \rightarrow cc \mid \varepsilon\}$ is of :

	Type 3 (regular on the right)
	Type 3 (regular on the left)
	Type 2
	Type 1
	Type 0

7) Two grammars are equivalent if and only if they :

	have the same axioms, non-terminals, terminals and rules
	generate the same language
	have the same non-terminals
	have the same terminals

8) $\forall w \in V^* : w^0 =$

	0
	1
	ε
	$\emptyset$
	w

9) Which of the following words belongs to the language denoted by $(\mathbf{ab} + \mathbf{ba})^*$ on the alphabet $V = \{\mathbf{a}, \mathbf{b}\}$:

	$abaabb$
	$abbaab$
	$aaabbb$
	$abbbba$

Exercise n°=2 (5 pts)

Let P be the set of production rules for a grammar G :

$P = \{\ S \rightarrow B \mid if\ (b)\{S\} \mid\ if\ (b)\{S\}\ else\ \{S\}\ ;\ \ B \rightarrow a\ ; B \mid a\ ;\ \}$

and the words : $w_1 = if\ (b)\ \{a\ ;\}\ else\ \{a\ ;\}$; $w_2 = if\ (b)\ \{if\ (b)\ \{a\ ;\}\ else\ \{a\ ;\}\}$

1. Give the formal description of G. **(1 pt)**
2. What is the type of G ? (Justify your answer) **(1 pt)**
3. Give a **<u>description in English</u>** of the language $L(G)$ generated by G. **(1 pt)**
4. Show that : $w_1 \in L(G)$; $w_2 \in L(G)$. **(2 pts)**

Exercise n°=3 (4 pts)

Let the following languages :

$L1 = \{all\ words\ on\ \{0,1\}\ of\ the\ form\ 1w_1 10 w_2\ where\ w_1 \in \{0\}^*\ and\ w_2 \in \{1\}^*\}.$

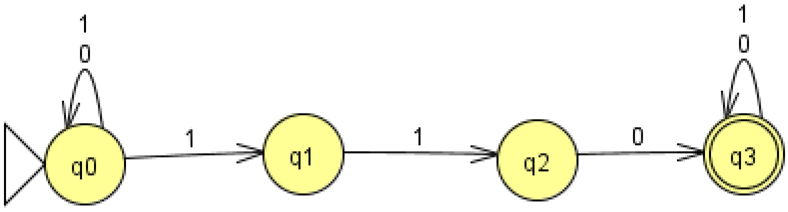
$L2 = \{all\ words\ on\ \{a,b,c\}\ starting\ with\ \boldsymbol{abc}\ and\ ending\ with\ a\ number\ \boldsymbol{multiple\ of}\ \mathbf{3}\ of\ \boldsymbol{b's}\}$

For each language :

1. Construct the corresponding automata **(<u>only give the transition graph</u>)** ;
2. and give a grammar that generates the language.

Exercise n°=4 (6 pts)

Let the following automaton ***A*** :



1. Explain why automaton ***A*** is not deterministic. **(0.5 pt)**
2. Determine ***A***. **(3 pts)**
3. Say if the following words can be recognized by the automaton ***A*** : ε , 01 , 1101 , 01101 **(2.5 pts)**

GOOD LUCK

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Language Theory Exam (LT)

Exercise n°=1 (5 pts = 0.5 pt * 10)

Choose the correct answer(s) :

1) Which of the following words is a sub-word of $abcabcabc$?

	$aabbcc$
	$abacbab$
	$acbba$
X	$acbac$

2) $\{a, b\}^*$ is an infinite language :

X	True
	False

3) two words u, w defined on the alphabet V are equal if and only if : $|u| = |w| = n$

	True
X	False

4) Let the word $w = aabcbbca$. The factor $u = bcbb$ is not proper.

	True
X	False

5) Languages equivalent to $\{a, b, aab, bab\}$ are :

X	$\{a, b\}.\{ab, \varepsilon\}$
	$\{a\}.\{ab, \varepsilon\}.\{b\}$
X	$\{\varepsilon\}.\{a, aab, b, bab\}$
	$\{a, b, aab, bab\}.\{\}$
	$\{a, aa\}\{\varepsilon, b, bb\}$

6) The grammar :
$G = (\{a, b, c\}, \{S, A, R, T\}, S, P)$ **with :**
$P = \{S \to aRbc \mid abc \mid \varepsilon$;
$R \to aRTb \mid aTb$;
$Tb \to bT$; $Tc \to cc \mid \varepsilon\}$ **is of :**

	Type 3 (regular on the right)
	Type 3 (regular on the left)
	Type 2
X	Type 1
	Type 0

7) Two grammars are equivalent if and only if they :

	have the same axioms, non-terminals, terminals and rules
X	generate the same language
	have the same non-terminals
	have the same terminals

8) $\forall w \in V^*$: $w^0 =$

	0
	1
X	ε
	$\emptyset$
	w

9) Which of the following words belongs to the language denoted by $(\mathrm{ab} + \mathrm{ba})^*$ on the alphabet $V = \{\mathrm{a}, \mathrm{b}\}$:

	$abaabb$
X	$abbaab$
	$aaabbb$
	$abbbba$

Exercise n°=2 (5 pts)

Let P be the set of production rules for a grammar G :

$P = \{ S \to B \mid if\ (b)\{S\} \mid if\ (b)\{S\}\ else\ \{S\}\ ;\ B \to a\ ; B \mid a\ ;\ \}$

and the words : $w_1 = if\ (b)\ \{a\ ;\}\ else\ \{a\ ;\}$; $w_2 = if\ (b)\ \{if\ (b)\ \{a\ ;\}\ else\ \{a\ ;\}\}$

1. $G = (\{a, b, if, else, \{ , \}, (,), ;\}\ ,\ \{S, B\}\ ,\ S\ ,\ P = \{ S \to B \mid if\ (b)\{S\} \mid if\ (b)\{S\}\ else\ \{S\}\ ;$
   $B \to a\ ; B \mid a\ ;\ \})$. **(1 pt)**
2. G is of type 2 **(0.25 pt)** because all the production rules are of the form :
   $A \to \alpha$ with $A \in V_N$ and $\alpha \in (V_T \cup V_N)^*$ **(0.75 pt)**
3. A **description in English** of the language $L(G)$ generated by G. **(1 pt)**
   The language is the set of words :
   a sequence of statements + ***if statement*** + ***if…else statement*** + ***Nested if statements*** + ***Nested if…else statement***
4. $(w_1 = if\ (b)\ \{a\ ;\}\ else\ \{a\ ;\}) \in L(G)$ **(1 pt)**
   $S \Longrightarrow if\ (b)\{S\}\ else\ \{S\} \Longrightarrow if\ (b)\{B\}\ else\ \{S\} \Longrightarrow if\ (b)\{B\}\ else\ \{B\} \Longrightarrow if\ (b)\{a\ ;\}\ else\ \{B\}$
   $\Longrightarrow if\ (b)\{a\ ;\}\ else\ \{a\ ;\}$
   Then $S \overset{*}{\Rightarrow} if\ (b)\ \{a\ ;\}\ else\ \{a\ ;\}$ so $w_1 \in L(G)$

$\left(\boldsymbol{w_2 = if\ (b)\ \{if\ (b)\ \{a\ ;\}\ else\ \{a\ ;\}\}}\right) \in \boldsymbol{L(G)}$. **(1 pt)**

$S \Longrightarrow if\ (b)\{S\} \Longrightarrow if\ (b)\{\ if\ (b)\{S\}\ else\ \{S\}\} \Longrightarrow if\ (b)\{\ if\ (b)\{B\}\ else\ \{S\}\} \Longrightarrow$

$if\ (b)\{\ if\ (b)\{B\}\ else\ \{B\}\} \Longrightarrow if\ (b)\{\ if\ (b)\{a\ ;\}\ else\ \{B\}\} \Longrightarrow \boldsymbol{if\ (b)\{\ if\ (b)\{a\ ;\}\ else\ \{a\ ;\}\}}$

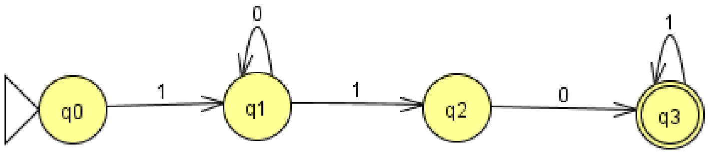
Then $S \overset{*}{\Rightarrow} if\ (b)\ \{if\ (b)\ \{a\ ;\}\ else\ \{a\ ;\}\}$ so $w_2 \in L(G)$

Exercice n°=3 (4 pts)

Let the following languages :

$L1 = \{all\ words\ on\ \{0,1\}\ of\ the\ form\ 1w_1 10w_2\ where\ w_1 \in \{0\}^*\ and\ w_2 \in \{1\}^*\}$.

1. Construct the corresponding automata (**<u>only give the transition graph</u>**) ; **(1 pt)**



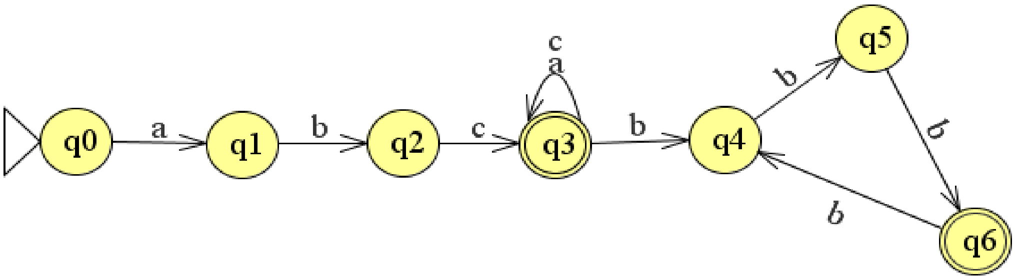
2. and give a grammar that generates the language. **(1 pt)**

$G1\ (V_T, V_N, S, P) = G1\ (\{0,1\}, \{S, A, B\}, S, P)\ ,\ P = \{S \rightarrow 1A10B\ ;\ A \rightarrow 0A|\varepsilon\ ;\ B \rightarrow 1B|\varepsilon\}$

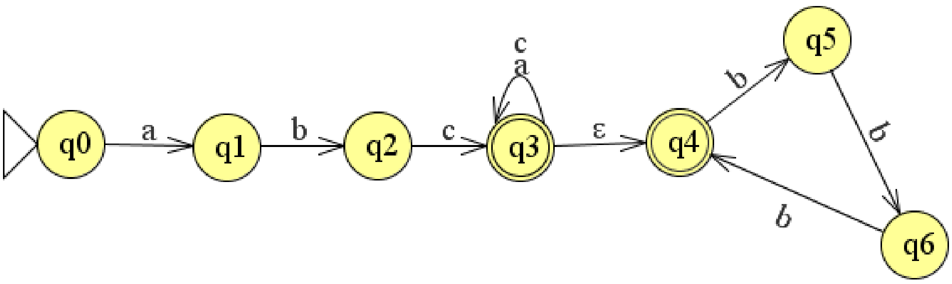
we can accept other answers

$L2 = \{all\ words\ on\ \{a, b, c\}\ starting\ with\ \boldsymbol{abc}\ and\ ending\ with\ a\ number\ \boldsymbol{multiple\ of}\ 3\ of\ \boldsymbol{b's}\}$

1. Construct the corresponding automata (**<u>only give the transition graph</u>**) ; **(1 pt)**



OR



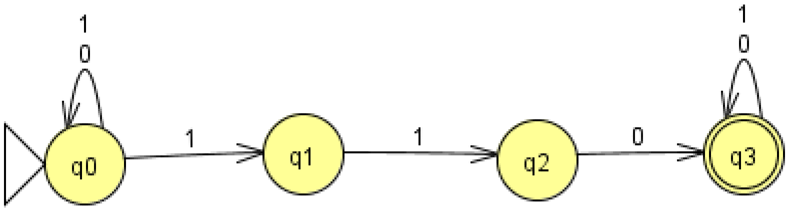
2. and give a grammar that generates the language. **(1 pt)**

$G2\ (V_T, V_N, S, P) = G2\ (\{a, b, c\}, \{S, A, B\}, S, P),\ P = \{S \rightarrow abcAB\ ;\ A \rightarrow aA|cA|\varepsilon\ ;\ B \rightarrow bbbB|\varepsilon\ \}$

we can accept other answers

Exercise n°=4 (6 pts)

Let the following automaton ***A*** :



1. Explain why automaton ***A*** is not deterministic. **(0.5 pt)**

The automaton A is not deterministic because : $\delta(q_0, 1) = \{q_0, q_1\}$, more than one state.

2. Determine ***A***.

Determinization of A (A_{det}) *:*

We apply the determinization algorithm of a NDFA without $\boldsymbol{\varepsilon - transitions}$ ***:***

Let's unroll the algorithm :

$Q^{(0)} = \boldsymbol{Q_I} = \{\boldsymbol{q_0}\}$

$\{\boldsymbol{q_0}\}$
- $0 : Q^{(1)} = \delta(q_0, 0) = \{q_0\}$
- $1 : Q^{(1)} = \delta(q_0, 1) = \{\boldsymbol{q_0}, \boldsymbol{q_1}\}$

0.25 pt

$\{\boldsymbol{q_0}, \boldsymbol{q_1}\}$
- $0 : Q^{(2)} = \delta(q_0, 0) \cup \delta(q_1, 0) = \{q_0\}$
- $1 : Q^{(2)} = \delta(q_0, 1) \cup \delta(q_1, 1) = \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_2}\}$

0.25 pt

$\{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_2}\}$
- $0 : Q^{(3)} = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0) = \{\boldsymbol{q_0}, \boldsymbol{q_3}\}$
- $1 : Q^{(3)} = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1) = \{q_0, q_1, q_2\}$

0.25 pt

$\{\boldsymbol{q_0}, \boldsymbol{q_3}\}$
- $0 : Q^{(4)} = \delta(q_0, 0) \cup \delta(q_3, 0) = \{q_0, q_3\}$
- $1 : Q^{(4)} = \delta(q_0, 1) \cup \delta(q_3, 1) = \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}$

0.25 pt

$\{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}$
- $0 : Q^{(5)} = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_3, 0) = \{q_0, q_3\}$
- $1 : Q^{(5)} = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_3, 1) = \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_2}, \boldsymbol{q_3}\}$

0.25 pt

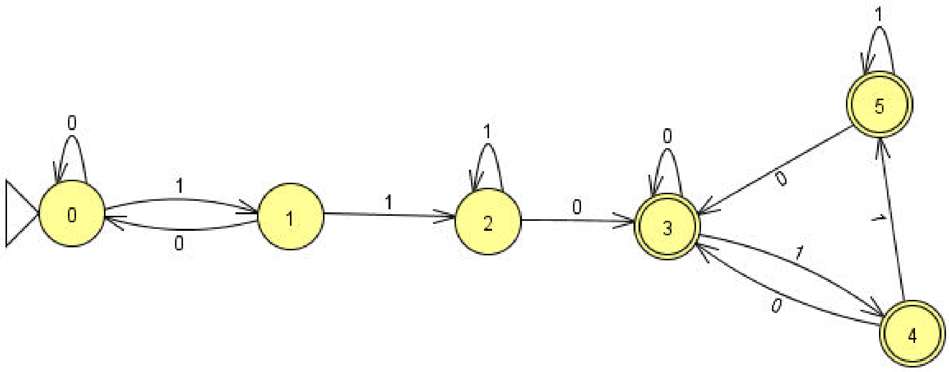
$\{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_2}, \boldsymbol{q_3}\}$
- $0 : Q^{(6)} = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0) \cup \delta(q_3, 0) = \{q_0, q_3\}$
- $1 : Q^{(6)} = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1) \cup \delta(q_3, 1) = \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_2}, \boldsymbol{q_3}\}$

0.25 pt

The new DFA $A_{det} = (V_T, Q', q_0, \delta_{det}, Q'_F) = (\{0,1\}, \{0,1,2,3,4,5\}, \{0\}, \delta_{det}, \{3,4,5\})$ with δ_{det} the transition function given by :

Q' \ V_T	Re-numbering of new states	0	1
$\Rightarrow \{q_0\}$	**$\Rightarrow$ 0**	**0**	**1**
$\{q_0, q_1\}$	**1**	**0**	**2**
$\{q_0, q_1, q_2\}$	**2**	**3**	**2**
$\{q_0, q_3\} \in Q'_F$	**3** $\in Q'_F$	**3**	**4**
$\{q_0, q_1, q_3\} \in Q'_F$	**4** $\in Q'_F$	**3**	**5**
$\{q_0, q_1,, q_2, q_3\} \in Q'_F$	**5** $\in Q'_F$	**3**	**5**

1 pt



0.5 pt

3. Say if the following words can be recognized by the automaton ***A*** : ε , 01 , 1101 , 01101 **(2.5 pts)**

$\boldsymbol{w = \varepsilon}$ **: (0.25 pt)**

$\delta^*(q_0, \varepsilon) = \delta(q_0, \varepsilon) = ?$

Or

$(q_0, \varepsilon) \vdash ?$

$So\ \varepsilon \notin L(A)$

w = 01 : (0.25 pt)

$\delta^*(q_0, 01) = \delta^*(q_0, 1) = \delta(q_0, 1) = \{q_0, q_1\}$

or

$(q_0, 01) \vdash (q_0, 1) \vdash \{q_0, q_1\}$

We have :

$(q_0, 01) \vdash^* \{q_0, q_1\}$ and $(\sigma = \{q_0, q_1\}) \in P(Q)$ and $\sigma \cap Q_F = \{q_0, q_1\} \cap \{q_3\} = \emptyset$

So 01 $\notin L(A)$

w = 1101 : (1 pt)

$$\delta^*(q_0, 1101) = \delta^*(\{q_0, q_1\}, 101)$$
$$= \delta^*(q_0, 101) \cup \delta^*(q_1, 101)$$
$$= \delta^*(\{q_0, q_1\}, 01) \cup \delta^*(q_2, 01)$$
$$= \delta^*(q_0, 01) \cup \delta^*(q_1, 01) \cup \delta^*(q_2, 01)$$
$$= \delta^*(q_0, 1) \cup \emptyset \cup \delta^*(q_3, 1)$$
$$= \delta(q_0, 1) \cup \delta(q_3, 1)$$
$$= \{q_0, q_1\} \cup \{q_3\}$$
$$= \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}$$

or

$$(q_0, 1101) \vdash (\{q_0, q_1\}, 101) \vdash (q_0, 101) \cup (q_1, 101) \vdash (\{q_0, q_1\}, 01) \cup (q_2, 01) \vdash (q_0, 01) \cup (q_1, 01) \cup (q_2, 01)$$
$$\vdash (q_0, 1) \cup \emptyset \cup (q_3, 1) \vdash (q_0, 1) \cup (q_3, 1) \vdash \{q_0, q_1\} \cup \{q_3\} \vdash \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}$$

We have :

$(q_0, 1101) \vdash^* \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}$ and $(\sigma = \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}) \in P(Q)$ and $\sigma \cap Q_F = \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\} \cap \{q_3\} \neq \emptyset$

So $1101 \in L(A)$

w = 01101 : (1 pt)

$$\delta^*(q_0, 01101) = \delta^*(q_0, 01101)$$
$$= \delta^*(q_0, 1101)$$
$$= \delta^*(\{q_0, q_1\}, 101)$$
$$= \delta^*(q_0, 101) \cup \delta^*(q_1, 101)$$
$$= \delta^*(\{q_0, q_1\}, 01) \cup \delta^*(q_2, 01)$$
$$= \delta^*(q_0, 01) \cup \delta^*(q_1, 01) \cup \delta^*(q_2, 01)$$
$$= \delta^*(q_0, 1) \cup \emptyset \cup \delta^*(q_3, 1)$$
$$= \delta(q_0, 1) \cup \delta(q_3, 1)$$
$$= \{q_0, q_1\} \cup \{q_3\}$$
$$= \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}$$

or

$$(q_0, 01101) \vdash (q_0, 1101) \vdash (\{q_0, q_1\}, 101) \vdash (q_0, 101) \cup (q_1, 101) \vdash (\{q_0, q_1\}, 01) \cup (q_2, 01)$$
$$\vdash (q_0, 01) \cup (q_1, 01) \cup (q_2, 01) \vdash (q_0, 1) \cup \emptyset \cup (q_3, 1) \vdash (q_0, 1) \cup (q_3, 1) \vdash \{q_0, q_1\} \cup \{q_3\} \vdash \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}$$

We have :

$(q_0, 01101) \vdash^* \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}$ and $(\sigma = \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\}) \in P(Q)$ and $\sigma \cap Q_F = \{\boldsymbol{q_0}, \boldsymbol{q_1}, \boldsymbol{q_3}\} \cap \{q_3\} \neq \emptyset$

So $01101 \in L(A)$