Larbi ben M'hidi Oum el Bouaghi University 2024/2025
1st Year Math

Descriptive Statistics and Probability
Final Exam

Exercise 1.(8pts) In a survey, the number of rooms, in 50 dwellings is given by the data

2	6	4	3	3	4	4	7	5	4
5	3	7	5	5	4	4	5	6	2
6	3	4	4	5	8	6	5	5	3
3	3	7	5	4	4	5	4	1	6
5	4	4	8	6	2	3	3	6	4

1. Determine what the population Ω, variable X for this data. What type of this data ? Determine the modalities set $X(\Omega)$.
2. Present the above data in a frequency table showing the following columns; relative frequency, cumulative frequency, relative cumulative frequency.
3. Construct a bar chart and a relative cumulative frequency curve (F_X) of this data.
4. Calculate $\overline{X}$, Q_1, $M\left(=Q_2\right)$, and Q_3 for this data.
5. Construct the box-plot for these data.

Exercise 2.(5pts) the ages of certain people are given by the following table

Ages	$[0,18[$	$[18,36[$	$[36,54[$	$[54,72[$	$[72,90[$
Frequency	20	36	20	15	9

1. Present the above data in a frequency table showing the following columns; relative frequency, cumulative frequency, relative cumulative frequency.
2. Construct a histogram and a relative cumulative frequency curve (F_X) of this data.
3. Calculate $\overline{X}$ and the median M for this data.
4. Calculate the number of people whose ages are in the interval $\left[M,\overline{X}\right]$.

Exercise 3. (7pts) (a) Let A and B be two events such that $P\left(A\right)=P\left(B\right)=0,5$.
Show that $P\left(\overline{A}\cap\overline{B}\right)=P\left(A\cap B\right)$
(b) Let A,B,C be three independent events with probabilities different from 0 and 1.

1. Show that A and $B\cup C$ are independent.
2. Show that $P(B\cup C)$ is strictly less than 1.

Larbi ben M'hidi Oum el Bouaghi University 2024/2025
1st Year Math

Descriptive Statistics and Probability
Final Exam (Correction)

Exercise 1.

1. The population Ω : 50 dwellings $(0,5)$

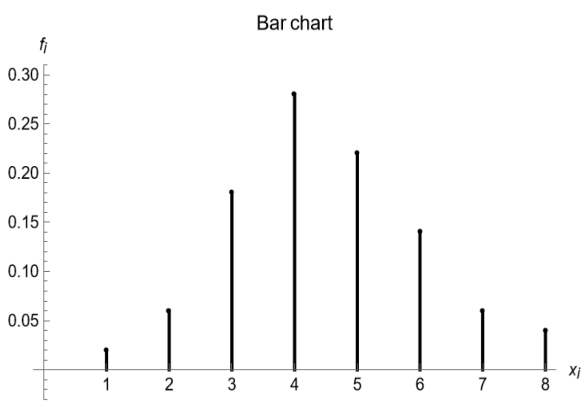
   Variable X : number of rooms in one dwelling $(0,5)$
   The data is quantitative discrete $(0,5)$
   $X(\Omega) = \{1,2,3,4,5,6,7,8\}$. $(0,5)$

2.

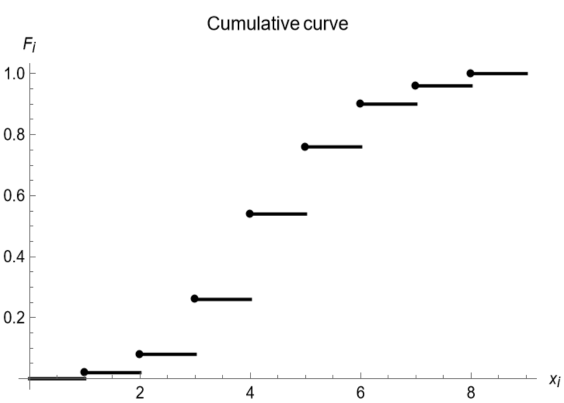
x_i	n_i	f_i	N_i	F_i	$n_i x_i$
1	1	$0,02$	1	0,02	1
2	3	$0,06$	4	0,08	6
3	9	$0,18$	13	0,26	27
4	14	$0,28$	27	0,54	56
5	11	$0,22$	38	0,76	55
6	7	$0,14$	45	0,9	42
7	3	$0,06$	48	0,96	21
8	2	$0,04$	50	1	16
Total	50	1			224

$(1pts)$

3.



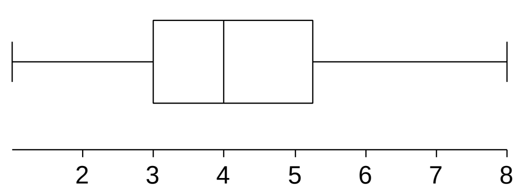
$(1pts)$



$(1pts)$

4.

$$\overline{X} = \frac{\sum_{i=1}^{8} n_i x_i}{n} = \frac{224}{50} = 4,48 \quad (0,5)$$
$n/4 = 12,5 \notin N_i \Longrightarrow Q_1 = 3$ $(0,5)$
$n/2 = 25 \notin N_i \Longrightarrow M = 4$ $(0,5)$
$3n/4 = 37,5 \notin N_i \Longrightarrow Q_3 = 5.$ $(0,5)$

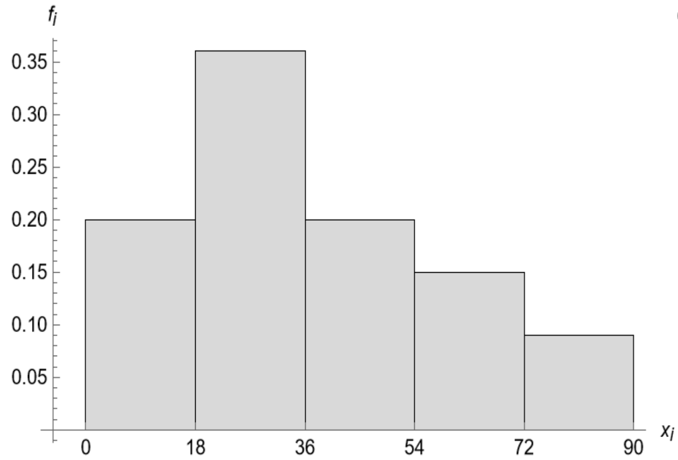


$(1pts)$

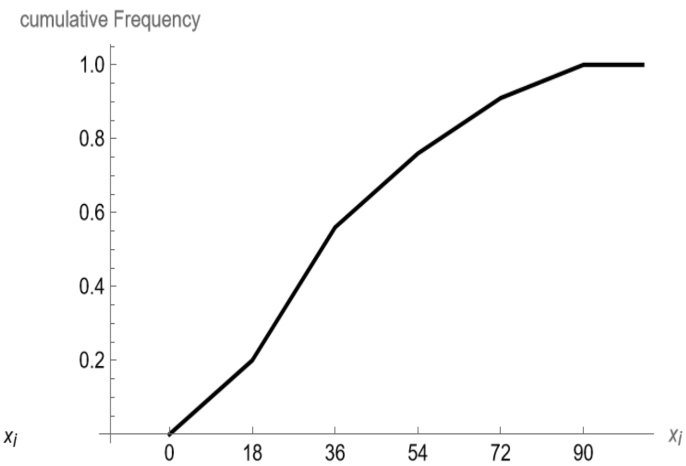
Exercise 2.

Bin	Bin center(x_i)	n_i	f_i	N_i	F_i	$n_i x_i$
$[0,18[$	9	20	$0,2$	20	$0,2$	180
$[18,36[$	27	36	$0,36$	56	$0,56$	972
$[36,54[$	45	20	$0,2$	76	$0,76$	900
$[54,72[$	63	15	$0,15$	91	$0,91$	945
$[72,90[$	81	9	$0,09$	100	1	729
Total		100	1			3726

$(1pts)$



$(0,75)$



$(0,75)$

2. $\overline{X} = \dfrac{\sum_{i=1}^{8} n_i x_i}{n} = \dfrac{3726}{100} = 37,26$ ans, $(0,5)$

median M : we have $M \in [18,36[$, so

$$M = L_1 + \frac{\frac{n}{2} - C}{n_m}(L_2 - L_1) = 18 + \frac{50-20}{36}(36-18) = 33 \text{ ans.} \quad (1pts)$$

3. Number of people whose ages $\in \left[M, \overline{X}\right] = N\left(\overline{X}\right) - N(M)$. We know that $N(M) = 50$, and from $N(x) = N_{i-1} + h_i(x - a_{i-1})$, $x \in [a_{i-1}, a_,[$, we have

$$N\left(\overline{X}\right) = 56 + \frac{20}{18}(37,26 - 36) = 57,4, \text{ so } N\left(\overline{X}\right) - N(M) = 7,4 \simeq 7 \text{ people.} \qquad (1pts)$$

Exercise 3.

(a) We have

$$\begin{aligned} P\left(\overline{A} \cap \overline{B}\right) &= P\left(\overline{A \cup B}\right) && (0,5) \\ &= 1 - P(A \cup B) && (0,5) \\ &= 1 - (P(A) + P(B) - P(A \cap B)) && (0,5) \\ &= 1 - (1 - P(A \cap B)) = P(A \cap B) && (0,5) \end{aligned}$$

(b)

1. We must show that $P(A \cap (B \cup C)) = P(A)\,P(B \cup C)$. $\quad (0,5)$

We have $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad (0,25)$, so

$$\begin{aligned} P(A \cap (B \cup C)) &= P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C)) \qquad (0,25) \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \qquad (0,5) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \qquad (0,5) \\ &= P(A)[P(B) + P(C) - P(B)P(C)] \qquad (0,5) \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \qquad (0,5) \\ &= P(A)P(A \cup C) \qquad (0,5) \end{aligned}$$

That is what is required.

2. We have $P(B \cup C) = P(B) + P(C)[1 - P(B)] \quad (0,5)$, so

$$1 - P(B \cup C) = [1 - P(B)][1 - P(C)] \qquad (0,5)$$

As $P(B) < 1$ and $P(C) < 1$, then $1 - P(B \cup C) > 0$, and so $P(B \cup C) < 1$. $\qquad (0,5)$