Larbi Ben M'hid University
Master 01 Mathematics
Exam: Optimization

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Exercise 01: First Necessary Condition of Optimality (**6 points**)

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a differentiable function.

Let $x^* \in \mathbb{R}$ be a local minimum of f.

State and prove the first-order necessary condition for optimality in equality constrained case.

Exercise 02: Optimization with Equality Constraints (Lagrange Multipliers) (**6 points**)

Use the method of Lagrange multipliers to solve the following optimization problem:

Minimize the function:

$$f(x, y, z) = x + z$$

Subject to the constraint:

$$g(x, y, z) = x^2 + y^2 + z^2 - 1$$

- Find the critical points of the Lagrangian.
- Determine the nature

Exercise 03: Optimization with Inequality Constraints (KKT Conditions) (**6 points**)

Let consider the problem

$$\begin{cases} \max \quad xy - x^2 - y^2 \\ \qquad s.c \\ .2x + y \geq 5, y \geq 3 \end{cases}$$

- Find all the critical points
- Determine the nature of the critical point(s)

Exercise 04: Optimization Theorems (**2 points**)

Fill in the Gaps

(1) Let the function f be a _______________ function, and let the feasible set be non-empty, closed, and _______________.

Then there exists at least one minimum point

This result is known as the Theorem of _______________.

(2) If the function f is _______________ convex, and the feasible set is convex, then the global minimum is _______________ if it exists.

This is known as the Theorem of _______________.

(3) If a function is both coercive and _______________, then it reaches its minimum on any closed set.