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Faculty of exact sciences, natural and life science

Department of Mathematics and Computer Science **Academic year: 2024/2025**

Level: L1 Computer Science **Duration:1h30**

Exam: Analysis 2

Exercise 1 (2.5+3+2.5=8 pts)

1) Let (u_n) be a sequence defined by:

$$\forall n \geq 1: \quad u_n = \sum_{k=1}^{n} \frac{n}{n^2+k^2}.$$

Using the Riemann sums calculate the limit: $\lim_{n\to\infty} u_n$.

2) a) Calculate the integral:

$$I = \int \frac{3t}{t^2+t-2}\,dt. \qquad \text{(Note that: } t^2+t-2=(t-1)(t+2)\text{)}$$

b) By integration by change of variable and using the integral I, calculate the integral:

$$J = \int \frac{3}{x+\sqrt{x+2}}\,dx.$$

Exercise 2 (3+1=4 pts)

1) Find the general solution of the differential equation: $y' - y = x \ldots\ldots (E)$.

2) Deduce the particular solution to equation (E) that achieves $y(0) = 1$.

Exercise 3 (3+2+1.5+1.5=8 pts)

1) a) Find a limited development of order 3 in a neighborhood of 0 for the functions

$$u(x) = e^x \tan x \quad , \quad v(x) = \frac{\sinh x}{1-x}.$$

b)Using the previous limited developments, calculate the following limit:

$$\lim_{x\to 0}\left(\frac{\frac{\sinh x}{1-x} - e^x \tan x}{x^3}\right).$$

2) Let g be a function defined by: $g(x) = x^2 \ln\left(1+\frac{2}{x}\right)$, we denote the graph that represents it by (Cg).

a) Find a limited development of order 2 in a neighborhood of ∞ for the function

$$h(x) = \ln\left(1+\frac{2}{x}\right).$$

b) Calculate the limit $\lim_{x\to\infty}(g(x) - 2x + 2)$ and conclude that the graph (Cg) has an asymptote (denoted (Δ)), write an equation for (Δ).

Given: $\sinh x = x + \frac{1}{6}x^3 + o(x^3)$; $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + o(x^3)$;

$\tan x = x + \frac{1}{3}x^3 + o(x^3)$; $\ln(1+x) = x - \frac{1}{2}x^2 + o(x^2)$.

Good luck.