Larbi Ben M'hid University
Master 01 Mathematics
Exam Correction: Optimization

Exercise 1 *Let $f(x_1, x_2, \ldots, x_n)$ be a differentiable objective function to be maximized or minimized, subject to the constraint*

$$g_i(x_1, x_2, \ldots, x_n) = 0, i = 1...p$$

where gg is a differentiable function.

Then, if $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ is a local extremum of ff subject to the constraint g =0, and the gradient of the constraint $\nabla g(x^*) \neq 0$, there exists a scalar λ_i $(i = 1...p) \in R$ such that:

$$\nabla f(x^*) = \sum_{i=1}^{p} \lambda_i \nabla g_i(x^*).$$

Proof. We have proved using the **Implicit Function** Theorem that at a local extremum of

$$f(x, y)$$

subject to

$$g(x, y) = 0$$

and under the condition

$$\nabla g(x^*, y^*) \neq 0$$

there exists $\lambda \in R$ such that:

$$\nabla f(x^*, y^*) = \lambda \nabla g(x^*, y^*).$$

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Exercise 2 *Problem Statement*

Set up the Lagrangian

Compute the Partial Derivatives

Solve the System

Corresponding Critical Points: $P_1 = (\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ *and* $P_2 = (-\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

Conclusion:: nature P_2*is minimum with* $f = -\sqrt{2}$ *and* P_1 *is maximum with* $f = \sqrt{2}$

Exercise 3 *Problem Statement*

KKT Conditions Setup

First-Order Conditions
$$\begin{cases} y - 2x - 2\lambda_1 = 0 \\ x - 2y - \lambda_1 - \lambda_2 = 0 \\ \lambda_1(2x + y - 5) = 0 \\ \lambda_2(y - 3) = 0 \end{cases}$$

Analyze the Cases

Critical point:$\left(\frac{3}{2}, 3\right)$

Nature: Local maximum under the KKT conditions.

Exercise 4 *(1) Let the function f be a* ***continuous*** *function, and let the feasible set be non-empty, closed, and* ***bounded****.*

Then there exists at least one minimum point.This result is known as the Theorem of ***Weierstrass****.*

(2) If the function f is ***strictly*** *convex, and the feasible set is convex, then the global minimum is* ***unique*** *if it exists.*

This is known as the Theorem of ***Uniqueness****.*

(3) If a function is both coercive and ***continuous****, then it reaches its minimum on any closed set.*