* variation of the Constant k:

$y_H = k(x) e^x$. 0,5

$y_H' = k'(x) e^x + k(x) e^x$. 0,5

Then:

$k'(x)e^x + k(x)e^x - k(x)e^x = x$ 0,5

$\Rightarrow k'(x) = x e^{-x}$. 0,5

$\Rightarrow k(x) = \int x e^{-x} dx$. 0,5

$\left.\begin{array}{l} u = x \Rightarrow u' = 1 \\ v' = e^{-x} \Rightarrow v = -e^{-x} \end{array}\right)$ 0,5

$k(x) = -x e^{-x} + \int e^{-x} dx$.

$= (-x-1) e^{-x}$. 0,5

$y_H = -x - 1$

Then: $\boxed{y = k e^x - x - 1}$ 0,5

$y(0) = 1 \Rightarrow 1 = k - 1 \Rightarrow k = 2$

$y = 2e^x - x - 1$ 0,5

$J = \int_0^3 \frac{1}{1+\sqrt{x+1}} dx$. 3

$u = \sqrt{x+1} \Rightarrow u^2 = x+1 \Rightarrow x = u^2 - 1$ 0,25 0,25

$dx = 2u\, du$. 0,25

$\left\{\begin{array}{l} x = 0 \Rightarrow u = 1 \\ x = 3 \Rightarrow u = 2. \end{array}\right)$ 0,25

$J = \int_1^2 \frac{2u\, du}{1+u} = \int_1^2 \frac{2u+2-2}{1+u} du$ 0,5

$= \int_1^2 2\, du - 2\int \frac{1}{1+u} du$ 0,5

$= \left[2u - 2\ln|1+u|\right]_1^2$ 0,5

$= 2(1 + \ln 2/3)$ 0,5

3

$y' - y = x$.

we solve:

$y' - y = 0$ (solution generale) 0,5

$y' - y = 0 \Rightarrow \frac{y'}{y} = 1$. 0,5

$\Rightarrow \ln|y| = x + C \Rightarrow \boxed{y_G = k e^x}$

0,5

$h(x) = 1 + \frac{2}{x} + \frac{3}{2x^2} + o\left(\frac{1}{x^2}\right)$ امتحان طلبة الديون

$g(x) = x\,h(x)$. الحل المقترح

$\boxed{g(x) = x + 2 + \frac{3}{2x} + o\left(\frac{1}{x}\right)}$ (1)

$\lim_{x\to\infty} \left(g(x) - (x+2)\right) = \lim_{x\to\infty} \left(\frac{3}{2x} + o\left(\frac{1}{x}\right)\right)$

$= 0$. (0,5)

$(\Delta): y = x + 2$ is an asymptote

(0,5) if $x < 0$ then $(C_g)$ is under $(\Delta)$

(0,5) if $x > 0$; then $(C_g)$ is below $(\Delta)$.

حل التمرين الثاني:

$I = \int \frac{4}{(x+1)^2(x-1)}\,dx$. (3)

$\frac{4}{(x+1)^2(x-1)} = \frac{a}{x+1} + \frac{b}{(x+1)^2} + \frac{c}{x-1}$ (0,5)

$a = 1$ (0,5); $B = -1$ (0,5); $c = -2$ (0,5)

$I = \int \frac{1}{x+1}\,dx - \int \frac{2}{(x+1)^2}\,dx - \int \frac{1}{x-1}\,dx$ (0,5)

$= \ln|x+1| + \frac{2}{x+1} - \ln|x-1| + c$ (0,5)

$= \ln\left|\frac{x+1}{x-1}\right| + \frac{2}{x+1} + c$

(08) التمرين الأول:

/1

$f(x) = \sqrt{1+x}\,\ln(1+x) - \sin x$.

$f(x) = \left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3\right)\left(x - \frac{1}{2}x^2 + \frac{1}{3}x^3\right)$

$-\left(x - \frac{1}{6}x^3\right) + o(x^3)$

$f(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{4}x^3 - x + \frac{1}{6}x^3$

$-\frac{1}{8}x^3 + o(x^3)$

$f(x) = \frac{1}{8}x^3 + o(x^3)$. (1,5 pts)

2) $\lim_{x\to 0} \frac{f(x)}{x^3} = \lim_{x\to 0} \frac{\frac{1}{8}x^3}{x^3} = \frac{1}{8}$ (1)

3) $g(x) = \frac{x^2}{x-1} e^{1/x}$.

$h(x) = \frac{x}{x-1} e^{1/x}$.

$h(x) = \frac{x}{x}\left(\frac{1}{1 - 1/x}\right) e^{1/x}$ (1)

we put $\frac{1}{x} = t$ / $x \to \infty$, $t \to 0$.

$h(t) = \frac{1}{1-t} e^{t}$. (1)

$h(t) = (1 + t + t^2)\left(1 + t + \frac{1}{2}t^2\right) + o(t^2)$

$h(t) = 1 + t + \frac{1}{2}t^2 + t + t^2 + o(t^2)$

$h(t) = 1 + 2t + \frac{3}{2}t^2 + o(t^2)$ (1,5)