Larbi Ben M'Hidi University –Oum El Bouaghi	2024/2025
Faculty of exact sciences, natural and life sciences	Duration: 1h30
Department: Matter Sciences	1st year MS

Exam: Mathematics 1

Exercice 01:

For $n \in \mathbb{N}^*$, let $S_n = \sum_{k=1}^{n} k(k+1)$ and $T_n = \sum_{k=1}^{n} k^2$.

1. Compute S_1, T_2.

2. Show by induction that $\forall n \in \mathbb{N}^*$: $S_n = \dfrac{n(n+1)(n+2)}{3}$.

3. Calculate $\sum_{k=1}^{n} k$, prove that $T_n = S_n - \sum_{k=1}^{n} k$, and deduce T_n from these results.

Exercice 02:

We define in $\mathbb{R} \times \mathbb{R}^+$ a relation $\mathcal{R}$ by:

$$\forall (x,y), (x',y') \in \mathbb{R} \times \mathbb{R}^+ : (x,y)\mathcal{R}(x',y') \iff x(1+y') = x'(1+y).$$

1. Show that $\mathcal{R}$ is an equivalence relation.

2. Determine the equivalence class of $\left(-1, \dfrac{1}{2}\right)$, respectively.

Exercice 03:

1. Let the function f be defined by:
$$f(x) = \frac{1}{1 - e^{2x-1}},$$

   (a) Determine D_f, the domain of definition of f.

   (b) Compute $f\left(\{3, -\dfrac{1}{2}\}\right)$.

   (c) Show that f is injective.

   (d) Is it surjective?

2. Let the function g defined by:
$$g(x) = \begin{cases} 2x - 4 \text{ if } x \leq 0, \\ \dfrac{1}{2}x - 2 \text{ if } 0 < x < 4, \\ 2x^2 - 5x + 4 \text{ if } x \geq 4. \end{cases}$$

   Study the continuity of this function.

Exercice 04:

We provide $A = \mathbb{R} \times \mathbb{R}$ with two operations defined by:
$$(x,y) + (x',y') = (x + x', y + y') \quad \text{and} \quad (x,y) \times (x',y') = (xx', xy' + x'y)$$

1 Show that $(A, +)$ is a commutative group.

2   a Show that the operation $\times$ is commutative.

    b Show that $\times$ is associative.

    c Find the neutral element of A for the operation $\times$.

    d Show that $(A, +, \times)$ is a commutative ring.