EXO 1 :

$$Z_1 = \boxed{\sum_{n_x, n_y, n_z = 1}^{\infty} e^{-\beta E}}$$

1 point

$$= \boxed{\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{\beta \pi^2 \hbar^2}{2m}\left(\frac{n_x^2}{L_1^2} + \frac{n_y^2}{L_2^2} + \frac{n_z^2}{L_3^2}\right)} dn_x\, dn_y\, dn_z}$$

1 point

$$= \int_0^\infty dn_x\, e^{-\frac{\beta\pi^2\hbar^2}{2m}\frac{n_x^2}{L_1^2}} \cdot \int_0^\infty dn_y\, e^{-\frac{\beta\pi^2\hbar^2}{2m}\frac{n_y^2}{L_2^2}} \cdot \int_0^\infty dn_z\, e^{-\frac{\beta\pi^2\hbar^2}{2m}\frac{n_z^2}{L_3^2}}$$

$$= \frac{1}{2}\sqrt{\frac{\pi 2m L_1^2}{\beta\pi^2\hbar^2}} \cdot \frac{1}{2}\sqrt{\frac{\pi 2m L_2^2}{\beta\pi^2\hbar^2}} \cdot \frac{1}{2}\sqrt{\frac{\pi^2 2m L_3^2}{\beta\pi^2\hbar^2}}$$

$$= \frac{1}{8} L_1 L_2 L_3 \cdot \left(\frac{2m\pi}{\beta\pi^2\hbar^2}\right)^{3/2} = \boxed{\frac{L_1 L_2 L_3}{8}\left(\frac{2m k_B T}{\pi\hbar^2}\right)^{3/2}}$$

3 point

on pose $L_1 L_2 L_3 = V$

$$= \frac{V}{2^3}\left[\frac{2m k_B T}{\pi \cdot \frac{h^2}{4\pi^2}}\right]^{3/2} = \frac{V}{2^3} \cdot \left[8 m k_B T \pi\right]^{3/2}$$

$= 2^{3/2}$

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$$= \frac{V}{2^3}\left[\left(2^3\right)^{3/2}\right] \cdot \left(m k_B T \pi\right)^{3/2}$$

$$Z(1) = V\left(\frac{2\pi m k_B T}{h^2}\right)^{3/2}$$

$$\boxed{Z(N) = \frac{V^N}{N!}\left(\frac{2\pi m k_B T}{h^2}\right)^{3N/2}} = \frac{V^N}{N!\, h^{3N}}\left(2m\pi k_B T\right)^{\frac{3N}{2}}$$

2 points

$$F = -k_B T \log Z(N)$$

2 points $\boxed{= -N k_B T\left\{1 + \ln \frac{V}{N}\left(\frac{2\pi m k_B T}{h^2}\right)^{3/2}\right\}}$

on a utilisé la f Stirling

$$S = -\frac{\partial F}{\partial T}\Bigg|_{V,N} = \boxed{N k_B\left\{\frac{5}{2} + \ln\left(\frac{V}{N}\left(\frac{2\pi m k_B T}{h^2}\right)^{3/2}\right)\right\}}$$

2 points

$$U = F + TS = \boxed{\frac{3}{2} N k_B T}$$

2 point

$$C_V = \frac{\partial U}{\partial T} = \boxed{\frac{3}{2} N k_B}$$

1 point

EXO2.

H pour N oscillateurs

$$\boxed{H = \sum_i \frac{P_i^2}{2} + \frac{X_i^2}{2}}$$ 1 point

$$\omega = \int_{H \leq E} d^N P \, d^N X = \int_{\sum_{i=1}^{N} \frac{P_i^2}{2} + \frac{X_i^2}{2} \leq E} dP_1 \dots dP_N \, dx_1 \dots dx_N$$

$$= \int_{\sum_i P_i^2 + X_i^2 \leq 2E} dP_1 \dots dP_N \, dx_1 \dots dx_N$$

$$= \int_{\sum_{i=1}^{2N} z_i^2 \leq 2E} dz_1 \dots dz_{2N} = \int_{\sum_{i=1}^{2N} z_i^2 \leq (\sqrt{2E})^2} dz_1 \dots dz_{2N}$$

$$= \frac{\pi^{\frac{2N}{2}} (2E)^{\frac{2N}{2}}}{\Gamma(\frac{2N}{2}+1)} = \boxed{\frac{\pi^N (2E)^N}{\Gamma(N+1)}}$$ 3 points

$$\Omega = \frac{1}{h^N} \frac{\partial \omega}{\partial E} \Big/ h = 1.$$

$$\Rightarrow \Omega = \frac{N \pi^N (2E)^{N-1} \cdot 2}{\Gamma(N+1)} = \boxed{\frac{\pi^N (2E)^N}{E \, \Gamma(N)}}$$

2 points