Corrigé

P1

Exercice 01 : 11 pts

I)

a) · Table de D.N.D

x_i	y_i	1ère D.N.D	2e D.N.D	3ème D.N.D	4ème D.N.D
0	0				
		12			
3	12		2		
		14		−2	
6	26		0		8
		14		6	
9	40		6		
		20			
12	60				

1 pt   1 pt

0,5

$$P_3(x) = y_0 + \frac{\Delta y_0}{h}(x-x_0) + \frac{\Delta^2 y_0}{2!\,h^2}(x-x_0)(x-x_1) + \frac{\Delta^3 y_0}{3!\,h^3}(x-x_0)(x-x_1)(x-x_2)$$

0,5

$$= 0 + \frac{12}{3}(x-0) + \frac{2}{2!\,3^2}(x-0)(x-3) + \frac{-2}{6\,3^3}(x-0)(x-3)(x-6)$$

$$P_3(x) = 4x + \frac{1}{9}x(x-3) - \frac{1}{81}x(x-3)(x-6).$$ 0,5

0,5

b) $V(5) \simeq P_3(5) = 4\times 5 + \frac{1}{9}5(5-3) - \frac{1\times 5}{81}(5-3)(5-6) = \frac{1720}{81} = 21{,}23\,m/s$

c) $E_3(x) \simeq \left| f[x_0,x_1,x_2,x_3,x_4](x-x_0)(x-x_1)(x-x_2)(x-x_3) \right|$

$$= \left| \frac{\Delta^4 y_0}{4!\,h^4}(x-x_0)(x-x_1)(x-x_2)(x-x_3) \right|$$ 0,5

$$E_3(5) = \left| \frac{8}{24\times 3^4}(5-0)(5-3)(5-6)(5-9) \right|$$ 0,5

$$= \frac{8}{1944}\times 5\times 2\times 1\times 3 = \frac{240}{1944} = 0{,}123$$ 0,5

II)

(a) $f(x+2h) = f(x) + 2h\,f'(x)$ ---- ①

$f(x+h) = f(x) + h\,f'(x)$ --- ②   0,25 × 4

$f(x-h) = f(x) - h\,f'(x)$ --- ③

$f(x-2h) = f(x) - 2h\,f'(x)$ --- ④

$-1\times ① + 8\times ② - 8\times ③ + ④ \Longleftrightarrow$

$-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)$

$= -f(x)+8f(x)-8f(x)+f(x)+2hf'(x)+8hf'(x)+8hf'(x)-2hf'(x)$ 0,8

$= 12h\,f'(x)$

c) $f'(x) = \dfrac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$ 0,8

b) $a(t) = v'(t) = \dfrac{-v(t+2h)+8v(t+h)-8v(t-h)+v(t-2h)}{12h}$ 0,8

$t=6$ et $h=3$

$a(6) = v'(6) = \dfrac{-v(12)+8v(9)-8v(3)+v(0)}{12\times 3}$

$= \dfrac{-60+8\times 40-8\times 12+0}{36}$ 0,6

$= \dfrac{164}{36} = 4,555$ m/s².

(C) $f(x+2h) = f(x)+2hf'(x)+\frac{(2h)^2}{2!}f''(x)+\frac{(2h)^3}{3!}f'''(x)+\frac{(2h)^4}{4!}f^{(4)}(x)+\frac{(2h)^5}{5!}f^{(5)}(\xi_1)$

$f(x+h) = f(x)+hf'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+\frac{h^4}{4!}f^{(4)}(x)+\frac{h^5}{5!}f^{(5)}(\xi_2)$

0,25 × 4 $f(x-h) = f(x)-hf'(x)+\frac{h^2}{2!}f''(x)-\frac{h^3}{3!}f'''(x)+\frac{h^4}{4!}f^{(4)}(x)-\frac{h^5}{5!}f^{(5)}(\xi_3)$

$f(x-2h) = f(x)-2hf'(x)+\frac{(-2h)^2}{2!}f''(x)+\frac{(-2h)^3}{3!}f'''(x)+\frac{(-2h)^4}{4!}f^{(4)}(x)+\frac{(-2h)^5}{5!}f^{(5)}(\xi)$

$-1\times(1)+8\times(2)-8\times(3)+(4) \Leftrightarrow$

$-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h) =$ 0,6

$12hf'(x)-\frac{48h^5}{5!}f^{(5)}(\xi)$.

$f'(x) = \dfrac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h} - \dfrac{48h^5}{12h\,5!}f^{(5)}(\xi)$ 0,6

donc la méthode est d'ordre 4

Exercice 02 (P3)

1) La méthode de Gauss :

$$\begin{cases} 4x_1 + x_2 + 2x_3 = 9 & --- L_1^{(0)} \\ 2x_1 + 4x_2 - x_3 = -5 & --- L_2^{(0)} \\ x_1 + x_2 - 3x_3 = -9 & --- L_3^{(0)} \end{cases}$$

$-\frac{1}{2} L_1^{(0)} + L_2^{(0)} \longrightarrow L_2^{(1)}$ (0,5)

et $-\frac{1}{4} L_1^{(0)} + L_3^{(0)} \longrightarrow L_3^{(1)}$ (0,5)

on obtient
$$\begin{cases} 4x_1 + x_2 + 2x_3 = 9 & --- L_1^{(0)} \\ \frac{7}{2}x_2 - 2x_3 = -\frac{19}{2} & --- L_2^{(1)} \\ \frac{3}{4}x_2 - \frac{7}{2}x_3 = -\frac{45}{4} & --- L_3^{(1)} \end{cases}$$
(0,5) (0,5)

$-\frac{3}{14} L_2^{(1)} + L_3^{(1)} \longrightarrow L_3^{(2)}$ (0,5)

on obtient
$$\begin{cases} 4x_1 + x_2 + 2x_3 = 9 & --- L_1^{(0)} \\ \frac{7}{2}x_2 - 2x_3 = -\frac{19}{2} & --- L_2^{(1)} \\ -\frac{43}{14}x_3 = -\frac{129}{14} & --- L_3^{(2)} \end{cases}$$
(0,5)

par la méthode de remontée on obtient alors,

$x_3 = 3$, $x_2 = -1$, $x_1 = 1$ (0,75)

2) Décomposition LU :

$$A = L\,U = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$ (0,5)

$$= \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{pmatrix}$$ (0,5)

$u_{11} = 4, u_{12} = 1 \quad u_{13} = 2$

par identification : $l_{21}u_{11} = 2 \Rightarrow l_{21} = \frac{1}{2}, \ldots$

(P4)

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & \frac{3}{14} & 1 \end{pmatrix} \text{ (0,75)}, \quad U = \begin{pmatrix} 4 & 1 & 2 \\ 0 & \frac{7}{2} & -2 \\ 0 & 0 & -\frac{43}{14} \end{pmatrix} \text{ (1,5)}$$

$\det(A) = \det(LU) = \det(L)\det(U)$ (0,5)

$= u_{11}u_{22}u_{33} = 4 \times \frac{7}{2} \times \left(-\frac{43}{14}\right)$

$= -43$ (0,5)

(3pts)

3) Jacobi

$$\begin{cases} 4x_1 + x_2 + 2x_3 = 9 \\ 2x_1 + 4x_2 - x_3 = -5 \\ x_1 + x_2 - 3x_3 = -9 \end{cases} \Rightarrow \begin{cases} x_1 = (9 - x_2 - 2x_3)/4 \\ x_2 = (-5 - 2x_1 + x_3)/4 \\ x_3 = (+9 + x_1 + x_2)/3 \end{cases} \text{ (0,5)}$$

Jacobi
$$\begin{cases} x_1^{(k+1)} = (9 - x_2^{(k)} - 2x_3^{(k)})/4 \\ x_2^{(k+1)} = (-5 - 2x_1^{(k)} + x_3^{(k)})/4 \\ x_3^{(k+1)} = (9 + x_1^{(k)} + x_2^{(k)})/3 \end{cases} \text{ (0,5)}$$

G-S
$$\begin{cases} x_1^{(k+1)} = (9 - x_2^{(k)} - 2x_3^{(k)})/4 \\ x_2^{(k+1)} = (-5 - 2x_1^{(k+1)} + x_3^{(k)})/4 \\ x_3^{(k+1)} = (9 + x_1^{(k+1)} + x_2^{(k+1)})/3 \end{cases} \text{ (0,5)}$$

Jacobi

$k=0$

$$\begin{cases} x_1^{(1)} = \frac{9}{4} = 2,25 \\ x_2^{(1)} = -\frac{5}{4} = -1,25 \\ x_3^{(1)} = 3 = 3 \end{cases} \text{ (0,5)}$$

$k=1$

$$\begin{cases} x_1^{(2)} = 1,0625 \\ x_2^{(2)} = -1,625 \\ x_3^{(2)} = 3,333 \end{cases} \text{ (0,5)}$$

G-S

$k=0$

$$\begin{cases} x_1^{(1)} = 2,25 \\ x_2^{(1)} = -2,375 \\ x_3^{(1)} = 2,9573 \end{cases} \text{ (0,5)}$$

$k=1$

$$\begin{cases} x_1^{(2)} = 1,3646 \\ x_2^{(2)} = -1,1927 \\ x_3^{(2)} = 3,0573 \end{cases} \text{ (0,5)}$$