Correction Exam

EX 01 :

1 • We use Gaussian elimination method to solve the system (1) :

Step 1 : express the coefficients and the right-hand side as an augmented

Matrix $\Rightarrow$
$$\begin{array}{c} \left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 3 & -\frac{3}{2} & 2 & 0 \\ 0 & 2 & 1 & 1 \end{array}\right] \end{array} \begin{array}{l} R_1^1 \\ R_2^1 \\ R_3^1 \end{array}$$

0,5

Step 2 : elimination of x_1 in R_2^1

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 2 & 1 & 1 \end{array}\right] \begin{array}{l} R_1^2 \leftarrow R_1^1 \\ R_2^2 \leftarrow R_2^1 - \frac{3}{2} R_1^1 \\ R_3^2 \leftarrow R_3^1 \end{array}$$

1

Step 3 : Permutation between R_2^2 and R_3^2 ($R_2^2 \longleftrightarrow$

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & \frac{1}{2} & 0 \end{array}\right] \quad \ldots (*)$$

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$$(*) \iff \begin{cases} 2x_1 - x_2 + x_3 = 0 \\ 2x_2 + x_3 = 1 \\ \frac{1}{2} x_3 = 0 \end{cases} \iff \begin{cases} x_3 = 0 \\ x_2 = \\ x_1 = \end{cases}$$

So $X = \left(0, \frac{1}{2}, \frac{1}{4}\right)^T$

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- We have $\det(A_1) = |2| = 2 \neq 0$

and $\det(A_3) = \det(A) = -2 \neq 0$ 1.1

but $\det(A_2) = \begin{vmatrix} 2 & -1 \\ 3 & -\frac{3}{2} \end{vmatrix} = 0$

Then, A does not have an LU decomposition

3) $\tilde{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 2 & 1 \\ 3 & -\frac{3}{2} & 2 \end{pmatrix}$ 1

LU decomposition of $\tilde{A}$:

$$\tilde{A} = LU \quad \dots (**)$$ 1

we have $U = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$ and $L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}$

From $(**)$, we get $l_{21} = 0$, $l_{31} = \frac{3}{2}$ and $l_{32} = 0$

$\Rightarrow$ $L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{3}{2} & 0 & 1 \end{pmatrix}$ 1

EX 03 :

1. The eigenvalues of the matrix A are the roots of the following equation : $\det(A - \lambda I_3) = 0$, where $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 0,5

So $\lambda_1 = 3$, $\lambda_2 = -\sqrt{2} + 1$ and $\lambda_3 = \sqrt{2} + 1$. 1,5

② . we choose the initial vector X^0 arbitrary such that $X^{(0)} \in \mathbb{R}^n$ 1

③ Matlab code to approximate the smallest eigenvalue of A ⟷ 3 pts

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EX 02 :

① • Jacobi Algorithm of System (2) is :

$$\begin{cases} x_1^{k+1} = \frac{1}{2} x^k \\ x_2^{k+1} = (x_1^k + x_2^k - 1)/4 \\ x_3^{k+1} = 1 + \frac{1}{2} x_2^k \end{cases}$$

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• Gauss-Seidel algorithm of System(2) is :

$$\begin{cases} x_1^{k+1} = \frac{1}{2} x^k \\ x_2^{k+1} = (x_1^{k+1} + x_2^k - 1)/4 \\ x_3^{k+1} = 1 + \frac{1}{2} x_2^{k+1} \end{cases}$$

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② we have (2) $\Leftrightarrow$ $Ax = b$ where, $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

and $b = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$

Since :

$|a_{11}| = 2 > |a_{12}| + |a_{13}| = 1$

$|a_{22}| = 4 > |a_{21}| + |a_{23}| = 2$

$|a_{33}| = 2 > |a_{31}| + |a_{32}| = 1$

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ien A is strictly diagonally dominant hence the Jacobi and Gauss-Seidel methods applied to the system 2) converge

3) Matlab code of Jacobi method $\longleftrightarrow$ 2

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