

L'arbi Ben M'hidi University

Faculty: Exact sciences and sciences of nature and life

Département: MI

Academic year : 2023/2024

Module: Algebra 2

Exam n 2

Exercise 1:

Let the following sets

$$\begin{aligned} F_1 &= \{(x, y, z) \in \mathbb{R}^3 / 2x - y + z = 0\} \\ F_2 &= \{(x, y) \in \mathbb{R}^2 / 3x - y = 0\} \end{aligned}$$

1. Prove that F_1, F_2 are vector subspaces.
2. Determine a basis for each vector subspace.
3. Deduce the dimension of F_1 and F_2 .

Exercise 2 :

Let the following vectors

$$v_1 = (0, 2, -4), v_2 = (2, 2, 0), v_3 = (-4, 0, -4)$$

1. Prove that v_1, v_2, v_3 form a basis of \mathbb{R}^3 .
2. Determine the coordinates of u_1, u_2 in the basis $\{v_1, v_2, v_3\}$ where

$$\begin{aligned} u_1 &= v_2 - 5v_3 \\ u_2 &= (-1, 3, 0) \end{aligned}$$

Exercise 3 : Let the matrix :

$$P = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$$

- (1) Find a and b where

$$P^2 + aP + bI_2 = 0_2$$

- (2) Deduce that P is an invertible matrix and calculate P^{-1}
- (3) Let the linear application $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ where

$$f(x, y) = (2x, x + 3y)$$

Prove that f^{-1} exists and find its formula.

Bonne chance.

Pr. Rezzag.S