Probability and Statistics for Computer Science (Final Exam) 2023-2024 (Correction)

Exercise 1. (12Marks)

(1) The population Ω : 99 employees in an industry (0.75)
Variable X : Hourly wages (paid in dollars) $(0, 75)$
The data is quantitative discrete $(0, 75)$
$X(\Omega) = \{7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ $(0, 75)$

(2) $(2Marks)$

Hourly Wages(x_i)	Frequency(f_i)	RF(f_i/99)	CF(F_i)	RCF	$f_i x_i$	$f_i x_i^2$
7	11	0.11	11	0.11	77	539
8	4	0.04	15	0.15	32	256
9	10	0.10	25	0.25	90	810
10	4	0.04	29	0.29	40	400
11	9	0.09	38	0.38	99	1089
12	8	0.08	46	0.46	96	1152
13	6	0.06	52	0.52	78	1014
14	5	0.05	57	0.57	70	980
15	17	0.17	74	0.74	255	3825
16	19	0.20	93	0.94	304	4864
17	3	0.03	96	0.97	51	867
18	1	0.01	97	0.98	18	324
19	2	0.02	99	1	38	722
Total	99	1			1248	16842

(3) $\overline{X} = \dfrac{\sum_{i=1}^{13} f_i x_i}{n} = \dfrac{1248}{99} = 12.60$ $(1Mark)$

$Var(X) = \dfrac{\sum_{i=1}^{13} f_i x_i^2}{n} - \left(\overline{X}\right)^2 = \dfrac{16842}{99} - (12.60)^2 = 11.36$ $(1Mark)$

$\sigma_X = \sqrt{Var(X)} = 3.37$ (0.5)

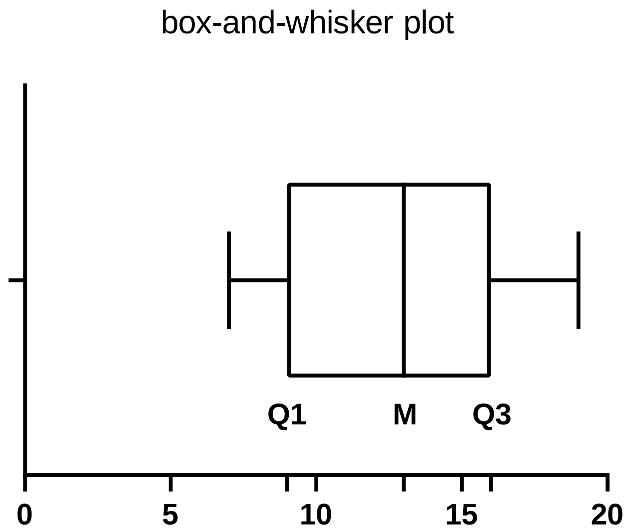
Since $n = 99$ is odd then $M = x_{\frac{n+1}{2}} = x_{50} = 13$ $(1Mark)$

For $Q_1 : \dfrac{n}{4} = 24.75$ then $Q_1 = x_{25} = 9$ $(1Mark)$

For $Q_3 : \dfrac{3n}{4} = 74.25$ then $Q_3 = x_{75} = 16$ $(1Mark)$

The mode for this data is 16 (0.5)

(4) The box-plot for these data $(1Mark)$



Exercice 2. (4Marks)

(1) We toss a coin 4 times then we have $2^4 = 16$ outcomes. $(1Mark)$

(2) We roll a die twice then $|\Omega| = 36$. Let E be the event "the product of the two numbers is 11", we have $E = \phi$ then $P(E) = \dfrac{|E|}{|\Omega|} = 0$. $(1Mark)$

(3) We have 4 unborn then $|\Omega| = 2^4 = 16$. Let F be the event "there will be at least 2 girls", we have $P(F) = \dfrac{|F|}{|\Omega|} = \dfrac{11}{16}$. $(1Mark)$

(4) Let G be the event "the ball is red", $P(G) = \dfrac{|G|}{\Omega} = \dfrac{C_{11}^1}{C_{44}^1} = \dfrac{11}{44} = \dfrac{1}{4}$. $(1Mark)$

Exercice 3. (4Marks)

(1) We have $P(A \mid B) = 0.4$ and $P(B) = 0.5$

(a) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A \cap B) = P(A \mid B) \times P(B) = 0.4 \times 0.5 = 0.2$ $(1Mark)$

(b) $P\left(\overline{A} \cap B\right) = P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3$ $(1Mark)$

2. We have $P(A \mid B) = 0.2$, $P\left(A \mid \overline{B}\right) = 0.3$ and $P(B) = 0.8$

$P(A \cap B) = P(A \mid B) \times P(B) = 0.2 \times 0.8 = 0.16$ (0.5)

$P\left(A \cap \overline{B}\right) = P\left(A \mid \overline{B}\right) \times P\left(\overline{B}\right) = P\left(A \mid \overline{B}\right)(1 - P(B)) = 0.3 \times 0.2 = 0.06$ (0.5)

Finally, we have $P(A) = P\left(A \cap \overline{B}\right) + P(A \cap B) = 0.06 + 0.16 = 0.22$. $(1Mark)$