Model Answer + Grading Rubric

Answer to exercise 01 : (Parallel machine : 05 Marks)

1. The appropriate queuing model for this system is $M/M/1$ because : **01.25**
   (a) Requests arrive according to the Poisson process
   (b) service time is distributed exponentially.
   (c) A single process web server
   (d) FIFO (default)
   (e) ∞ (default)
   (f) ∞ (default)

   $\lambda = 6$ requests/second and $\mu = \frac{1}{s} = \frac{1}{0.125} = 8$ requests/second. $\varrho = \frac{\lambda}{\mu} = \frac{6}{8} = \frac{3}{4} = 0.75 < 1$ and the system is **stable**.
2. The average server activity time in a day (24 hours) is : **01.25**
   Activity time $= 24 * \varrho = 24 * 0.75 = 18$ hours
3. The average number of requests on the web server is : **00.75**
   $\overline{N} = \frac{\varrho}{1-\varrho} = \frac{0.75}{0.25} = 3$ requests
4. The average number of pending requests on the web server is : **00.75**
   $\overline{Q} = \frac{\varrho^2}{1-\varrho} = \frac{0.75^2}{0.25} = 2.25$ requests
5. The average waiting time on the web server is : **00.75**
   $\overline{W} = \frac{\overline{Q}}{\lambda} = \frac{2.25}{6} = 0.375$ seconds
6. The average residance time in the web server is : **00.75**
   $\overline{T} = \frac{\overline{N}}{\lambda} = \frac{3}{6} = 0.5$ seconds
7. The probability that the server receives 5 requests in a second is :
   We have : $P_n(t) = \frac{(\lambda t)^n}{n!}e^{-\lambda t}$. For $n = 5$ and $t = 1$: $P_5(1) = \frac{(6\times1)^5}{5!}e^{-6\times1} = \frac{7776}{120}e^{-6}$.
   Hence, $P_5(1) = 0.16062314$ **00.75**
8. The probability that the server does not receive any requests for 1.5 seconds is :
   We have : $P_n(t) = \frac{(\lambda t)^n}{n!}e^{-\lambda t}$. For $n = 0$ and $t = 1.5$: $P_0(1.5) = \frac{(6\times1.5)^0}{0!}e^{-6\times1.5}e^{-9}$.
   Hence, $P_0(1.5) = 0.00012340$ **00.75**
9. The probability that the server becomes idle between the arrival of two requests, knowing that it was unoccupied when the first of the two arrived is :
   We have : $P(T_{n+1} - T_n \leq t) = 1 - e^{-\lambda t}$. To become idle, the interarrival time must be greater than the mean service time. $P(T_{n+1} - T_n > t) = e^{-\lambda t}$.
   $P(T_{n+1} - T_n > 0.125) = e^{-6\times0.125} = 0.47236655$ **00.75**
10. The average number of daily processed requests is :
    $Nb = 24 \times 3600 \times \varrho \times \mu = 86400 \times 0.75 \times 8 = 518400$ requests **00.75**

We want to lower the utilization rate to 35% (maximum) :

11. The number of parallel processes that should be executed on the server side is :
    $\varrho' = \frac{\lambda}{m\mu} = 0.35 \implies m = \frac{\lambda}{0.35\times\mu} = \frac{6}{0.35\times8}$
    hence, $m = 2.14285714$. The minimal number of parallel processes is 3 **00.75**
12. In this case, the average number of requests in service is :
    $\overline{R} = m\varrho = m \times \frac{\lambda}{m\mu} = \frac{\lambda}{\mu} = \frac{6}{8} = 0.75$ requests **00.75**

Answer to exercise 02 : (Queueing network : 10 Marks)

Nous avons : $\gamma = 3, m_1 = +\infty, m_2 = 2, m_3 = 1, \mu_1 = 1, \mu_2 = 5, \mu_3 = 10$.

1. Internal routing probability matrix : $\begin{pmatrix} 0 & 1 & 0 \\ 0.4 & 0 & 0.6 \\ 0.4 & 0 & 0 \end{pmatrix}$, External routing probability matrix : $\begin{pmatrix} 0 \\ 0 \\ 0.6 \end{pmatrix}$ 01.25 01.25

2. Effective arrival rates λ_i : $\begin{cases} \lambda_1 = \gamma + 0.4\lambda_2 + 0.4\lambda_3 \\ \lambda_2 = \lambda_1 \\ \lambda_3 = 0.6\lambda_2 \end{cases} \Rightarrow \begin{cases} 0.6\lambda_1 = 5 + 0.4\lambda_3 \\ \lambda_2 = \lambda_1 \\ \lambda_3 = 0.6\lambda_2 \end{cases}$

$\Rightarrow \begin{cases} 0.6\lambda_1 = 5 + 0.4 \times 0.6\lambda_1 \\ \lambda_2 = \lambda_1 \\ \lambda_3 = 0.6\lambda_2 \end{cases} \Rightarrow \begin{cases} 0.36\lambda_1 = 5 \\ \lambda_2 = \lambda_1 \\ \lambda_3 = 0.6\lambda_1 \end{cases} \Rightarrow \begin{cases} \lambda_1 = 8.33333333 \\ \lambda_2 = 8.33333333 \\ \lambda_3 = 5 \end{cases}$ 01.00

3. The average number of clients waiting at each station and in the network :

$\begin{cases} FA_1 \text{ Stable} \\ FA_2 \text{ Stable} \\ FA_3 \text{ Stable} \end{cases} \Rightarrow \begin{cases} \varrho_1 = 0 \\ \varrho_2 = \frac{\lambda_2}{m_2\mu_2} = \frac{8.33333333}{2\times5} \\ \varrho_3 = \frac{\lambda_1}{m_1\mu_1} = \frac{5}{1\times10} \end{cases} \Rightarrow \begin{cases} \varrho_1 = 0 < 1 \\ \varrho_2 = 0.83333333 < 1 \\ \varrho_3 = 0.5 < 1 \end{cases}$ 01.00

The network is **stable**.
Queue 1 : of type $M/M/+\infty$. Queue 2 : de type $M/M/2$. Queue 3 : de type $M/M/1$.

(a) $\overline{Q_1} = 0$ 00.25

(b) $P_0(2) = \left[\frac{(m_2\rho_2)^{m_2}}{m_2!(1-\rho_2)} + \sum_{k=0}^{m_2-1}\frac{(m_2\rho_2)^k}{k!}\right]^{-1} = \left[\frac{(2\times0.83333333)^2}{2!(1-0.83333333)} + 1 + 2\times 0.83333333\right]^{-1}$ 00.25

$P_0(2) = [8.33333333 + 1 + 1.6666666666]^{-1} \Rightarrow P_0(2) = 0.09090909$ 00.25

$\zeta_2 = \frac{(m_2\rho_2)^{m_2}}{m_2!(1-\rho_2)}P_0(2) = \frac{(2\times0.83333333)^2}{2!(1-0.83333333)}P_0(2) = 8.33333333 \times 0,09090909 \Rightarrow \zeta_2 = 0.75757575$

$\overline{Q_2} = \frac{\zeta_2\rho_2}{1-\rho_2} = \frac{0.75757575\times0.83333333}{1-0.83333333} \Rightarrow \overline{Q_2} = 3.78787878$ 00.25

(c) $\overline{Q_3} = \overline{N_3} - \overline{R_3} = \frac{\varrho_1}{1-\varrho_1} - m_1\varrho_1 = \frac{0.5}{1-0.5} - 0.5 = 1 - 0.5 \Rightarrow \overline{Q_3} = 0.5$ 00.25

(d) $\overline{Q_R} = \sum_{i=1}^{3}\overline{Q_i} = 0 + 3.78787878 + 0.5 \Rightarrow \overline{Q_R} = 4.28787878$ 00.25

4. The average number of customers in each queue and in the network :

(a) $\overline{N_1} = \overline{R_1} = \frac{\lambda_1}{\mu_1} = \frac{8.33333333}{1} \Rightarrow \overline{N_1} = 8.33333333$ 00.25

(b) $\overline{N_2} = \overline{Q_2} + \overline{R_2} = 3.78787878 + 2\times0.83333333 \Rightarrow \overline{N_2} = 5.45454545$ 00.25

(c) $\overline{N_3} = \frac{\varrho_3}{1-\varrho_3} = \frac{0.5}{1-0.5} \Rightarrow \overline{N_3} = 1$ 00.25

(d) $\overline{N_R} = \sum_{i=1}^{3}\overline{N_i} = 8.33333333 + 5.45454545 + 1 \Rightarrow \overline{N_R} = 14.78787878$ 00.25

5. The average residence time in each queue and in the network :

(a) $\overline{T_1} = \frac{\overline{N_1}}{\lambda_1} = \frac{8.33333333}{8.33333333} \Rightarrow \overline{T_1} = 1$ 00.25

(b) $\overline{T_2} = \frac{\overline{N_2}}{\lambda_2} = \frac{5.45454545}{8.33333333} \Rightarrow \overline{T_2} = 0.65454545$ 00.25

(c) $\overline{T_3} = \frac{\overline{N_3}}{\lambda_3} = \frac{1}{5} \Rightarrow \overline{T_3} = 0.2$ 00.25

(d) $\overline{T_R} = \frac{\overline{N_R}}{\lambda_R} = \frac{\overline{N_R}}{\sum_{i=1}^{2}\gamma_i} = \frac{14.78787878}{3} \Rightarrow \overline{T_R} = 4,92929292$ 00.25

6. The average waiting time in each queue and in the network :

(a) $\overline{W_1} = 0$ 00.25

(b) $\overline{W_2} = \frac{\overline{Q_2}}{\lambda_2} = \frac{3.78787878}{8.33333333} \Rightarrow \overline{W_2} = 0.45454545$ 00.25

(c) $\overline{W_3} = \frac{\overline{Q_3}}{\lambda_3} = \frac{0.5}{5} \Rightarrow \overline{W_3} = 0.1$ 00.25

(d) $\overline{W_R} = \frac{\overline{Q_R}}{\lambda_R} = \frac{\overline{Q_R}}{\sum_{i=1}^{2}\gamma_i} = \frac{4.28787878}{3} \Rightarrow \overline{W_R} = 1.42929292$ 00.25

7. The probability that the network is empty :

Pr(Network empty) = (Pr(Queue$_1$ empty and Queue$_2$ empty and Queue$_3$ empty)) = P_0(Queue$_1$) $\times$ P_0(Queue$_2$) $\times$ P_0(Queue$_3$)

$Pr(\text{Network empty}) = e^{\frac{-\lambda_1}{\mu_1}} \times P_0(2) \times P_0(3) = e^{\frac{-8.33333333}{1}} \times P_0(2) \times (1-\rho_3)$
$= 0.00024036 \times 0.09090909 \times 0.5 = 1 - 0.00001863$

Hence : $Pr(\text{Network not empty}) = 0.0000109258$ 01.00