Exercice 03 (5pt)

Méthode Runge-Kutta d'ordre 3

$$K_1 = f(x_0, y_0) = 0 \qquad (0.5\text{pt})$$
$$K_2 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}K_1 h\right) = f(0.5, 1) = 0.0075, \qquad (0.5\text{pt})$$
$$K_3 = f(x_0 + h, y_0 - K_1 h + 2K_2 h) = f(0.1, 1, 0015) = 0.03. \qquad (0.5\text{pt})$$

$$\Longrightarrow \quad y_1 = y_0 + \frac{1}{6}h(K_1 + 4K_2 + K_3) \qquad (0.5\text{pt})$$
$$= 1 + \frac{1}{6}(0.1)(4 \times 0.0075 + 0.03) = 1.0010. \qquad (0.5\text{pt})$$

f. *Méthode Runge-Kutta d'ordre 4*

$$K_1 = f(x_0, y_0), = 0 \qquad (0.5\text{pt})$$
$$K_2 = f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}K_1 h\right) = f(0.05, 1) = 0.0075, \qquad (0.5\text{pt})$$
$$K_3 = f(x_0 + h, y_0 - K_1 h + 2K_2 h) = f(0.05, 1, 0004) = 0.0075, \qquad (0.5\text{pt})$$
$$K_4 = f(x_0 + h, y_0 + K_3 h) = f(0.1, 1.0008) = 0.0300. \qquad (0.5\text{pt})$$

$$\Longrightarrow \quad y_1 = y_0 + \frac{1}{6}h(K_1 + 2K_2 + 2K_3 + K_4) = 1.0010. \qquad (0.5\text{pt})$$