Larbi Ben M'hidi University
Physical measurements 1st Year

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Exam of mathematics 2 (duration 1h30min)

Exercise 1.

1. Use the numbers (called nodes) $x_0 = \mathbf{2}$, $x_1 = \mathbf{2.75}$ and $x_2 = \mathbf{4}$ to find the second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$

2. Use this polynomial to approximate f(**3**), compare with the exact value

Exercise 2.

1. Construct the vector field of : $\vec{v}_1(x, y) = x\vec{i} + y\vec{j}$, $\vec{v}_2(x, y) = x\vec{i} - y\vec{j}$,

$$\vec{v}_3(x, y) = y\vec{i} + x\vec{j} \text{ , } \vec{v}_4(x, y) = y\vec{i} - x\vec{j}$$

2. Determine the coordinates of grad f where f is the following scalar field:
   a. $f(x,y,z) = xy^2 - yz^2$
   b. $f(x,y,z) = xyz sin(xy)$
3. Determine div f where f is the following vector field:
   a. $f(x,y,z) = (2x^2y, 2xy^2, xy)$
   b. $f(x,y,z) = (sin(xy), 0, cos(xz))$

Exercise 3.

1. If $A = [2x \quad 1]\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ -1 \end{bmatrix} = 0$, find the value of x

2. Express the matrix C as the sum of a symmetric and a skew symmetric matrix, where

$$C = \begin{pmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{pmatrix} = \ldots\ldots\ldots\ldots\ldots\ldots + \ldots\ldots\ldots\ldots\ldots\ldots$$

3. Calculate the determinant and the inverse of $A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 3 \\ 3 & 2 & 2 \end{pmatrix}$, can

   we perform the product AC and CA?

Ex 1 6points

1. $f(x_0) = \frac{1}{2} = 0,5$ (0,5pt) ; $f(x_1) = \frac{4}{11} = 0,3636$ (0,5pt)

$f(x_2) = \frac{1}{4} = 0,25$ (0,5pt)

$$L_0 = \frac{(x-2,75)(x-4)}{(2-2,75)(2-4)} = \frac{2}{3}(x-2,75)(x-4)$$ (0,5pt)

$$L_1 = \frac{(x-2)(x-4)}{(2,75-2)(2,75-4)} = \frac{-(x-2)(x-4)}{\frac{15}{16}} = -\frac{16}{15}(x-2)(x-4)$$ (0,5pt)

$$L_2 = \frac{(x-2)(x-2,75)}{(4-2)(4-2,75)} = \frac{2}{5}(x-2)(x-2,75)$$ (0,5pt)

$$P(x) = \sum_{i=0}^{2} L_i f(x_i) = L_0 f(x_0) + L_1 f(x_1) + L_2 f(x_2)$$ (1pt)

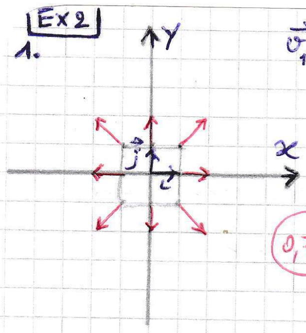
$$P(x) = \frac{1}{3}(x-2,75)(x-4) - \frac{64}{165}(x-2)(x-4) + \frac{1}{10}(x-2)(x-2,75)$$ (0,5pt)

$$\left.\begin{array}{l} P(x) = \frac{1}{22}x^2 - \frac{35}{88}x + \frac{49}{44} \\ \text{or } P(x) = 0,0455x^2 - 0,3977x + 1,1136 \end{array}\right\}$$ (1pt)

$P(3) = 0,32955$ (0,5pt) $f(3) = 0,333$ (0,5pt)

$\boxed{\text{Ex2}}$

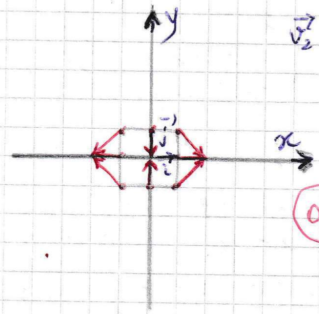
1.



$$\vec{v}_1 = x\vec{i} + y\vec{j}$$

x	y	$\vec{v}_1$
1	0	$\vec{i}$
0	1	$\vec{j}$
1	1	$\vec{i}+\vec{j}$
-1	1	$-\vec{i}+\vec{j}$
-1	0	$-\vec{i}$
-1	-1	$-\vec{i}-\vec{j}$
0	-1	$-\vec{j}$
+1	-1	$\vec{i}-\vec{j}$

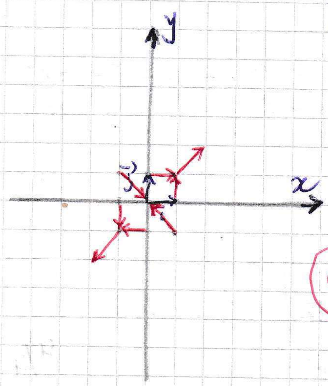
0,75 pt



$$\vec{v}_2 = x\vec{i} - y\vec{j}$$

x	y	$\vec{v}_2$
1	0	$\vec{i}$
0	1	$-\vec{j}$
1	1	$\vec{i}-\vec{j}$
-1	1	$-\vec{i}-\vec{j}$
-1	0	$-\vec{i}$
-1	-1	$-\vec{i}+\vec{j}$
0	-1	$\vec{j}$
+1	-1	$\vec{i}+\vec{j}$

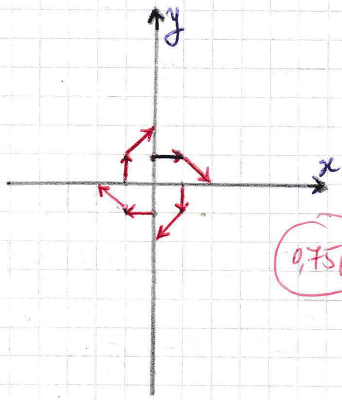
0,75 pt



$$\vec{v}_3 = y\vec{i} + x\vec{j}$$

x	y	$\vec{v}_3$
1	0	$\vec{j}$
0	1	$\vec{i}$
1	1	$\vec{i}+\vec{j}$
-1	1	$\vec{i}-\vec{j}$
-1	0	$-\vec{j}$
-1	-1	$-\vec{i}-\vec{j}$
0	-1	$-\vec{i}$
+1	-1	$-\vec{i}+\vec{j}$

0,75 pt



$\vec{v}_4 = y\vec{i} - x\vec{j}$

x	y	$\vec{v}_4$
1	0	$-\vec{j}$
0	1	$\vec{i}$
1	1	$\vec{i} - \vec{j}$
−1	1	$\vec{i} + \vec{j}$
−1	0	$\vec{j}$
−1	−1	$-\vec{i} + \vec{j}$
0	−1	$-\vec{i}$
+1	−1	$-\vec{i} - \vec{j}$

0,75 pt

2. The coordinates of $\overrightarrow{grad} f$

$$\overrightarrow{grad} f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

a. $\overrightarrow{grad} f = (y^2, 2xy - z^2, -2yz)$ 0,75 pt

b. $\overrightarrow{grad} f = (yz\sin(xy) + xy^2z\cos(xy), xz\sin(xy) + x^2yz\cos(xy), xy\sin(xy))$ 0,75 pt

3. The $div\,\vec{v}$ (the vector here is $\vec{f}$)

$$div\,\vec{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

a. $div\,\vec{f} = 8xy$ 0,75 pt

b. $div\,\vec{f} = y\cos(xy) - x\sin(xz)$ 0,75 pt

$\boxed{\text{Ex3}}$ 7,5pt

1. $[2x \quad 1]\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix}\begin{bmatrix}x\\ -1\end{bmatrix} = 0$

$[2x-1 \quad 4x]\begin{bmatrix}x\\ -1\end{bmatrix} = (2x-1)x + 4x(-1) = 0$

So : $2x^2 - x - 4x = 2x^2 - 5x = 0$

Then : $\begin{cases} x = 0 & \text{0,5pt} \\ x = 5/2 & \text{0,5pt} \end{cases}$

2. $C = \begin{pmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{pmatrix}$; $C' = \begin{pmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{pmatrix}$ 0,5pt

$\dfrac{C+C'}{2} = \begin{pmatrix} 2 & 5,5 & -2,5 \\ 5,5 & 3 & 1,5 \\ -2,5 & 1,5 & 4 \end{pmatrix}$ 0,5pt $\longrightarrow$ symmetric matrix

and let calculate $\dfrac{C-C'}{2} = \begin{pmatrix} 0 & 1,5 & -3,5 \\ 1,5 & 0 & 3,5 \\ 3,5 & -3,5 & 0 \end{pmatrix}$ skew sym. mat. 0,5pt

$\dfrac{C+C'}{2} + \dfrac{C-C'}{2} = C$ verified

3. $\det(A) = 2 - 6 = -4$, 1pt 0,5pt

$\det(C) = 1\times\begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = -4 + 15 + 1 = 12$ 0,5pt 1pt

$\text{inv}(A) = \begin{pmatrix} -1/2 & 3/4 \\ 1/2 & -1/4 \end{pmatrix}$ 1pt

$\text{inv}(C) = \begin{pmatrix} -1/3 & -1/3 & 2/3 \\ 5/12 & -1/12 & -1/12 \\ 1/12 & 7/12 & -5/12 \end{pmatrix}$ 1pt