Univ_Oeb.

Departement of: MS_L1

Correction of the exam

"Mathematatic 1"

Exercise 1: (5pts)

1) $*$ $\sup(A) = 7 \in A \Rightarrow Max = 7$ (0,25) (0,25), $Inf(A) = \pi/2 \notin A \Rightarrow Min$ does not exist (0,25) (0,25)

$*$ $B = \left\{ \frac{n-1}{n+1}, n \in \mathbb{N} \right\}$.

If $n = 0 \Rightarrow \frac{n-1}{n+1} = \frac{0-1}{0+1} = -1$ (0,5) then we get:

If $n \to +\infty \Rightarrow \frac{n-1}{n+1} \to 1$ (0,5)

$Sup(B) = 1 \notin B \Rightarrow Max(B)$: does not exist (0,25)

(0,25)

$Inf(B) = -1 \in B$ (0,25) $\Rightarrow Min(B) = -1$ (0,25).

2). prove that: $f^{-1}(A \cup B) \subset f^{-1}(A) \cup f^{-1}(B)$.

$\forall x \in f^{-1}(A \cup B) \overset{?}{\Rightarrow} x \in f^{-1}(A) \cup f^{-1}(B)$ (0,5)

$\forall x \in f^{-1}(A \cup B)$ Then $\exists y \in A \cup B : f^{-1}(y) = x$. (0,5)

$$\Rightarrow \begin{cases} \exists y \in A : f^{-1}(y) = x \\ \text{or} \\ \exists y \in B : f^{-1}(y) = x \end{cases} \text{(0,5)} \Rightarrow \begin{cases} x \in f^{-1}(A) \\ \text{or} \\ x \in f^{-1}(B) \end{cases} \text{(0,5)}.$$

$$\Rightarrow x \in f^{-1}(A) \cup f^{-1}(B) \ \#.$$

Exercise 2:

$$f(x) = \begin{cases} -2x & \text{if } x \leq -\pi/2 \\ a\sin(x) + b & \text{if } -\pi/2 < x < \pi/2 \\ \cos(x) & \text{if } x \geq \pi/2 \end{cases}$$

$$(f \text{ is continous at } x_0) \Leftrightarrow \left(\lim_{x \overset{>}{\to} x_0} f(x) = \lim_{x \overset{<}{\to} x_0} f(x) = f(x_0) \right).$$ (0,5)

we have: $f(-\pi/2) = -2 \cdot -\pi/2 = \pi$, $f(\pi/2) = \cos(\pi/2) = 0$.

($f(x)$ is continous at $\pi/2$) $\Leftrightarrow$ (0,5) $\left(\lim\limits_{x \overset{>}{\to} \pi/2} f(x) = \lim\limits_{x \overset{<}{\to} \pi/2} f(x) = f(\pi/2) \right)$.

then we get: $\lim\limits_{x \overset{<}{\to} \pi/2} a\sin(x) + b = 0$.

$\Rightarrow a\sin(\pi/2) + b = 0 \Rightarrow a \underbrace{\sin(\pi/2)}_{=1} + b = 0 \Rightarrow \boxed{a + b = 0}$ (0,25)

($f(x)$ is continous at $-\pi/2$) $\Leftrightarrow \left(\lim\limits_{x \overset{<}{\to} -\pi/2} f(x) = \lim\limits_{x \overset{>}{\to} -\pi/2} f(x) = f(-\pi/2) \right)$

then we get:

$\lim\limits_{x \overset{>}{\to} -\pi/2} a\sin(x) + b = \pi \Rightarrow a\sin(-\pi/2) + b = \pi$. (0,5)

$\Rightarrow \boxed{-a + b = \pi}$ (0,25)

$\begin{cases} a + b = 0 \quad (1) \\ -a + b = \pi \quad (2) \end{cases} \Rightarrow$ we add (1) and (2) we get. $2b = \pi \Rightarrow b = \pi/2$.

(0,25)

then we get: $\boxed{b = \pi/2} \Rightarrow \boxed{a = -\pi/2}$.

- Is f continous on $\mathbb{R}$? (2pts).

we have $f(x) = 2x$ if $x \in]-\infty, -\pi/2]$ then $f(x)$ is defined and continous at $]-\infty, -\pi/2]$ (0,5)

- $f(x) = a\sin(x) + b$ if $x \in]-\pi/2, \pi/2[$ in this interval $f(x)$ is defined and continous (0,5)

(0,25)

- $f(x) = \cos(x)$ if $x \in [\pi/2, +\infty[$: $f(x)$ is defined and continous

if $a = -\pi/2$ and $b = +\pi/2$: $f(x)$ is continous at $-\pi/2$, $\pi/2$ (0,5).

then we get $f(x)$ is continous on $\mathbb{R}$.

②. Study the extension by continuity of g: $g(x) = \frac{x^2-a^2}{x-a}$.

* $D_g = \{x \in \mathbb{R} / x - a \neq 0\} \Rightarrow D_g = \mathbb{R} / \{a\}$ (0,5)

* $\lim_{x\to a} g(x) = \lim_{x\to a} \frac{x^2-a^2}{x-a} = \lim_{x\to a} \frac{\cancel{(x-a)}(x+a)}{\cancel{(x-a)}} = 2a$ (1).

Then we get: the extension by continuity of g is:

$$\tilde{g}(x) = \begin{cases} \frac{x^2-a^2}{x-a} & \text{if } x \in D_g \\ 2a & \text{if } x = a \end{cases}$$ (0,5).

Exercise 3: $\frac{4}{4}$.

Let $F = \{(x.y.z) \in \mathbb{R}^3 / x+y+z=0\}$.

1- prove that F is subspace of $\mathbb{R}^3$ (2pts).

* $0_{\mathbb{R}^3} = (0,0,0) \in F$ because $0+0+0=0$ ... (0,5)

* $\forall \alpha.\beta \in \mathbb{R}, \forall u.v \in F$: $\alpha u + \beta v \in F$.

$$\alpha u + \beta v = \alpha(x_1, y_1, z_1) + \beta(x_2, y_2, z_2)$$
$$= (\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2, \alpha z_1 + \beta z_2) \overset{?}{\in} F \text{ ... } (0,5)$$

$$(\alpha x_1 + \beta x_2) + (\alpha y_1 + \beta y_2) + (\alpha z_1 + \beta z_2) = \alpha(x_1 + y_1 + z_1) + \beta(x_2 + y_2 + z_2)$$
$$= \alpha \times 0 + \beta \times 0 = 0 \text{ .. } (0,5)$$

we get $\alpha u + \beta v \in F \Rightarrow$ F is subspace of $\mathbb{R}^3$ ... (0,5)

2- prove that $F = vect(u.v)$ (2pts)

$F = \{(x.y.z) \in \mathbb{R}^3 / x+y+z=0\} \Leftrightarrow F = \{(x.y.z) \in \mathbb{R}^3 : x+y=-z$

$\Rightarrow F = \{(x.y.z) \in \mathbb{R}^3 : z = -x-y\}$ ... (0,5).

$F = \{ (x, y, -x-y) \ , (x, y) \in \mathbb{R}^2 \}$ --- 0,5

$F = \{ (x, 0, -x) + (0, y, -y) \ , (x, y) \in \mathbb{R}^2 \}$ -- 0,5

$F = \{ x(1, 0, -1) + y(0, 1, -1) \}$ then we get.

$F = \text{vect}\{ \underbrace{(1, 0, -1)}_{u}, \underbrace{(0, 1, -1)}_{v} \}$ --- 0,5

Exercise 4: 4/4

Find the kernel of f: 2pts

$Ker(f) = \{ (x, y) \in \mathbb{R}^2 : f(x, y) = 0_{\mathbb{R}^3} \}$

$= \{ (x, y) \in \mathbb{R}^2 : (x+y, x-y, x+y) = 0_{\mathbb{R}^3} \}$ --- 0,5

$$\begin{cases} x+y=0 \\ x-y=0 \\ x+y=0 \end{cases} \Rightarrow \begin{cases} x+y=0 \\ x-y=0 \end{cases} \Rightarrow \begin{cases} 2x=0 \Rightarrow \boxed{x=0} \text{ --- } 0,5 \\ \text{then we get } \boxed{y=0} \ 0,5 \end{cases}$$

$Ker(f) = \{ 0_{\mathbb{R}^2} \}$ --- 0,5

Find the range of f: 2pts

$rg(f) = \{ (x, y) \in \mathbb{R}^2 \ / \ f(x, y) = (x+y, x-y, x+y) \}$. 0,5

$range(f) = \{ (x, y) \in \mathbb{R}^2 \ / \ (x, x, x) + (y, -y, y) \}$ 0,5

$range(f) = \{ (x, y) \in \mathbb{R}^2 \ / \ x(1, 1, 1) + y(1, -1, 1) \}$ 0,5

$range(f) = \text{vect}\{ (1, 1, 1), (1, -1, 1) \}$ 0,5