

### Final Exam in OPM2

#### Exercise 1:

Consider a matrix:

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & -1 \\ -3 & 1 & 2 & 0 & 0 \\ 0 & -3 & 1 & 2 & 0 \\ 0 & 0 & -3 & 1 & 2 \\ 1 & 0 & 0 & -3 & 1 \end{bmatrix}$$

1. How to quickly construct matrix A
2. Write the Matlab command to extract the elements of the diagonal of A
3. Write the Matlab command to delete the first row of A
4. Write the Matlab command to delete the first column of A

#### Exercise 2:

Given the decimal number  $(1.3)_{10}$ :

1. Determine its single-precision IEEE-754 floating-point representation with truncation.
2. Calculate the absolute value of the resulting error.

#### Exercise 3:

- Plot the function  $f(x)$  using both vectorized and for-loop methods, where  $f(x)$  is given by:

$$f(x) = \begin{cases} e^x / \sin(x), & \text{if } x \leq 0 \\ \cos(x\sqrt{x}), & \text{if } x > 0 \end{cases}$$



## Sample solution for Final Exam

### Exercise 1:

Consider a matrix:

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & -1 \\ -3 & 1 & 2 & 0 & 0 \\ 0 & -3 & 1 & 2 & 0 \\ 0 & 0 & -3 & 1 & 2 \\ 1 & 0 & 0 & -3 & 1 \end{bmatrix}$$

1. How to quickly construct matrix A
2. Write the Matlab command to extract the elements of the diagonal of A
3. Write the Matlab command to delete the first row of A
4. Write the Matlab command to delete the first column of A

### Solution:

- ② 1- `A = toeplitz([1 -3 0 0 1],[1 2 0 0 -1])`
- ② 2- `diagonal_elements = diag(A)`
- ① 3- `A(1,:) = []` or `A = A(2:end, :)`
- ① 4- `A(:,1) = []` or `A = A(:,2:end)`



### Exercise 2:

Given the decimal number  $(1.3)_{10}$ :

1. Determine its single-precision IEEE-754 floating-point representation with truncation.
2. Calculate the absolute value of the resulting error.

**Solution:**

$$x = 1.3$$

$$= (1.0100110011\dots)_2$$

$$= (-1)^S * (1.F) * 2^E$$

Then,

$$E = 0$$

$$S = 0$$

$$SE = E + 127$$

$$= 127$$

$$= (01111111)_2 \text{ and}$$

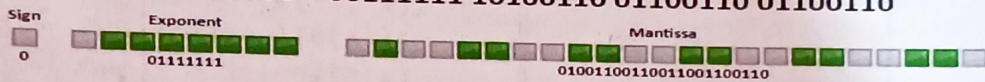
$$F = 0100\ 1100\ 1100\ 1100\ 1100\ 110$$

After truncation, we get the same F

Thus,

$$x =$$

$$0x3FA66666 = 00111111\ 10100110\ 01100110\ 01100110$$



In [12]:

```
format long
```

```
x = 1.3
```

```
x_truncated = bin2dec('10100110011001100110') * 2^-23  
error_ = abs(x - x_truncated)
```

```
x = 1.3000000000000000
```

```
x_truncated = 1.299999952316284
```

```
error_ = 4.768371586472142e-08
```



### Exercise 3:

- Plot the function  $f(x)$  using both vectorized and for-loop methods, where  $f(x)$  is given by:

$$f(x) = \begin{cases} e^x / \sin(x), & \text{if } x \leq 0 \\ \cos(x\sqrt{x}), & \text{if } x > 0 \end{cases}$$

### Solution:

```
clc, clear all, close all
% Vectorized Method
x = linspace(-1, 1);
f = @(x) (x <= 0) .* (exp(x) ./ sin(x)) + (x > 0) ...
.* cos(x .* sqrt(x));
figure
plot(x, f)
title('Piecewise Function: f(x) = {e^x/\sin(x), ...
x <= 0; \cos(x\sqrt{x}), x > 0}')
xlabel('x')
ylabel('f(x)')
```

```
clc, clear all, close all
% for-loop Method
x = linspace(-1, 1);
f = zeros(size(x));
for i = 1:length(x)
    if x(i) <= 0
        f(i) = exp(x(i))./sin(x(i));
    else
        f(i) = cos(x(i).*sqrt(x(i)));
    end
end
figure
plot(x, f)
title('Piecewise Function: f(x) = {e^x/\sin(x), ...
x <= 0; \cos(x\sqrt{x}), x > 0}')
xlabel('x')
ylabel('f(x)')
```