Exercice 01 : (10 pt)

| $x_i$ | $y_i$ | $1^{re}$ D.D | $2^{ème}$ D.D | $3^{ème}$ D.D | $4^{ème}$ D.D |
|---|---|---|---|---|---|
| 1 | 220 | | | | |
| | | 210 | | | 1,5 pt |
| 3 | 640 | | 0 | | |
| | | 210 | | 0 | |
| 5 | 1060 | | 0 | | -0,2604 |
| | | 210 | | $\frac{-12,5}{6}$ | 0,5 pt |
| 7 | 1480 | | -12,5 | | |
| | | 160 | | | |
| 9 | 1800 | | | | |

1) $P_3(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$
$+ f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2)$. (0,5 pt)

$P_3(x) = 220 + 210(x-1)$ (0,5)

2) $y(6) = P_3(6) = 1270$ (0,5 pt) (0,5)

3) $E_3(x) \simeq f[x_0,x_1,x_2,x_3,x_4](x-x_0)(x-x_1)(x-x_2)(x-x_3)$ (0,5)

$\simeq -0,2604(x-1)(x-3)(x-5)(x-7)$ (0,5)

$E_3(6) = -0,2604(5)(3)(1)(-1) = 3,906$ (0,5)

II) (a) $f(x+2h) = f(x) + 2h f'(x) + \frac{(2h)^2}{2!} f''(x)$ (0,5 pt)

$f(x-2h) = f(x) - 2h f'(x) + \frac{(2h)^2}{2!} f''(x)$ (0,5 pt)

donc $f''(x) = \frac{f(x+2h) - 2f(x) + f(x-2h)}{4h^2}$ (0,5 pt)

(b) $y''(t) = \frac{y(t+2h) - 2y(t) + y(t-2h)}{4h^2}$ (0,5)

(0,5) $h = 2$

$a(5) = y''(5) = \frac{y(5+4) - 2y(5) + y(5-4)}{4 \times 2^2} = \frac{1800 - 2(1060) + 220}{16}$

(0,5 pt)

$= -6,25$ (0,5 pt)

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Corrigé

Exercice 01 :

I) 1) On a $\det(A_{[1]}) = 4 \neq 0$

$$\det(A_{[2]}) = \begin{vmatrix} 4 & -2 \\ -2 & 8 \end{vmatrix} = 28 \neq 0$$ (01pt)

$$\det(A_{[3]}) = \begin{vmatrix} 4 & -2 & 0 \\ -2 & 8 & -2 \\ 0 & -2 & 4 \end{vmatrix} = 96 \neq 0$$

2) $$A = LU = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$ (0,5pt)

$$= \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{pmatrix}$$ (0,5pt)

par identification on trouve : (01,5pt)

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{2}{7} & 1 \end{pmatrix}$$ (1pt) $$U = \begin{pmatrix} 4 & -2 & 0 \\ 0 & 7 & -2 \\ 0 & 0 & \frac{24}{7} \end{pmatrix}$$

3) $Ax = b \iff LUX = b \iff \begin{cases} LY = b \\ UX = Y \end{cases}$ (0,5pt)

ⓐ $LY = b \iff \begin{cases} y_1 = 4 \\ y_2 = 10 \\ y_3 = \frac{48}{7} \end{cases}$ (1pt)

ⓑ $UX = Y \iff \begin{cases} x_1 = 2 \\ x_2 = 2 \\ x_3 = 2 \end{cases}$ (1pt)

II) On a $\begin{cases} x_1 = \dfrac{(4+2x_2)}{4} = 1 + \frac{1}{2}x_2 \\ x_2 = \dfrac{(8+2x_1+2x_3)}{8} = 1 + \frac{1}{4}x_1 + \frac{1}{4}x_3 \\ x_3 = (4+2x_2) = 1+\frac{1}{}x \end{cases}$

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l'algorithme de Jacobi :

$$\begin{cases} x_1^{(k+1)} = 1 + \frac{1}{2} x_2^{(k)} \\ x_2^{(k+1)} = 1 + \frac{1}{4} x_1^{(k)} + \frac{1}{4} x_3^{(k)} \\ x_3^{(k+1)} = 1 + \frac{1}{2} x_2^{(k)} \end{cases}$$ 0,5pt

Gauss-Seidel :

$$\begin{cases} x_1^{(k+1)} = 1 + \frac{1}{2} x_2^{(k)} \\ x_2^{(k+1)} = 1 + \frac{1}{4} x_1^{(k+1)} + \frac{1}{4} x_3^{(k)} \\ x_3^{(k+1)} = 1 + \frac{1}{2} x_2^{(k)} \end{cases}$$ 0,5pt

(b) deux itérations de Jacobi :

$\underline{k=0}$ $\begin{cases} x_1^{(1)} = 1 \\ x_2^{(1)} = 1 \\ x_3^{(1)} = 1 \end{cases}$ 0,5pt $\underline{k=1}$ $\begin{cases} x_1^{(2)} = \frac{3}{2} \\ x_2^{(2)} = \frac{3}{2} \\ x_3^{(2)} = \frac{3}{2} \end{cases}$ 0,5pt

deux itérations de G.S

$k=0$ $\begin{cases} x_1^{(1)} = 1 \\ x_2^{(1)} = \frac{5}{4} \\ x_3^{(1)} = \frac{13}{8} \end{cases}$ 0,5pt $\underline{k=1}$ $\begin{cases} x_1^{(2)} = \frac{13}{8} \\ x_2^{(2)} = \frac{58}{32} \\ x_3^{(3)} = \frac{122}{64} \end{cases}$. 0,5pt

(c) calcul de l'erreur : 0,5pt

Jacobi $\|X^{(2)} - X\| = \left\| \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \\ \frac{3}{2} \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left|\frac{1}{2}\right|^2 + \left|\frac{1}{2}\right|^2}$

$= 1,81$ 0,5pt

G.S : $\|X^{(2)} - X\| = \left\| \begin{pmatrix} \frac{13}{8} \\ \frac{58}{32} \\ \frac{122}{64} \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right\| = 0,42$ 0,5pt

0,5pt