Exam

Exercise 1

Given A, B and C three parts of a set E,

1) Show that :

(a) $(A \cap B) \cup \complement_E B = A \cup \complement_E B$.

(b) $(A \setminus B) \setminus C = A \setminus (B \cup C)$.

(c) $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$.

2) Simplify :

(a) $\overline{(A \cup B)} \cap \overline{(C \cup \overline{A})}$.

(b) $\overline{(A \cap B)} \cup \overline{(C \cap \overline{A})}$.

Exercise 2

We define the relation $\mathcal{R}$ on $\mathbb{R}^2$ by :

$$(x, y)\mathcal{R}(x', y') \Leftrightarrow x + y = x' + y'$$

1) Show that $\mathcal{R}$ is an equivalence relation.

2) Find the equivalence class of the couple $(0, 0)$.

Exercise 3

1) Determine if part H is a subgroup of group G.

a) $G = (\mathbb{Z}, +)$; $H = \{\text{even numbers}\}$

b) $G = (\mathbb{Z}, +)$; $H = \{\text{odd numbers}\}$.

2) Show that $U = \{z \in \mathbb{C}, |z| = 1\}$ equipped with multiplication is a subgroup of $(\mathbb{C}^*, \times)$.