Oum El Bouaghi University Department of mathematics and informatics L3 Mathematics 14-01-2024

Exercise 01

Let f(x) be a differentiable function on a convex set $C \subset \mathbb{R}^n$. - Prove that

 $f(x) \text{ convex} \Rightarrow f(y) \ge f(x) + \langle \nabla f(x) | (y-x) \rangle, \ \forall x, y \in C$

- Deduce the following

$$\langle \nabla f(y) - \nabla f(x) | y - x \rangle \ge 0, \ \forall x, y \in C$$

Exercise 02

Consider the function

$$f(x,y) = e^{x-y}(x^2 - 2y^2).$$

- Calculate the gradient
- Give the critical points
- Determine the nature

Exercise 03

Consider the function

$$f(x,y) = x^2 + 2y^2 - 2xy + 2x + 1$$

Find the iterations

$$X_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \quad X_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \quad X_3 = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$$

by Gradient method with fixed step

(Put
$$X_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \rho = 0.1, \quad \varepsilon = 10^{-7})$$