Exercice 3 **(02.5 points).**

On a

$$\begin{aligned} G_T(u) &= \sum_{k=0}^{\infty} u^k P(T=k) \qquad \textbf{(0.25)} \\ &= \sum_{k=0}^{\infty} u^k \frac{k^2+1}{\lambda k!} = \frac{1}{\lambda}\left(\sum_{k=0}^{\infty} \frac{k^2}{k!}u^k + \sum_{k=0}^{\infty} \frac{u^k}{k!}\right) \\ &= \frac{1}{\lambda}\left(u^2 e^u + u e^u + e^u\right) \qquad \textbf{(0.25)} \\ &= \frac{1}{\lambda}\left(u^2+u+1\right)e^u, \qquad \textbf{(0.25)} \end{aligned}$$

alors

$$G_T(1) = \frac{1}{\lambda} 3e = 1 \qquad \textbf{(0.25)} \Longrightarrow \lambda = 3e. \qquad \textbf{(0.25)}$$

c'est-à-dire

$$G_T(u) = \frac{(u^2+u+1)}{3} e^{u-1}.$$

On a

$$E(T) = G'_T(1), \qquad \textbf{(0.25)}$$

où

$$G'_T(u) = \frac{(2u+1)}{3} e^{u-1} + \frac{(u^2+u+1)}{3} e^{u-1} \qquad \textbf{(0.5)} \Longrightarrow G'_T(1) = 2. \qquad \textbf{(0.25)}$$

Donc

$$E(T) = 2. \qquad \textbf{(0.25)}$$